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INTERVAL VALUED NEUTROSOPHIC VAGUE SEMI GENERALISED CLOSED SETS IN INTERVAL VALUED NEUTROSOPHIC VAGUE TOPOLOGICAL SPACES

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Abstract: This paper focuses on the introduction and to develop a new class of sets namely Interval valued Neutrosophic Vague Semi Generalised Closed Sets in an Interval valued Neutrosophic Vague Topological spaces and the characterizations are investigated. Further we have analyzed the properties of an Interval valued Neutrosophic Vague Semi Generalised Open sets. Also some applications of an Interval valued Neutrosophic Vague semi $T_{1/2}$ spaces are introduced and discussed.

***Index Terms* - Interval Valued Neutrosophic Vague topology (IVNVT), Interval Valued Neutrosophic Vague Semi Generalised Closed Sets (IVNVSGCS), Interval Valued Neutrosophic Vague Semi Generalised Open Sets (IVNVSGOS), Interval Valued Neutrosophic Vague Semi $T_{1/2}$ space (IVNV semi $T_{1/2}$ space).**

I. INTRODUCTION

The complexity generally arises from uncertainity in the form of ambiguity in real world. Researchers in economy, sociology, medical science and many other several fields deals daily with the complexities of modeling uncertain data. Therefore, many different theories were developed to solve uncertainty and vagueness including the fuzzy set theory [52], intuitionistic fuzzy set [15], rough sets theory [40], soft set [34], vague sets [18, 24], soft expert set [33] and some other mathematical tools. There are many real applications were solved using these theories related to the uncertainty of these applications [50], [21], [25], [45] . However, these theories cannot deals with indeterminacy and consistent information. Furthermore, all these theories have their inherent difficulties and weakness. Therefore, neutrosophic set is developed by Smarandache in 1998 which is generalization of probability set, fuzzy set and intuitionistic fuzzy set [46]. The neutrosophic set contains three independent membership functions. Unlike fuzzy and intuitionistic fuzzy sets, the memberships in neutrosophic sets are truth, indeterminacy and falsity. The neutrosophic set has received moreand more attention since its appearance. Hybrid neutrosophic set were introduced by many researchers [49], [30],[43], [9], [8]. In line with these developments, these extensions have been used in multi criteria decision making problem such as ANP, VIKOR, TOPSIS and DEMATEL with different application [1]–[4].

By considering the concept of semi-open sets due to Levine[32] instead of open sets, many concepts of topology have been generalized. Norman Levine[31] initiated generalized closed (briefly g-closed) sets in 1970. Bhattacharyya and Lahiri[17] have introduced semi-generalised closed sets with the help of semi-openness. Arya and Nour[14] have introduced generalized semi-open sets in topological spaces. The concept of fuzzy sets was introduced by Zadeh[52] in 1965. The theory of fuzzy topology was introduced by C.L. Chang[19] in 1967. The concepts of neutrosophic semi-open sets and neutrosophic semi-closed sets in neutrosophic topological spaces were introduced by Iswarya et.al.[29] in 2016. Also intuitionistic fuzzy semi-generalised closed sets and its applications were introduced by Santhi et.al [41] in 2010. Vague sets have been introduced by Gau and Buehrer in 1993 as an extension of fuzzy set theory [24]. The concept of Vague semi generalized closed sets in Vague topological spaces were introduced by Vakithabegam and Dr.M.Helen [48] in 2022.

Vague sets is considered as an effective tool to deal with uncertainty since it provides more information as compared to fuzzy sets [53]. Several studies have revealed that, many researchers have combined vague sets with others theories. Xu et al. proposed vague soft sets and examined its properties [51]. Later, Hassan [26] have combined vague set with soft expert set and its operations were introduced. In addition, others hybrid theories such as complex vague soft set [42], interval valued vague soft set [6], generalized interval valued vague soft set [5] and possibility vague soft set [7] were presented to solve uncertainty problem in decision making.

Shawkat Alkhazaleh introduced the concept of Neutrosophic vague set [44] which is a combination of Neutrosophic set and vague set. Neutrosophic vague set theory is an effective tool to process incomplete, indeterminate and inconsistent information. Recently,Al- Quran and Hassan proposed new hybrid of neutrosophic vague such as [10], [11], [12] and [27]. Hazwani Hashim, Lazim Abdullah and Ashraf Al- Quran [28] introduced Interval neutrosophic vague set (INVS) and its combination of vague sets and interval neutrosophic set and as a generalization of interval valued neutrosophic vague set. This set theory provides an interval-based membership structure to handle the neutrosophic vague data. This feature allows users to record their hesitancy in assigning membership values which in turn better capture the vagueness and uncertainties of these data.

We introduce the Interval valued neutrosophic vague semi generalised closed sets, Interval valued neutrosophic vague semi generalised open sets and their properties are obtained. Also its relationship with other existing sets are compared and discussed with an examples.

II. PREAMBLES

2.1.1 Definition [44]

A Neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as $\{< x; \hat{T}_{A_{NV}}(x); \hat{I}_{A_{NV}}(x); \hat{F}_{A_{NV}}(x) >; x \in X\}$, whose truth membership, indeterminacy membership and false membership functions is defined as:

$$\hat{T}_{A_{NV}}(x) = [T^-, T^+], \hat{I}_{A_{NV}}(x) = [I^-, I^+], \hat{F}_{A_{NV}}(x) = [F^-, F^+]$$

Where,

(1) $T^+ = 1 - F^-$

(2) $F^+ = 1 - T^-$ and

(3) $^-0 \le T^- + I^- + F^- \le 2^+$

2.1.2 Definition [44]

Let A_{NV} and B_{NV} be two Neutrosophic vague sets of the Universe U. If $\forall u_i \in U, \hat{T}_{A_{NV}}(u_i) \le \hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i) \ge \hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i) \ge \hat{F}_{B_{NV}}(u_i)$, then the Neutrosophic vague set A_{NV} is included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$, where $1 \le i \le n$.

2.1.3 Definition [44]

The Complement of Neutrosophic vague set A_{NV} is denoted by A_{NV}^c and is defined by

$$\hat{T}_{A_{NV}}^c(x) = [1 - T^+, 1 - T^-], \hat{I}_{A_{NV}}^c(x) = [1 - I^+, 1 - I^-], \hat{F}_{A_{NV}}^c(x) = [1 - F^+, 1 - F^-].$$

2.1.4 Definition [44]

Let Ψ_{NV} be a Neutrosophic vague set of the Universe U, where $\forall u_i \in U, \hat{T}_{A_{NV}}(u_i) = [1,1]; \hat{I}_{A_{NV}}(u_i) = [0,0]; \hat{F}_{A_{NV}}(u_i) = [0,0]$. Then Ψ_{NV} is called unit Neutrosophic Vague set, where $1 \le i \le n$.

2.1.5 Definition [44]

Let Φ_{NV} be a Neutrosophic vague set of the Universe U, where $\forall u_i \in U, \hat{T}_{A_{NV}}(u_i) = [0,0]; \hat{I}_{A_{NV}}(u_i) = [1,1]; \hat{F}_{A_{NV}}(u_i) = [1,1]$. Then Φ_{NV} is called zero Neutrosophic Vague set, where $1 \le i \le n$.

2.1.6 Definition [44]

The union of two Neutrosophic vague sets A_{NV} and B_{NV} is Neutrosophic vague set C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false- membership functions are related to those of A_{NV} and B_{NV} given by,

$$\hat{T}_{C_{NV}}(x) = [\max(T^-_{A_{NVx}}, T^-_{B_{NVx}}), max(T^+_{A_{NVx}}, T^+_{B_{NVx}})],$$
$$\hat{I}_{C_{NV}}(x) = [\min(I^-_{A_{NVx}}, I^-_{B_{NVx}}), \min(I^+_{A_{NVx}}, I^+_{B_{NVx}})],$$
$$\hat{F}_{C_{NV}}(x) = [\min(F^-_{A_{NVx}}, F^-_{B_{NVx}}), min(F^+_{A_{NVx}}, F^+_{B_{NVx}})].$$

2.1.7 Definition [44]

The intersection of two Neutrosophic vague sets A_{NV} and B_{NV} is Neutrosophic vague set C_{NV}, written as $C_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false- membership functions are related to those of A_{NV} and B_{NV} given by,

$$\hat{T}_{C_{NV}}(x) = [\min(T^-_{A_{NVx}}, T^-_{B_{NVx}}), min(T^+_{A_{NVx}}, T^+_{B_{NVx}})],$$
$$\hat{I}_{C_{NV}}(x) = [\max(I^-_{A_{NVx}}, I^-_{B_{NVx}}), \max(I^+_{A_{NVx}}, I^+_{B_{NVx}})],$$
$$\hat{F}_{C_{NV}}(x) = [\max(F^-_{A_{NVx}}, F^-_{B_{NVx}}), max(F^-_{A_{NVx}}, F^-_{B_{NVx}})].$$

2.1.8 Definition [44]

Let A_{NV} and B_{NV} be two Neutrosophic vague sets of the universe U. If $\forall u_i \in U$, $\hat{T}_{A_{NV}}(u_i) = \hat{T}_{B_{NV}}(u_i); \hat{I}_{A_{NV}}(u_i) = \hat{I}_{B_{NV}}(u_i)$;

$\hat{F}_{A_{NV}}(u_i) = \hat{F}_{B_{NV}}(u_i)$, then the Neutrosophic vague sets A_{NV} and B_{NV} are called equal, where $1 \le i \le n$.

