An Approach On Fuzzy Decision Making Problem In Icosikaitetragonal Fuzzy Number

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Abstract

The principal aim of this article is to fetch the fuzzy decision making problem by Icosikaitetragonal fuzzy number. We initiate the value of Payoff matrix by Icosikaitetragonal fuzzy number. We modify the fuzzy decision making problem into crisp valued decision making problem by means of ranking to pay off. The crisp valued decision making problem can be efficiently demonstrated with savage mini max regret criterion.

Keywords: Icosikaitetragonal fuzzy number, fuzzy game problem, fuzzy ranking.

1. Introduction

Fuzzy decision making has been introduced by Bellman and Zadeh [1]. In the decision making problem the alternatives from which the decision has to be taken must be determined. There are different types of decision making. A single person is responsible for taking decisions are individual decision making. Several persons are gathering and transforming expert knowledge from various persons are utilized to make decisions are Multi person decision making. Proper information are to be collected for better decision making. Decision making is an activity which includes the steps to be taken for choosing a suitable alternative from those that needs for realizing a certain plan. Jain [2] was the first to propose method of ranking fuzzy numbers for decision making in fuzzy related situation. Raju and Jayagopal [3] was the first to introduce the Icosikaitetragonal fuzzy number. Some decisions are taken based on the collection of data. Decision makers are applying Icosikaitetragonal fuzzy numbers rather than real numbers to express their judgments. Solving problems and making decisions are essential skills for business and life. Problem solving shows the importance of decision making. Decision making
is very important for management and leadership. In today’s scenario there are many process and techniques to improve the decision making and the quality of decision making.

In this paper, we have taken decision making problem in which imprecise values are Icosikaiettragonal fuzzy numbers. We have made clear it with converting to crisp valued decision making problem using ranking technique. We have expounded fuzzy decision making problem using Icosikaiettragonal fuzzy number with illustrations

2. PRELIMINARIES

In this section, we furnish the preliminaries that are required for this study.

**Definition 2.1.** A fuzzy set $A$ is defined by $A = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0, 1]\}$. Here $x$ is crisp set $A$ and $\mu_A(x)$ is membership function in the interval $[0,1]$.

**Definition 2.2.** The fuzzy number $A$ is a fuzzy set whose membership function must satisfy the following conditions.

(i) A fuzzy set $A$ of the universe of discourse $X$ is convex

(ii) A fuzzy set $A$ of the universe of discourse $X$ is a normal fuzzy set if $x_I \in X$ exists

(iii) $\mu_A(x)$ is piecewise continuous

**Definition 2.3.** A fuzzy number $A = (a, b, c)$, where $a \leq b \leq c$, is triangular fuzzy number and its membership function is given by

$$
\mu_A(x) = \begin{cases}
\frac{x-a}{b-a} & \text{for } a \leq x \leq b \\
\frac{c-x}{c-b} & \text{for } b \leq x \leq c \\
0 & \text{for } x > c
\end{cases}
$$

**Definition 2.4**

A fuzzy number $A = (a, b, c, d)$, where $a \leq b \leq c \leq d$, is trapezoidal fuzzy number and its membership function is given by

$$
\mu_A(x) = \begin{cases}
0 & \text{for } x < a \\
\frac{x-a}{b-a} & \text{for } a \leq x \leq b \\
1 & \text{for } b \leq x \leq c \\
\frac{d-x}{d-c} & \text{for } c \leq x \leq d \\
0 & \text{for } x > d
\end{cases}
$$

**Definition 2.5**

An $\alpha$-cut of fuzzy set $A$ is classical set defined as $^\alpha A = \{x \in X | \mu_A(x) \geq \alpha \}$

**Definition 2.6**

A fuzzy set $A$ is a convex fuzzy set iff each of its $\alpha$-cut $^\alpha A$ is a convex set.
Definition 2.7 [3] A fuzzy number $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, \ldots, a_{24})$ is Icosikaitetragonal fuzzy number and its membership function is given by