2.1.9 Definition

Let (X,τ) be a topological space. A subset A of X is called,

i) a semi closed set (SCS in short) [16,20] if int(cl(A) ⊆ A

ii) a pre - closed set (PCS in short) [37] if cl(int(A)) ⊆ A

iii) a semi-pre closed set (SPCS in short) [13] if int(cl(int(A))) ⊆ A

iv) a α - closed set (α-CS in short) [38] ifcl(int(cl(A))) ⊆ A

v) a regular closed set (RCS in short) [47] if A = cl (int(A))

2.1.10 Definition

Let (X,τ) be a topological space. A subset A of X is called

i)a generalised closed set (briefly g - closed) [31] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.

ii) a generalised semi - closed set (briefly gs - closed)[14] if scl(A) ⊆ U whenever A ⊆ U and U is open in X.

iii) a semi generalised closed set (briefly sg - closed) [17] if scl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.

iv) a generalized semi pre closed (briefly gsp - closed) [22] if $spcl$ ⊆ U whenever A ⊆ U and U is open in X.

v) a generalized pre closed (briefly gp - closed) [23,38] if pcl ⊆ U whenever A ⊆ U and U is open in X.

vi) a α-generalized closed (briefly αg - closed) [35] if αcl(A) ⊆ U whenever A ⊆ U and U is open in X.

vii) a generalized α closed (briefly gα - closed) [36] if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in X.

viii) a regular generalised closed set (briefly rg- closed) [39] if cl(A) ⊆ U whenever A ⊆ U and U is regular open in X.

2.2 INTERVAL VALUED NEUTROSOPHIC VAGUE SETS

2.2.1 Definition [28]

An Interval valued neutrosophic vague set B_{IVNV} (IVNVS in short) on the universe of discourse Y. An IVNVS is characterized by truth membership, indeterminacy membership and falsity membership functions are defined as:

$$A_{IVNV} = \{y, \langle[\tilde{V}_B^L(y), \tilde{V}_B^U(y)], [\tilde{W}_B^L(y), \tilde{W}_B^U(y)], [\tilde{X}_B^L(y), \tilde{X}_B^U(y)]\rangle / y \in Y\},$$

$$\tilde{V}_B^L(y) = [V^{L^-}, V^{L^+}],\ \tilde{V}_B^U(y) = [V^{U^-}, V^{U^+}],$$

$$\tilde{W}_B^L(y) = [W^{L^-}, W^{L^+}],\ \tilde{W}_B^U(y) = [W^{U^-}, W^{U^+}],$$

$$\tilde{X}_B^L(y) = [X^{L^-}, X^{L^+}],\ \tilde{X}_B^U(y) = [X^{U^-}, X^{U^+}].$$

Where,

(1) $V^{L^+} = 1 - X^{L^-}, X^{L^+} = 1 - V^{L^-}, V^{U^+} = 1 - X^{U^-}, X^{U^+} = 1 - V^{U^-}$ *and*

(2) $-0 \le V^{L^-} + V^{U^-} + W^{L^-} + W^{U^-} + X^{L^-} + X^{U^-} \le 4^+$

(3) $-0 \le V^{L^+} + V^{U^+} + W^{L^+} + W^{U^+} + X^{L^+} + X^{U^+} \le 4^+$

When Y is continuous, an IVNVS B_{IVNV} can be represented as follows:

$$B_{IVNV} = \int_Y \langle[\tilde{V}_B^L(y), \tilde{V}_B^U(y)], [\tilde{W}_B^L(y), \tilde{W}_B^U(y)], [\tilde{X}_B^L(y), \tilde{X}_B^U(y)]\rangle / y; \quad y \in Y$$

and when Y is discrete, an IVNVS B_{IVNV} can be represented as follows:

$$B_{IVNV} = \sum_{i=1}^{n} \langle[\tilde{V}_B^L(y), \tilde{V}_B^U(y)], [\tilde{W}_B^L(y), \tilde{W}_B^U(y)], [\tilde{X}_B^L(y), \tilde{X}_B^U(y)]\rangle / y_i; \quad y_i \in Y$$

and $0 \le \sup \tilde{V}_B(y) + \sup \tilde{W}_B(y) + \sup \tilde{X}_B(y) \le 3$

2.2.2 Example [28]

Let $Y = \{y_1, y_2, y_3\}$. Then $B_{IVNV} = \left\{ \frac{y_1}{\langle\{[0.2,0.3],[0.2,0.5]\},\{[0.1,0.6],[0.3,0.6]\},\{[0.7,0.8],[0.5,0.8]\}\rangle}, \frac{y_2}{\langle\{[0.4,0.5],[0.1,0.7]\},\{[0.5,0.5],[0.1,0.3]\},\{[0.5,0.6],[0.3,0.9]\}\rangle}, \frac{y_3}{\langle\{[0.3,0.7],[0.2,0.9]\},\{[0.3,0.7],[0.4,0.7]\},\{[0.3,0.7],[0.1,0.8]\}\rangle} \right\}$ is an IVNVS of Y.

2.2.3 Definition [28]

Let Φ_{IVNV} be an IVNVS of the universe Y where $\forall y_n \in Y$, $\tilde{V}_{\Phi_{IVNV}}^L(y) = [1,1], \tilde{V}_{\Phi_{IVNV}}^U(y) = [1,1]$, $\tilde{W}_{\Phi_{IVNV}}^L(y) = [0,0], \tilde{W}_{\Phi_{IVNV}}^U(y) = [0,0]$, $\tilde{X}_{\Phi_{IVNV}}^L(y) = [0,0], \tilde{X}_{\Phi_{IVNV}}^U(y) = [0,0]$. Then Φ_{IVNV} is called unit IVNVS, where $1 \le n \le m$

2.2.4 Definition [28]

Let δ_{IVNV} be an IVNVS of the universe Y where $\forall y_n \in Y$, $\tilde{V}_{\delta_{IVNV}}^L(y) = [0,0], \tilde{V}_{\delta_{IVNV}}^U(y) = [0,0]$, $\tilde{W}_{\delta_{IVNV}}^L(y) = [1,1], \tilde{W}_{\delta_{IVNV}}^U(y) = [1,1]$, $\tilde{X}_{\delta_{IVNV}}^L(y) = [1,1], \tilde{X}_{\delta_{IVNV}}^U(y) = [1,1]$. Then δ_{IVNV} is called zero IVNVS, where $1 \le n \le m$

2.2.5 Definition [28]

The complement of an IVNVS B_{IVNV} is denoted by B_{IVNV}^c and is defined by

$$(\tilde{V}_B^L)^c(y) = [1 - V^{L^+}, 1 - V^{L^-}],\ (\tilde{V}_B^U)^c(y) = [1 - V^{U^+}, 1 - V^{U^-}],$$

$$(\tilde{W}_B^L)^c(y) = [1 - W^{L^+}, 1 - W^{L^-}],\ (\tilde{W}_B^U)^c(y) = [1 - W^{U^+}, 1 - W^{U^-}],$$

$$(\tilde{X}_B^L)^c(y) = [1 - X^{L^+}, 1 - X^{L^-}],\ (\tilde{X}_B^U)^c(y) = [1 - X^{U^+}, 1 - X^{U^-}].$$

2.2.6 Example [28]

Considering Example 2.2.2, by using Definition 2.2.5, we have the complement of B_{IVNV} is

$$B^c_{IVNV} = \left\{ \frac{y_1}{\langle\{[0.7,0.8],[0.5,0.8]\},\{[0.4,0.9],[0.4,0.7]\},\{[0.2,0.3],[0.2,0.5]\}\rangle}, \frac{y_2}{\langle\{[0.5,0.6],[0.3,0.9]\},\{[0.5,0.5],[0.7,0.9]\},\{[0.4,0.5],[0.1,0.7]\}\rangle}, \frac{y_3}{\langle\{[0.3,0.7],[0.1,0.8]\},\{[0.3,0.7],[0.3,0.6]\},\{[0.3,0.7],[0.2,0.9]\}\rangle} \right\}$$

2.2.7 Definition [28]

Let B_{IVNV} and C_{IVNV} be two IVNVS of the universe Y. If $\forall y_n \in Y, \tilde{V}^L_A(y_n) = \tilde{V}^L_B(y_n), \tilde{V}^U_A(y_n) = \tilde{V}^U_B(y_n)$, $\widetilde{W}^L_A(y_n) = \widetilde{W}^L_B(y_n), \widetilde{W}^U_A(y_n) = \widetilde{W}^U_B(y_n), \tilde{X}^L_A(y_n) = \tilde{X}^L_B(y_n)$ *and* $\tilde{X}^U_A(y_n) = \tilde{X}^U_B(y_n)$. Then the IVNVS B_{IVNV} and C_{IVNV} are equal, where $1 \le n \le m$

2.2.8 Definition [28]

Let B_{IVNV} and C_{IVNV} be two IVNVS of the universe Y. If $\forall y_n \in Y$, $\tilde{V}^L_A(y_n) \le \tilde{V}^L_B(y_n)$ *and* $\tilde{V}^U_A(y_n) \le \tilde{V}^U_B(y_n)$, $\widetilde{W}^L_A(y_n) \ge \widetilde{W}^L_B(y_n)$ *and* $\widetilde{W}^U_A(y_n) \ge \widetilde{W}^U_B(y_n)$, $\tilde{X}^L_A(y_n) \ge \tilde{X}^L_B(y_n)$ *and* $\tilde{X}^U_A(y_n) \ge \tilde{X}^U_B(y_n)$.

Then the IVNVS B_{IVNV} are included by C_{IVNV} denoted by $B_{IVNV} \subseteq C_{IVNV}$, where $1 \le n \le m$.

2.2.9 Definition [28]

The union of two IVNVS B_{IVNV} and C_{IVNV} is denoted by an IVNVS D_{IVNV}, written as $D_{IVNV} = B_{IVNV} \cup C_{IVNV}$ and it is defined as follows:

$\tilde{V}^L_A(y_n) = [max(V^{L^-}_A, V^{L^-}_B), max(V^{L^+}_A, V^{L^+}_B)]$ *and* $\tilde{V}^U_A(y_n) = [max(V^{U^-}_A, V^{U^-}_B), max(V^{U^+}_A, V^{U^+}_B)]$,

$\widetilde{W}^L_A(y_n) = [min(W^{L^-}_A, W^{L^-}_B), min(W^{L^+}_A, W^{L^+}_B)]$ *and* $\widetilde{W}^U_A(y_n) = [min(W^{U^-}_A, W^{U^-}_B), min(W^{U^+}_A, W^{U^+}_B)]$,

$\tilde{X}^L_A(y_n) = [min(X^{L^-}_A, X^{L^-}_B), min(X^{L^+}_A, X^{L^+}_B)]$ *and* $\tilde{X}^U_A(y_n) = [min(X^{U^-}_A, X^{U^-}_B), min(X^{U^+}_A, X^{U^+}_B)]$

2.2.10 Definition [28]

The intersection of two IVNVS B_{IVNV} and C_{IVNV} is denoted by an IVNVS D_{IVNV}, written as $D_{IVNV} = B_{IVNV} \cap C_{IVNV}$ and it is defined as follows:

$\tilde{V}^L_A(y_n) = [min(V^{L^-}_A, V^{L^-}_B), min(V^{L^+}_A, V^{L^+}_B)]$ *and* $\tilde{V}^U_A(y_n) = [min(V^{U^-}_A, V^{U^-}_B), min(V^{U^+}_A, V^{U^+}_B)]$,

$\widetilde{W}^L_A(y_n) = [max(W^{L^-}_A, W^{L^-}_B), max(W^{L^+}_A, W^{L^+}_B)]$ *and* $\widetilde{W}^U_A(y_n) = [max(W^{U^-}_A, W^{U^-}_B), max(W^{U^+}_A, W^{U^+}_B)]$,

$\tilde{X}^L_A(y_n) = [max(X^{L^-}_A, X^{L^-}_B), max(X^{L^+}_A, X^{L^+}_B)]$ *and* $\tilde{X}^U_A(y_n) = [max(X^{U^-}_A, X^{U^-}_B), max(X^{U^+}_A, X^{U^+}_B)]$

III. INTERVAL VALUED NEUTROSOPHIC VAGUE TOPOLOGICAL SPACE

In this section we introduce an Interval valued neutrosophic vague topological space and some of its properties.

3.1 Definition

An Interval valued neutrosophic vague topology (IVNVT in short) on a non-empty set Y is a family τ of an Interval valued neutrosophic vague sets (IVNVS in short) in Y satisfying the following axioms:

i) $0_{IVNV}, 1_{IVNV} \in \tau$

ii) $\cup H_i \in \tau$, for any $\{H_i : i \in J\} \subseteq \tau$

iii) $A \cap B \in \tau$ for any $A, B \in \tau$

In this case the pair (Y, τ) is called an Interval valued neutrosophic vague topological space (IVNVTS for short) and any Interval valued neutrosophic vague set (IVNVS in short) in τ is known as an Interval valued neutrosophic vague open set (IVNVOS in short) in Y.

The complement of an IVNVOS in an IVNVTS (Y, τ) is called an Interval valued neutrosophic vague closed set (IVNVCS for short) in Y.

3.2 Definition

Let (Y, τ) be an IVNVTS and $B = \{\langle y, [\tilde{V}_B, \widetilde{W}_B, \tilde{X}_B]\rangle\}$ be an IVNVS in Y. Then the interval valued neutrosophic vague interior and interval valued neutrosophic vague closure are defined by,

(1) IVNV int $(B) = \cup\{H /$ H is an IVNVOS in Y and $H \subseteq B\}$

(2) $IVNV$ cl $(B) = \cap\{K /$ K is an IVNVCS in Y and $B \subseteq K\}$

3.3 Remark

For any IVNVS B in (Y, τ), we have

IVNVcl $(B^c) = (\text{IVNVint}(B))^c$ and $IVNV$int $(B^c) = (\text{IVNVcl}(B))^c$.

It can be also shown that $IVNV$ cl (B) is IVNVCS and IVNV int(B) is IVNVOS in Y.

(1) B is an IVNVCS in Y if and only if IVNVcl (B) = B.

(2) B is an IVNVOS in Y if and only if IVNVint (B) = B.

3.4 Definition

Let (Y, τ) be an IVNVTS and $B = \{\langle y, [\tilde{V}_B, \widetilde{W}_B, \tilde{X}_B]\rangle\}$ be an IVNVS in Y. Then the interval valued neutrosophic vague semi interior of B

(IVNV sint (B) for short) and interval valued neutrosophic vague semi closure of B (IVNV scl (B) in short) are defined by,

(1) IVNV sint $(B) = \cup\{H/$ H is an IVNVSOS in Y and $H \subseteq B\}$

(2) $IVNV$ scl $(B) = \cap\{K/$ K is an IVNVSCS in Y and $B \subseteq K\}$

3.5 Remark

For any IVNVS B in (Y, τ), we have IVNV scl(B^c) = (IVNV sint $(B))^c$ and $IVNV$ sint(B^c) = (IVNV scl $(B))^c$.

It can be also shown that $IVNV$scl (B) is an IVNVSCS and IVNVsint(B) is an IVNVSOS in Y.

(1) B is an IVNVSCS in Y if and only if IVNV scl (B) = B.

(2) B is an IVNVSOS in Y if and only if IVNV sint (B) = B.

3.6 Result

Let (Y, τ) be an IVNVTS and $B = \{\langle y, [\tilde{V}_B, \widetilde{W}_B, \tilde{X}_B]\rangle\}$ be an IVNVS in Y. Then

(1) IVNV scl (B) = B ∪ IVNV int (IVNV cl (B))

(2) IVNV sint (B) = B ∩ IVNV cl (IVNV int (B))

Proof: The proof is obvious.