$$
\mu_A(x) = \begin{cases} 
0, & \text{for } x < a_1 \\
\frac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2 \\
k_1, & \text{for } a_2 \leq x \leq a_3 \\
k_1 + (k_2 - k_1) \frac{x - a_3}{a_4 - a_3}, & \text{for } a_3 \leq x \leq a_4 \\
k_2, & \text{for } a_4 \leq x \leq a_5 \\
k_2 + (k_3 - k_2) \frac{x - a_5}{a_6 - a_5}, & \text{for } a_5 \leq x \leq a_6 \\
k_3, & \text{for } a_6 \leq x \leq a_7 \\
k_3 + (k_4 - k_3) \frac{x - a_7}{a_8 - a_7}, & \text{for } a_7 \leq x \leq a_8 \\
k_4, & \text{for } a_8 \leq x \leq a_9 \\
k_4 + (k_5 - k_4) \frac{x - a_9}{a_{10} - a_9}, & \text{for } a_9 \leq x \leq a_{10} \\
k_5, & \text{for } a_{10} \leq x \leq a_{11} \\
k_5 + (1 - k_5) \frac{x - a_{11}}{a_{12} - a_{11}}, & \text{for } a_{11} \leq x \leq a_{12} \\
k_5, & \text{for } a_{12} \leq x \leq a_{13} \\
k_5 + (1 - k_5) \frac{x - a_{13}}{a_{14} - a_{13}}, & \text{for } a_{13} \leq x \leq a_{14} \\
k_5, & \text{for } a_{14} \leq x \leq a_{15} \\
k_4 + (k_3 - k_4) \frac{x - a_{15}}{a_{16} - a_{15}}, & \text{for } a_{15} \leq x \leq a_{16} \\
k_4, & \text{for } a_{16} \leq x \leq a_{17} \\
k_3 + (k_4 - k_3) \frac{x - a_{17}}{a_{18} - a_{17}}, & \text{for } a_{17} \leq x \leq a_{18} \\
k_3, & \text{for } a_{18} \leq x \leq a_{19} \\
k_3 + (k_2 - k_3) \frac{x - a_{19}}{a_{20} - a_{19}}, & \text{for } a_{19} \leq x \leq a_{20} \\
k_2, & \text{for } a_{20} \leq x \leq a_{21} \\
k_1 + (k_2 - k_1) \frac{x - a_{21}}{a_{22} - a_{21}}, & \text{for } a_{21} \leq x \leq a_{22} \\
k_1, & \text{for } a_{22} \leq x \leq a_{23} \\
k_1 \frac{a_{23} - x}{a_{24} - a_{23}}, & \text{for } a_{23} \leq x \leq a_{24} \\
0, & \text{for } x > a_{24}
\end{cases}
$$
3. Mathematical formulation of Fuzzy Decision making problem:

Consider a fuzzy decision making problem in which all the entries of the payoff matrix are Icosikaitetagonal fuzzy numbers. Let us obtain the problem R has m strategies and problem S has n strategies. Then the payoff matrix m x n is

\[
A = \begin{pmatrix}
  r_{11} & r_{12} & \cdots & r_{1n} \\
  r_{21} & r_{22} & \cdots & r_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{m1} & r_{m2} & \cdots & r_{mn} \\
\end{pmatrix}
\]

3.1 Procedures for solving Savage Minimax regret criterion:

Step 1: Construct a regret (opportunity loss) table of each alternative for every state of nature from the given payoff matrix.

Step 2: Pick out the maximum pay off in each column and subtract all the elements in that column from this maximum value.

Step 3: For each decision alternative (row), pick out the maximum row value and enter this in the last decision column.

Step 4: Choose the decision alternative with the smallest value in the decision column.
3.2 Numerical Example:

Consider the fuzzy game problem with payoff matrix as Icosikaitetragonal fuzzy numbers. This problem is worked out by taking the values \( k_1 = \frac{1}{6}, k_2 = \frac{2}{6}, k_3 = \frac{3}{6}, k_4 = \frac{4}{6}, k_5 = \frac{5}{6} \)

We get the values of \( \mu_{icsktetra}(a_{ij}) \)

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Expected level of sale ( Rupees )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>1.67</td>
</tr>
<tr>
<td>C</td>
<td>14.54</td>
</tr>
</tbody>
</table>

Step 1: Fuzzy decision making problem is abridged to the following payoff profit matrix

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Expected level of sale ( Rupees )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>A</td>
<td>12.54</td>
</tr>
<tr>
<td>B</td>
<td>12.87</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
</tbody>
</table>

Step 2: The opportunity loss table for each alternative with the states of nature is noted below

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Expected level of sale ( Rupees )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>A</td>
<td>14.54</td>
</tr>
</tbody>
</table>
Step 3: The opportunity loss table and the maximum loss in each row is entered and shown in the below table

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Expected level of sale (Rupees)</th>
<th>Decision Column (Maximum Loss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12.54</td>
<td>12.54</td>
</tr>
<tr>
<td>B</td>
<td>12.87</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Result: Since the minimum of maximum loss is in alternative C = 1 rupee, this alternative should be selected.