3.7 Definition

Let (Y, τ) be an IVNVTS and $B = \{\langle y, [\tilde{V}_B, \widetilde{W}_B, \tilde{X}_B]\rangle\}$ be an IVNVS in Y. Then the interval valued neutrosophic alpha interior of B (IVNV αint (B) in short) and interval valued neutrosophic vague alpha closure of B (IVNV αcl (B) in short) are defined by,

(1) IVNV αint $(B) = \cup\{H/$ H is an IVNV αOS in Y and $H \subseteq B\}$

(2) $IVNV$ αcl $(B) = \cap\{K/$ K is an IVNV αCS in Y and $B \subseteq K\}$

3.8 Result

Let (Y, τ) be an IVNVTS and $B = \{\langle y, [\tilde{V}_B, \widetilde{W}_B, \tilde{X}_B]\rangle\}$ be an IVNVS in Y. Then

(1) IVNV αcl (B) = B ∪ IVNV cl (IVNV int (IVNV cl (B))),

(2) IVNV αint (B) = B ∩ IVNV int (IVNV cl (IVNV int (B))).

Proof: The proof is obvious.

3.9 Definition

An Interval valued neutrosophic vague set $B = \{\langle y, [\tilde{V}_B, \widetilde{W}_B, \tilde{X}_B]\rangle\}$ in IVNVTS (Y, τ) is said to be

i) An Interval valued neutrosophic vague semi closed set (IVNVSCS in short) if IVNVint(IVNVcl(B)) ⊆ B.

ii) An Interval valued neutrosophic vague semi open set (IVNVSOS in short) if B ⊆ IVNVcl(IVNVint(B)).

iii) An Interval valued neutrosophic vague pre-closed set (IVNVPCS in short) if IVNVcl(IVNVint(B)) ⊆ B.

iv) An Interval valued neutrosophic vague pre-open set (IVNVPOS in short) if B ⊆ IVNVint(IVNVcl(B)).

v) An Interval valued neutrosophic vague α-closed set (IVNVαCS in short) if IVNVcl(IVNVint(IVNVcl(B))) ⊆ B.

vi) An Interval valued neutrosophic vague α-open set (IVNVαOS in short) if B ⊆ IVNVint(IVNVcl(IVNVint(B))).

vii) An Interval valued neutrosophic vague regular closed set (IVNVRCS in short) if B = IVNVcl(IVNVint(B)).

viii) An Interval valued neutrosophic vague regular open set (IVNVROS in short) if B = IVNVint(IVNVcl(B)).

3.10 Definition

i) An IVNVS B of (Y, τ) is said to be an Interval valued neutrosophic vague generalized closed set (IVNVGCS in short) if $IVNV$cl(B) ⊆ H whenever B ⊆ H and H is an IVNVOS in Y.

ii) An IVNVS B of (Y, τ) is said to be an Interval valued neutrosophic vague generalized pre-closed set (IVNVGPCS in short) if $IVNV$pcl(B) ⊆ H whenever B ⊆ H and H is an IVNVOS in Y .

iii) An IVNVS B of (Y, τ) is said to be an Interval valued neutrosophic vague alpha generalized closed set (IVNVαGCS in short) if $IVNV$αcl (B) ⊆ H whenever B ⊆ H and H is an IVNVOS in Y.

iv) An IVNVS B of (Y, τ) is said to be an Interval valued neutrosophic vague generalized α closed (IVNVGαCS in short) if $IVNV$αcl (B) ⊆ H whenever B ⊆ H and H is an IVNVα-open in Y.

v) An IVNVS B of (Y, τ) is said to be an Interval valued neutrosophic vague regular generalised closed set (IVNVRGCS in short)

if $IVNV$cl(B) ⊆ H whenever B ⊆ H and H is an IVNVROS in Y.

vi) An IVNVS B of (Y, τ) is said to be an Interval valued neutrosophic vague generalized semi-closed set (IVNVGSCS in short) if $IVNV$scl(B) ⊆ H whenever B ⊆ H and H is an IVNVOS in Y.

vii) An IVNVS B of (Y, τ) is said to be an Interval valued neutrosophic vague semi generalized closed set (IVNVSGCS in short) if $IVNV$scl(B) ⊆ H whenever B ⊆ H and H is an IVNVSOS in Y.

3.11 Proposition

Let B be any Interval valued neutrosophic vague set in (Y, τ). Then

(1) $IVNV\ int(1 - B) = 1 - (IVNVcl(B))$ and

(2) $IVNVcl(1 - B) = 1 - (IVNV\ int(B))$

Proof:

(1). By definition $IVNV$ cl $(B) = \cap$ {K/ K is an IVNVCS in Y and B ⊆ K }

$$1 - (IVNV\text{ cl }(B)) = 1 - \cap\{K/\ K \text{ is an IVNVCS in Y and } B \subseteq K\}$$
$$= \cup\{1 - K/\ K \text{ is an IVNVCS in Y and } B \subseteq K\}$$
$$= \cup\{H/\ H \text{ is an IVNVOS in Y and } H \subseteq 1 - B\}$$
$$= IVNVint(1 - B)$$

(2). The proof is similar to (1).

3.12 Proposition

Let (Y, τ) be an IVNVTS. For any IVNVS B $= \{\langle y, [\tilde{V}_B, \widetilde{W}_B, \tilde{X}_B]\rangle\}$ the following results holds

(i) $0_{IVNV} \subseteq B$, $0_{IVNV} \subseteq 0_{IVNV}$

(ii) $1_{IVNV} \subseteq 1_{IVNV}$, $B \subseteq 1_{IVNV}$

Proof: The proof is obvious.

3.13 Theorem

Let (Y, τ) be an IVNVTS and B, C be an IVNVSs in Y. Then the following properties hold:

(a) IVNVint (B) ⊆ B, (a') B⊆ IVNVcl (B)

(b) $B \subseteq C \Longrightarrow IVNV$int (B) ⊆ $IVNV$int (C), (b') $B \subseteq C \Longrightarrow IVNV$cl (B) ⊆ $IVNV$cl (C),

(c) IVNVint (IVNVint (B)) = IVNVint (B) (c') IVNVcl (IVNVcl(B)) = IVNVcl(B)

(d) IVNVint $(1_{IVNV}) = 1_{IVNV}$ (d') IVNVcl $(0_{IVNV}) = 0_{IVNV}$

(e) IVNVint $(B \cap$ C) = IVNVint (B) ∩ IVNVint (C) (e') IVNVcl $(B \cup$ C)=IVNVcl (B) ∪ IVNVcl (C)

Proof :

The proof of (a), (b) and (d) are obvious, (c) follows from (a).

(e) We know that IVNVint $(B \cap$ C) ⊆ IVNVint (B) and IVNVint $(B \cap$ C) ⊆ IVNVint (C) we obtain

IVNVint $(B \cap$ C) ⊆ IVNVint (B) ∩ IVNVint (C) -------(1)

On the other hand, from the facts that IVNVint (B) ⊆ B and IVNVint (C) ⊆ C

⟹ IVNVint (B) ∩ IVNVint (C) ⊆ B ∩ C and IVNVint (B) ∩ IVNVint (C) ∈ τ, we get

IVNVint (B) ∩ IVNVint (C) ⊆ IVNVint $(B \cap$ C) ------(2)

From (1) and (2) we obtain the result (e).

(a') – (e') can be easily deduced from (a) – (e).

IV. INTERVAL VALUED NEUTROSOPHIC VAGUE SEMI GENERALIZED CLOSED SETS

In this section we introduce an Interval valued neutrosophic vague semi generalized closed sets and some of its properties.

4.1 Definition

An Interval valued neutrosophic vague set B of (Y, τ) is said to be an Interval valued neutrosophic vague semi generalized closed set (IVNVSGCS) if $IVNV$scl $(B) \subseteq$ H whenever B ⊆ H and H is an IVNVSOS in (Y, τ).

4.2 Definition

An Interval valued neutrosophic vague set B of (Y, τ) is said to be an Interval valued neutrosophic vague generalized semi closed set (IVNVGSCS) if IVNVscl (B) ⊆ H whenever B ⊆ H and H is an IVNVOS in (Y, τ).

4.3 Example

Let $Y = \{p, q\}, \tau = \{0, H, 1\}$ is an IVNVT on Y, where $H = \left\{\begin{matrix} < y, \frac{p}{\{[0.4,0.5],[0.4,0.9]\},\{[0.5,0.7],[0.6,0.8]\},\{[0.5,0.6],[0.1,0.6]\}}, \\ \frac{q}{\{[0.4,0.6],[0.2,0.8]\},\{[0.5,0.7],[0.6,0.8]\},\{[0.4,0.6],[0.2,0.8]\}} > \end{matrix}\right\}$.

Then the IVNVS $B = \left\{\begin{matrix} < y, \frac{p}{\{[0.3,0.4],[0.3,0.7]\},\{[0.2,0.3],[0.1,0.5]\},\{[0.6,0.7],[0.3,0.7]\}}, \\ \frac{q}{\{[0.3,0.4],[0.3,0.8]\},\{[0.2,0.3],[0.1,0.5]\},\{[0.6,0.7],[0.2,0.7]\}} > \end{matrix}\right\}$ is an IVNVSGCS in Y .

4.4 Theorem

Every IVNVCS is an IVNVSGCS but not conversely.

Proof: Let B ⊆ H and H is an IVNVSOS in (Y, τ). Since IVNVscl(B) ⊆ IVNVcl(B) and B is an IVNVCS, IVNVscl(B) ⊆ IVNVcl(B)=B ⊆ H ⇒ $IVNV$scl $(B) \subseteq$ H whenever B ⊆ H and H is an IVNVSOS in (Y, τ).

Therefore B is an IVNVSGCS in Y.

4.5 Example

Let $Y = \{p, q\}, \tau = \{0, H, 1\}$ is an IVNVT on Y, where $H = \left\{\begin{matrix} < y, \frac{p}{\{[0.4,0.5],[0.1,0.7]\},\{[0.7,0.8],[0.7,0.9]\},\{[0.5,0.6],[0.3,0.9]\}}, \\ \frac{q}{\{[0.4,0.5],[0.1,0.8]\},\{[0.7,0.8],[0.7,0.9]\},\{[0.5,0.6],[0.2,0.9]\}} > \end{matrix}\right\}$. Then the Interval valued neutrosophic vague set $B = \left\{\begin{matrix} < y, \frac{p}{\{[0.3,0.4],[0.1,0.6]\},\{[0.1,0.3],[0.1,0.3]\},\{[0.6,0.7],[0.4,0.9]\}}, \\ \frac{q}{\{[0.3,0.4],[0.1,0.7]\},\{[0.1,0.3],[0.1,0.4]\},\{[0.6,0.7],[0.3,0.9]\}} > \end{matrix}\right\}$ is an IVNVSGCS in Y but not an IVNVCS in Y.

4.6 Theorem

Every IVNVSCS is an IVNVSGCS but not conversely.

Proof: Let $B \subseteq H$ and H is an IVNVSOS in (Y, τ). By hypothesis, IVNVscl(B) = B.

Hence $IVNV\text{scl}(B) \subseteq H$ whenever $B \subseteq H$ and H is an IVNVSOS in (Y, τ). Therefore B is an IVNVSGCS in Y.

4.7 Example

Let $Y = \{p, q\}, \tau = \{0, H, 1\}$ is an IVNVT on Y, where $H = \left\{ \begin{array}{c} < y, \frac{p}{\{[0.4,0.5],[0.1,0.5]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.6],[0.5,0.9]\}}, \\ \frac{q}{\{[0.4,0.5],[0.1,0.6]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.6],[0.4,0.9]\}} > \end{array} \right\}$. Then the Interval valued neutrosophic vague set $B = \left\{ \begin{array}{c} < y, \frac{p}{\{[0.3,0.5],[0.1,0.8]\},\{[0.2,0.3],[0.1,0.4]\},\{[0.5,0.7],[0.2,0.9]\}}, \\ \frac{q}{\{[0.3,0.6],[0.1,0.7]\},\{[0.2,0.3],[0.1,0.4]\},\{[0.4,0.7],[0.3,0.9]\}} > \end{array} \right\}$ is an IVNVSGCS in Y but not an IVNVSCS in Y.

4.8 Theorem

Every IVNVαCS is an IVNVSGCS but not conversely.

Proof: The proof is obvious.

4.9 Example

Let $Y = \{p, q\}, \tau = \{0, H, 1\}$ is an IVNVT on Y, where $H = \left\{ \begin{array}{c} < y, \frac{p}{\{[0.4,0.5],[0.1,0.9]\},\{[0.2,0.3],[0.2,0.4]\},\{[0.5,0.6],[0.1,0.9]\}}, \\ \frac{q}{\{[0.4,0.5],[0.2,0.5]\},\{[0.1,0.3],[0.1,0.4]\},\{[0.5,0.6],[0.5,0.8]\}} > \end{array} \right\}$.

Then the Interval valued neutrosophic vague set $B = \left\{ \begin{array}{c} < y, \frac{p}{\{[0.3,0.6],[0.2,0.7]\},\{[0.6,0.7],[0.6,0.9]\},\{[0.4,0.7],[0.3,0.8]\}}, \\ \frac{q}{\{[0.3,0.4],[0.1,0.8]\},\{[0.6,0.8],[0.6,0.9]\},\{[0.6,0.7],[0.2,0.9]\}} > \end{array} \right\}$

is an IVNVSGCS in Y but not an IVNVαCS in Y.

4.10 Theorem

Every IVNVSGCS is an IVNVSPCS but not conversely.

Proof: The proof is obvious.

4.11 Example

Let $Y = \{p, q\}, \tau = \{0, H, 1\}$ is an IVNVT on Y, where $H = \left\{ \begin{array}{c} < y, \frac{p}{\{[0.4,0.5],[0.2,0.6]\},\{[0.6,0.8],[0.7,0.8]\},\{[0.5,0.6],[0.4,0.8]\}}, \\ \frac{q}{\{[0.4,0.5],[0.2,0.7]\},\{[0.6,0.8],[0.7,0.8]\},\{[0.5,0.6],[0.3,0.8]\}} > \end{array} \right\}$. Then the Interval valued neutrosophic vague set $B = \left\{ \begin{array}{c} < y, \frac{p}{\{[0.5,0.6],[0.4,0.8]\},\{[0.2,0.4],[0.3,0.4]\},\{[0.4,0.5],[0.2,0.6]\}}, \\ \frac{q}{\{[0.5,0.6],[0.3,0.8]\},\{[0.2,0.4],[0.3,0.4]\},\{[0.4,0.5],[0.2,0.7]\}} > \end{array} \right\}$ is an IVNVSPCS in Y but not an IVNVSGCS in Y.

4.12 Theorem

Every IVNVRCS is an IVNVSGCS but not conversely.

Proof: The proof is obvious.

4.13 Example

Let $Y = \{p, q\}, \tau = \{0, H, 1\}$ is an IVNVT on Y, where $H = \left\{ \begin{array}{c} < y, \frac{p}{\{[0.4,0.5],[0.1,0.5]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.6],[0.5,0.9]\}}, \\ \frac{q}{\{[0.4,0.5],[0.1,0.6]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.6],[0.4,0.9]\}} > \end{array} \right\}$.

Then the Interval valued neutrosophic vague set $B = \left\{ \begin{array}{c} < y, \frac{p}{\{[0.3,0.5],[0.1,0.8]\},\{[0.2,0.3],[0.1,0.4]\},\{[0.5,0.7],[0.2,0.9]\}}, \\ \frac{q}{\{[0.3,0.6],[0.1,0.7]\},\{[0.2,0.3],[0.1,0.4]\},\{[0.4,0.7],[0.3,0.9]\}} > \end{array} \right\}$ is an IVNVSGCS in Y but not an IVNVRCS in Y.

4.14 Remark

The concepts of IVNVGCS and IVNVSGCS are independent of each other as seen from the following two examples.

4.15 Example

Let $Y = \{p, q\}, \tau = \{0, H, 1\}$ is an IVNVT on Y, where $H = \left\{ \begin{array}{c} < y, \frac{p}{\{[0.4,0.5],[0.1,0.5]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.6],[0.5,0.9]\}}, \\ \frac{q}{\{[0.4,0.5],[0.1,0.6]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.6],[0.4,0.9]\}} > \end{array} \right\}$.

Then the Interval valued neutrosophic vague set $B = \left\{ \begin{array}{c} < y, \frac{p}{\{[0.3,0.5],[0.1,0.8]\},\{[0.2,0.3],[0.1,0.4]\},\{[0.5,0.7],[0.2,0.9]\}}, \\ \frac{q}{\{[0.3,0.6],[0.1,0.7]\},\{[0.2,0.3],[0.1,0.4]\},\{[0.4,0.7],[0.3,0.9]\}} > \end{array} \right\}$ is an IVNVSGCS in Y but not an IVNVGCS in Y.

4.16 Example

Let Y = {p, q}, τ = {0, H, 1} is an IVNVT on Y, where $H = \left\{ < y, \frac{p}{\{[0.4,0.5],[0.2,0.6]\},\{[0.6,0.8],[0.7,0.8]\},\{[0.5,0.6],[0.4,0.8]\}}, \frac{q}{\{[0.4,0.5],[0.2,0.7]\},\{[0.6,0.8],[0.7,0.8]\},\{[0.5,0.6],[0.3,0.8]\}} > \right\}$.

Then the Interval valued neutrosophic vague set $B = \left\{ < y, \frac{p}{\{[0.5,0.6],[0.4,0.8]\},\{[0.2,0.4],[0.3,0.4]\},\{[0.4,0.5],[0.2,0.6]\}}, \frac{q}{\{[0.5,0.6],[0.3,0.8]\},\{[0.2,0.4],[0.3,0.4]\},\{[0.4,0.5],[0.2,0.7]\}} > \right\}$ is an IVNVGCS in Y but not an IVNVSGCS in Y.

4.17 Remark

The concepts of IVNVPCS and IVNVSGCS are independent of each other as seen from the following two examples.

4.18 Example

Let Y = {p, q}, τ = {0, H, 1} is an IVNVT on Y, where $H = \left\{ < y, \frac{p}{\{[0.3,0.4],[0.1,0.5]\},\{[0.5,0.8],[0.6,0.8]\},\{[0.6,0.7],[0.5,0.9]\}}, \frac{q}{\{[0.3,0.4],[0.1,0.9]\},\{[0.5,0.8],[0.6,0.9]\},\{[0.6,0.7],[0.1,0.9]\}} > \right\}$. Then the Interval valued neutrosophic vague set $B = \left\{ < y, \frac{p}{\{[0.4,0.5],[0.2,0.8]\},\{[0.5,0.6],[0.5,0.7]\},\{[0.5,0.6],[0.2,0.8]\}}, \frac{q}{\{[0.4,0.5],[0.2,0.9]\},\{[0.5,0.6],[0.5,0.8]\},\{[0.5,0.6],[0.1,0.8]\}} > \right\}$ is an IVNVSGCS in Y but not an IVNVPCS in Y.

4.19 Example

Let Y = {p, q}, τ = {0, H, 1} is an IVNVT on Y, where $H = \left\{ < y, \frac{p}{\{[0.4,0.5],[0.3,0.5]\},\{[0.6,0.7],[0.6,0.9]\},\{[0.5,0.6],[0.5,0.7]\}}, \frac{q}{\{[0.4,0.5],[0.3,0.6]\},\{[0.6,0.8],[0.6,0.9]\},\{[0.5,0.6],[0.4,0.7]\}} > \right\}$. Then the Interval valued neutrosophic vague set $B = \left\{ < y, \frac{p}{\{[0.3,0.5],[0.1,0.5]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.7],[0.5,0.9]\}}, \frac{q}{\{[0.3,0.5],[0.1,0.6]\},\{[0.6,0.7],[0.6,0.9]\},\{[0.5,0.8],[0.4,0.9]\}} > \right\}$ is an IVNVPCS in Y but not an IVNVSGCS in Y.

4.20 Remark

The concepts of IVNVRGCS and IVNVSGCS are independent of each other as seen from the following two examples.

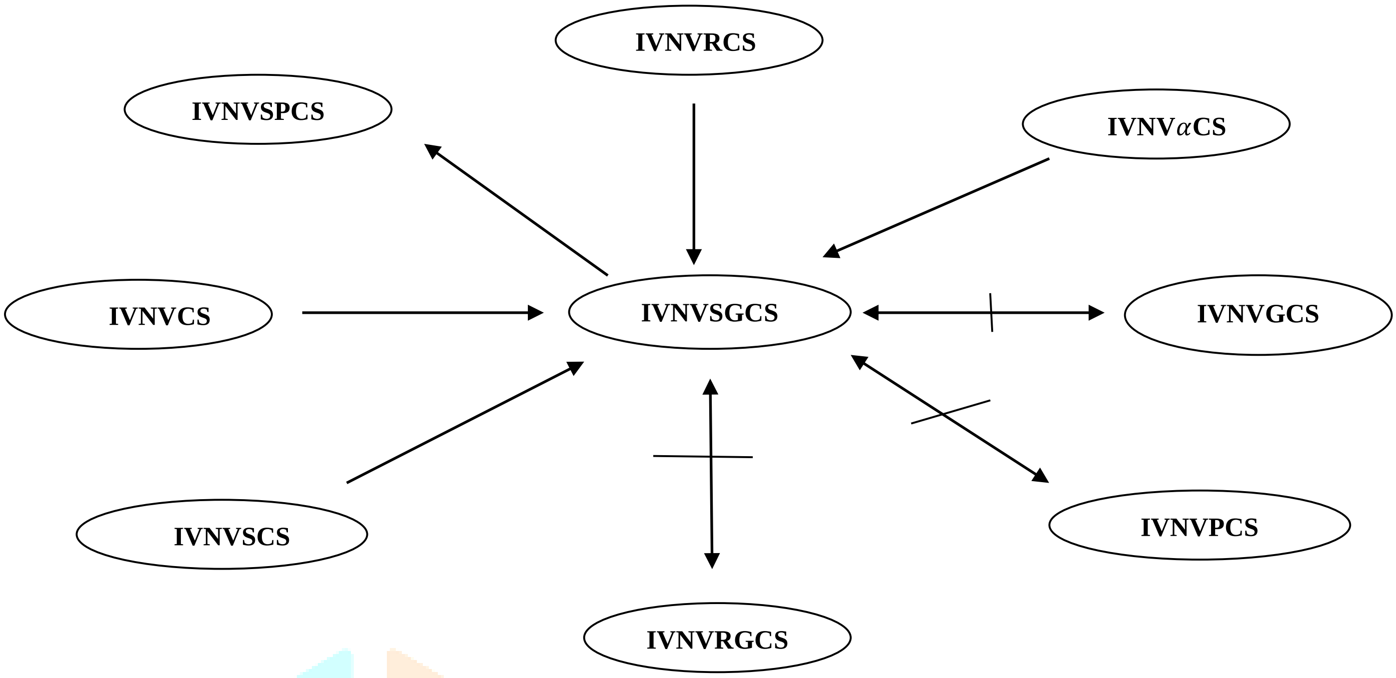
4.21 Example

Let Y = {p, q}, τ = {0, H, 1} is an IVNVT on Y, where $H = \left\{ < y, \frac{p}{\{[0.4,0.5],[0.3,0.5]\},\{[0.6,0.7],[0.6,0.9]\},\{[0.5,0.6],[0.5,0.7]\}}, \frac{q}{\{[0.4,0.5],[0.3,0.6]\},\{[0.6,0.8],[0.6,0.9]\},\{[0.5,0.6],[0.4,0.7]\}} > \right\}$. Then the Interval valued neutrosophic vague set $B = \left\{ < y, \frac{p}{\{[0.3,0.5],[0.1,0.5]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.7],[0.5,0.9]\}}, \frac{q}{\{[0.3,0.5],[0.1,0.6]\},\{[0.6,0.7],[0.6,0.9]\},\{[0.5,0.8],[0.4,0.9]\}} > \right\}$ is an IVNVRGCS in Y but not an IVNVSGCS in Y.

4.22 Example

Let Y = {p, q}, τ = {0, H, 1} is an IVNVT on Y, where $H = \left\{ < y, \frac{p}{\{[0.4,0.5],[0.3,0.7]\},\{[0.5,0.6],[0.5,0.7]\},\{[0.5,0.6],[0.3,0.7]\}}, \frac{q}{\{[0.4,0.5],[0.3,0.8]\},\{[0.5,0.7],[0.5,0.8]\},\{[0.5,0.6],[0.2,0.7]\}} > \right\}$. Then the Interval valued neutrosophic vague set $B = \left\{ < y, \frac{p}{\{[0.3,0.5],[0.1,0.7]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.7],[0.3,0.9]\}}, \frac{q}{\{[0.3,0.5],[0.1,0.9]\},\{[0.6,0.8],[0.6,0.9]\},\{[0.5,0.7],[0.1,0.9]\}} > \right\}$ is an IVNVSGCS in Y but not an IVNVRGCS in Y.

Summing up the above theorems, we have the following implication:



In this diagram " B $\longrightarrow$ C" means that B implies C and "B $\nleftrightarrow$ C" means that B and C are independent of each other.

4.23 Remark

The union of any two IVNVSGCS is not IVNVSGCS in (Y, τ).

4.24 Theorem

Intersection of any two IVNVSGCS is again an IVNVSGCS in (Y, τ).

Proof: The proof is obvious.

4.25 Example

Let $\mathrm{Y} = \{p, q\}, \tau = \{0, \mathrm{H}, 1\}$ is an IVNVT on Y, where $\mathrm{H} = \left\{ < y, \frac{p}{\{[0.3,0.4],[0.3,0.5]\},\{[0.2,0.5],[0.3,0.6]\},\{[0.6,0.7],[0.5,0.7]\}}, \frac{q}{\{[0.3,0.4],[0.3,0.6]\},\{[0.2,0.6],[0.2,0.7]\},\{[0.6,0.7],[0.4,0.7]\}} > \right\}$ .Consider the IVNVSs

$\mathrm{A} = \left\{ < y, \frac{p}{\{[0.3,0.4],[0.2,0.7]\},\{[0.4,0.6],[0.4,0.7]\},\{[0.6,0.7],[0.3,0.8]\}}, \frac{q}{\{[0.3,0.4],[0.2,0.8]\},\{[0.4,0.6],[0.4,0.8]\},\{[0.6,0.7],[0.2,0.8]\}} > \right\}$ and $\mathrm{B} = \left\{ < y, \frac{p}{\{[0.4,0.6],[0.2,0.6]\},\{[0.3,0.5],[0.3,0.7]\},\{[0.4,0.6],[0.4,0.8]\}}, \frac{q}{\{[0.4,0.6],[0.2,0.7]\},\{[0.3,0.6],[0.3,0.8]\},\{[0.4,0.6],[0.3,0.8]\}} > \right\}$. Then

$\mathrm{A} \cup \mathrm{B} = \left\{ < y, \frac{p}{\{[0.4,0.6],[0.2,0.7]\},\{[0.3,0.5],[0.3,0.7]\},\{[0.4,0.6],[0.3,0.8]\}}, \frac{q}{\{[0.4,0.6],[0.2,0.8]\},\{[0.3,0.6],[0.3,0.8]\},\{[0.4,0.6],[0.2,0.8]\}} > \right\}$ is not an IVNVSGCS in Y but $\mathrm{A} \cap \mathrm{B}$ is an IVNVSGCS in Y.

4.26 Theorem

Let B and C be IVNVSGCS in (Y, τ) such that IVNV $\mathrm{cl}(B) = IVNV\ scl(B)$ and IVNV $\mathrm{cl}(C) = IVNV\ scl(C)$. Then $\mathrm{B} \cup \mathrm{C}$ is an Interval valued Neutrosophic vague semi-generalised closed set in (Y, τ).

Proof: The proof is obvious.

4.27 Theorem

If B is an IVNVSGCS in a space (Y, τ) and $\mathrm{B} \subseteq \mathrm{C} \subseteq$ IVNV scl(B) then C is also an IVNVSGCS in Y.

Proof:

Let B be an IVNVSGCS. So $\mathrm{B} \subseteq$ H and H is an IVNVSOS in (Y, τ). Since $\mathrm{C} \subseteq$ IVNV scl(B), then $IVNV$ scl(C) $\subseteq$ IVNVscl(B) $\subseteq$ H. Therefore C is also an IVNVSGCS.

4.28 Theorem

Suppose that $\mathrm{C} \subseteq \mathrm{B} \subseteq \mathrm{Y}$, C is an Interval valued Neutrosophic vague semi-generalised closed set relative to B and that B is an Interval valued Neutrosophic vague semi-generalised closed set in (Y, τ). Then C is an Interval valued Neutrosophic vague semi-generalised closed set in (Y, τ).

Proof: The proof is obvious.

4.29 Theorem

Let B be an IVNVSGCS in (Y, τ). Then IVNV scl(B) – B does not contain any non- empty IVNVSCS.

Proof: The proof is obvious.

4.30 Theorem

Let B be an IVNVSGCS in (Y, τ). Then IVNV scl(B) – B contains no non-empty IVNVCS.

Proof: The proof is obvious.

4.31 Corollary

An IVNVSGCS 'B' is IVNVSCS if and only if IVNV scl(B) – B is an IVNVSCS.

Proof: The proof is obvious.

V. INTERVAL VALUED NEUTROSOPHIC VAGUE SEMI GENERALISED OPEN SETS

In this section we introduce an Interval valued neutrosophic vague semi generalized open sets and some of its properties.

5.1 Definition

An IVNVS 'B' is said to be an Interval valued neutrosophic vague semi generalized open set (IVNVSGOS in short) in (Y, τ) if the complement B^c is an IVNVSGCS in (Y, τ).

The family of all IVNVSGOS of a IVNVTS (Y, τ) is denoted by IVNVSGOS(Y).

5.2 Theorem

For any IVNVTS (Y, τ), we have the following

(i) Every IVNVOS is an IVNVSGOS

(ii) Every IVNVSOS is an IVNVSGOS

(iii) Every IVNVαOS is an IVNVSGOS

(iv) Every IVNVSGOS is an IVNVSPOS

(v) Every IVNVROS is an IVNVSGOS, But the converses need not be true.

Proof:

(i) Let B be an IVNVOS in Y. Then B^c is an IVNVCS in Y. Therefore B^c is an IVNVSGCS in Y by theorem 4.4. Hence B is an IVNVSGOS in Y.

The proof of (ii) , (iii) , (iv) and (v) are obvious.

5.3 Example

Let $\mathrm{Y} = \{p, q\}, \tau = \{0, \mathrm{H}, 1\}$ is an IVNVT on Y, where $\mathrm{H} = \left\{ \begin{array}{l} < \mathrm{y}, \frac{p}{\{[0.4,0.5],[0.4,0.5]\},\{[0.7,0.8],[0.7,0.9]\},\{[0.5,0.6],[0.5,0.6]\}}, \\ \frac{q}{\{[0.4,0.5],[0.4,0.6]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.6],[0.4,0.6]\}} > \end{array} \right\}$. Then the Interval valued neutrosophic vague set $\mathrm{B} = \left\{ \begin{array}{l} < \mathrm{y}, \frac{p}{\{[0.3,0.5],[0.2,0.5]\},\{[0.4,0.5],[0.4,0.6]\},\{[0.5,0.7],[0.5,0.8]\}}, \\ \frac{q}{\{[0.3,0.5],[0.2,0.6]\},\{[0.5,0.6],[0.5,0.7]\},\{[0.5,0.7],[0.4,0.8]\}} > \end{array} \right\}$ is an IVNVSGOS in Y but not an IVNVOS in Y.

5.4 Example

Let $\mathrm{Y} = \{p, q\}, \tau = \{0, \mathrm{H}, 1\}$ is an IVNVT on Y, where $\mathrm{H} = \left\{ \begin{array}{l} < \mathrm{y}, \frac{p}{\{[0.4,0.5],[0.4,0.5]\},\{[0.7,0.8],[0.7,0.9]\},\{[0.5,0.6],[0.5,0.6]\}}, \\ \frac{q}{\{[0.4,0.5],[0.4,0.6]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.6],[0.4,0.6]\}} > \end{array} \right\}$. Then the Interval valued neutrosophic vague set $\mathrm{B} = \left\{ \begin{array}{l} < \mathrm{y}, \frac{p}{\{[0.3,0.5],[0.2,0.5]\},\{[0.4,0.5],[0.4,0.6]\},\{[0.5,0.7],[0.5,0.8]\}}, \\ \frac{q}{\{[0.3,0.5],[0.2,0.6]\},\{[0.5,0.6],[0.5,0.7]\},\{[0.5,0.7],[0.4,0.8]\}} > \end{array} \right\}$ is an IVNVSGOS in Y but not an IVNVSOS in Y.

5.5 Example

Let $\mathrm{Y} = \{p, q\}, \tau = \{0, \mathrm{H}, 1\}$ is an IVNVT on Y, where $\mathrm{H} = \left\{ \begin{array}{l} < \mathrm{y}, \frac{p}{\{[0.4,0.5],[0.4,0.7]\},\{[0.5,0.7],[0.5,0.8]\},\{[0.5,0.6],[0.3,0.6]\}}, \\ \frac{q}{\{[0.4,0.5],[0.4,0.8]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.6],[0.2,0.6]\}} > \end{array} \right\}$. Then the Interval valued neutrosophic vague set $\mathrm{B} = \left\{ \begin{array}{l} < \mathrm{y}, \frac{p}{\{[0.3,0.5],[0.2,0.7]\},\{[0.3,0.6],[0.3,0.7]\},\{[0.5,0.7],[0.3,0.8]\}}, \\ \frac{q}{\{[0.3,0.5],[0.2,0.8]\},\{[0.3,0.5],[0.3,0.8]\},\{[0.5,0.7],[0.2,0.8]\}} > \end{array} \right\}$ is an IVNVSGOS in Y but not an IVNVαOS in Y.

5.6 Example

Let $\mathrm{Y} = \{p, q\}, \tau = \{0, \mathrm{H}, 1\}$ is an IVNVT on Y, where $\mathrm{H} = \left\{ \begin{array}{l} < \mathrm{y}, \frac{p}{\{[0.4,0.6],[0.1,0.6]\},\{[0.5,0.6],[0.5,0.7]\},\{[0.4,0.6],[0.4,0.9]\}}, \\ \frac{q}{\{[0.4,0.6],[0.1,0.7]\},\{[0.5,0.7],[0.5,0.8]\},\{[0.4,0.6],[0.3,0.9]\}} > \end{array} \right\}$. Then the Interval valued neutrosophic vague set B = H = $\left\{\begin{matrix} < y, \frac{p}{\{[0.3,0.5],[0.2,0.7]\},\{[0.3,0.6],[0.3,0.7]\},\{[0.5,0.7],[0.3,0.8]\}}, \\ \frac{q}{\{[0.3,0.5],[0.2,0.8]\},\{[0.3,0.5],[0.3,0.8]\},\{[0.5,0.7],[0.2,0.8]\}} > \end{matrix}\right\}$ is an IVNVSPOS in Y but not an IVNVSGOS in Y.

5.7 Example

Let Y = {p, q}, τ = {0, H, 1} is an IVNVT on Y, where H = $\left\{\begin{matrix} < y, \frac{p}{\{[0.4,0.5],[0.4,0.7]\},\{[0.5,0.7],[0.5,0.8]\},\{[0.5,0.6],[0.3,0.6]\}}, \\ \frac{q}{\{[0.4,0.5],[0.4,0.8]\},\{[0.6,0.7],[0.6,0.8]\},\{[0.5,0.6],[0.2,0.6]\}} > \end{matrix}\right\}$. Then the Interval valued neutrosophic vague set B = $\left\{\begin{matrix} < y, \frac{p}{\{[0.3,0.5],[0.2,0.7]\},\{[0.3,0.6],[0.3,0.7]\},\{[0.5,0.7],[0.3,0.8]\}}, \\ \frac{q}{\{[0.3,0.5],[0.2,0.8]\},\{[0.3,0.5],[0.3,0.8]\},\{[0.5,0.7],[0.2,0.8]\}} > \end{matrix}\right\}$ is an IVNVSGOS in Y but not an IVNVROS in Y.

5.8 Theorem

If *B* is an IVNVSGOS in a space (Y, τ) and IVNVsint(B) ⊆ C ⊆ B then C is also an IVNVSGOS in Y.

Proof: The proof is obvious.

5.9 Theorem

If *B* and C are an IVNVSGOS, then B∪ C is also an IVNVSGOS in (Y, τ).

Proof: The proof is obvious.

5.10 Remark

Intersection of any two IVNVSGOS in (Y, τ) need not be an IVNVSGOS.

Proof: The proof is obvious.

5.11 Example

Consider example 4.25, the union A ∪ B is an IVNVSGOS but the intersection A ∩ B is not an IVNVSGOS.

5.12 Theorem

Let B and C be an IVNVSGOS in (Y, τ) such that IVNV int$(B) = IVNV\ sint(B)$ and IVNV int$(C) = IVNV\ sint(C)$. Then B ∩ C is an Interval valued Neutrosophic vague semi-generalised open set in (Y, τ).

Proof: The proof is obvious.

5.13 Theorem

An IVNVS 'B' is an IVNVSGOS of an Interval valued neutrosophic vague topological space (Y, τ) if and only if C ⊆ IVNV sint(B) whenever C is an IVNVSCS and C ⊆ B .

Proof: The proof is obvious.

5.14 Theorem

If *B* is an IVNVSGOS in a space (Y, τ) and IVNV sint(B) ⊆ C ⊆ B, then C is also an IVNVSGOS in Y.

Proof: The proof is obvious.

5.15 Theorem

An IVNVS 'B' is an IVNVSGOS of an Interval valued neutrosophic vague topological space (Y, τ) if and only if C ⊆ IVNV sint(B) whenever C is an IVNVSCS and C ⊆ B .

Proof: The proof is obvious.

5.16 Theorem

An IVNVS 'B' is an IVNVSGCS if and only if IVNV scl(B) – B is an IVNVSGOS.

Proof: The proof is obvious.

VI. APPLICATION OF AN INTERVAL VALUED NEUTROSOPHIC VAGUE SEMI GENERALISED CLOSED SETS

In this section, we introduce an Interval valued neutrosophic vague semi-$T_{1/2}$ space, which utilizes Interval valued neutrosophic vague semi generalized closed sets and its characterizations are proved.

6.1 Definition

An IVNVTS (Y, τ) is called an Interval valued neutrosophic vague semi-$T_{1/2}$ space, if every IVNVSGCS is IVNVSCS.

6.2 Theorem

An IVNVTS (Y, τ) is called an Interval valued neutrosophic vague semi-$T_{1/2}$ space if and only if IVNVSOS(Y) = IVNVSGOS(Y)

Proof: The proof is obvious.

6.3 Theorem

For an IVNVTS (Y, τ) the following conditions are equivalent:

i) (Y, τ) is an Interval valued neutrosophic vague semi- $T_{1/2}$ space

ii) Every singleton set of Y is either an IVNVSCS or IVNVSOS.

Proof: The proof is obvious.

6.4 Theorem

Every Interval valued neutrosophic vague - $T_{1/2}$ space is an Interval valued neutrosophic vague semi-$T_{1/2}$ space.

Proof: The proof is obvious.

But the converse need not be true.

6.5 Example

Let $Y = \{p, q\}, \tau = \{0, H, 1\}$ is an IVNVT on Y, where $H = \left\{\begin{matrix} < y, \frac{p}{\{[0.4,0.6],[0.1,0.8]\},\{[0.4,0.5],[0.3,0.5]\},\{[0.4,0.6],[0.2,0.9]\}}, \\ \frac{q}{\{[0.4,0.6],[0.1,0.9]\},\{[0.4,0.5],[0.3,0.6]\},\{[0.4,0.6],[0.1,0.9]\}} > \end{matrix}\right\}$ and an IVNVS $B = \left\{\begin{matrix} < y, \frac{p}{\{[0.3,0.4],[0.2,0.5]\},\{[0.4,0.5],[0.3,0.5]\},\{[0.6,0.7],[0.5,0.8]\}}, \\ \frac{q}{\{[0.4,0.6],[0.1,0.9]\},\{[0.5,0.6],[0.4,0.7]\},\{[0.4,0.6],[0.1,0.9]\}} > \end{matrix}\right\}$. Then (Y, τ) is an Interval valued neutrosophic vague semi - $T_{1/2}$ space but not an Interval valued neutrosophic vague - $T_{1/2}$ space, Since B is an IVNVSGCS but not IVNVCS in Y.

6.6 Theorem

Every Interval valued neutrosophic vague regular- $T_{1/2}$ space is an Interval valued neutrosophic vague semi-$T_{1/2}$ space.

Proof: The proof is obvious.

But the converse may not be true which is shown by the following example.

6.7 Example

Consider an example 6.5, Already we have proved that (Y, τ) is an Interval valued neutrosophic vague semi - $T_{1/2}$ space but not an Interval valued neutrosophic vague regular - $T_{1/2}$ space, since B is an IVNVSGCS but not an IVNVRCS in Y.

6.8 Theorem

Let (Y, τ) be an IVNVTS and Y is an Interval valued neutrosophic vague semi-$T_{1/2}$ space, then the following conditions are equivalent:

i) $B \in IVNVSGO(Y)$ ii) $B \subseteq IVNV\ cl(IVNV\ int(B))$ iii) $B \in IVNVRC(Y)$

Proof: The proof is obvious.

VII. CONCLUSION

In this article, we have developed the Interval Valued Neutrosophic Vague Semi Generalised Closed Sets in an Interval Valued Neutrosophic Vague Topological spaces. Also its relationship with other existing sets are compared and validated with suitable examples. The idea of an Interval Valued Neutrosophic Vague Set was taken from the theory of vague sets and interval neutrosophic. Neutrosophic set theory is concerned with indeterminate and inconsistent information. However, interval neutrosophic vague sets were developed to improvise results in decision making problem. Meanwhile, vague set capturing vagueness of data. So interval neutrosophic vague sets, can be utilize in solving decision making problems that inherited uncertainties.

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