3.3 Ranking of Icosikaitetragonal fuzzy number:

Let I be a normal Icosikaitetragonal fuzzy number. The value $M(I)$, called as measure of I is calculated as

$$M(I) = \frac{1}{2} \int_{k_1}^{k_2} (\epsilon_1 + \epsilon_2) d\epsilon + \frac{1}{2} \int_{k_2}^{k_3} (m_1 + m_2) dm + \int_{k_3}^{k_4} (n_1 + n_2) dn + \int_{k_4}^{k_5} (o_1 + o_2) do + \int_{k_5}^{k_6} (p_1 + p_2) dp + \int_{k_6}^{k_7} (q_1 + q_2) dq$$

where $0 \leq k_1 \leq k_2 \leq k_3 \leq k_4 \leq k_5 \leq 1$

$$M(L) = \frac{1}{4} \left[ (a_1 + a_2 + a_3 + a_4)k_1 + (a_3 + a_4 + a_1 + a_2)(k_2 - k_1) + (a_5 + a_6 + a_9 + a_{10})(k_3 - k_2) + (a_7 + a_8 + a_{12} + a_{13})(k_4 - k_3) + (a_{11} + a_{12} + a_{13} + a_{14})(k_5 - k_4) + (a_{11} + a_{12} + a_{13} + a_{14})(1 - k_5) \right]$$

where $0 \leq k_1 \leq k_2 \leq k_3 \leq k_4 \leq k_5 \leq 1$

we take the values for $k_1 = \frac{1}{6}, k_2 = \frac{2}{6}, k_3 = \frac{3}{6}, k_4 = \frac{4}{6}, k_5 = \frac{5}{6}$

3.4 Numerical Example:

Let us consider the matrix

\[
\begin{pmatrix}
-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17 \\
6,7,8,9,10,11,12,13,14,15,16,17 \\
-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17 \\
6,7,8,9,10,11,12,13,14,15,16,17 \\
0,1,2,3,4,5,6,7,9,10,11,13,14,15,17,19,21,22,24,25,26,27,28,29 \\
17,19,21,22,24,25,26,27,28,29 \\
-8,-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24 \\
10,11,12,13,14,15,16,17,18,19,20,21,22,23,24 \\
1,2,3,6,8,9,10,12,13,15,16,17,19,20,22,23,25,28,30,32,34,35,37,39 \\
-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30 \\
-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100 \\
8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100
\end{pmatrix}
\]
Step 1:
We obtain the values of $\mu_{\text{lskoc}}(a_{ij})$ of the given fuzzy game problem and convert the fuzzy game problem into crisp valued problem which is shown in the given table.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Expected level of Sale (in Rupees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>A</td>
<td>5.5</td>
</tr>
<tr>
<td>B</td>
<td>5.54</td>
</tr>
<tr>
<td>C</td>
<td>14.25</td>
</tr>
</tbody>
</table>

Step 2: The given fuzzy decision making problem is reduced to the following payoff profit matrix

\[
\begin{array}{c|ccc}
\text{Alternatives} & \text{Expected level of Sale (in Rupees)} & \text{I} & \text{II} & \text{III} \\
\hline
a_{11} & -6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17 & \mu_R(a_{11}) = 5.5 \\
a_{12} & -8,-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15 & \mu_R(a_{12}) = 3.5 \\
a_{13} & 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24 & \mu_R(a_{13}) = 12.5 \\
a_{21} & -6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,18 & \mu_R(a_{21}) = 5.54 \\
a_{22} & -4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19 & \mu_R(a_{22}) = 7.5 \\
a_{23} & 0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,20,21,22,24,26,27,28,29,30 & \mu_R(a_{23}) = 14.3 \\
a_{31} & 0,1,2,3,4,5,6,7,9,10,11,12,13,14,15,16,17,19,21,22,24,25,26,27,28,29,30 & \mu_R(a_{31}) = 14.25 \\
a_{32} & 1,2,3,6,8,9,10,12,13,15,16,17,19,20,22,23,25,28,30,32,34,35,37,39 & \mu_R(a_{32}) = 19 \\
a_{33} & -5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18 & \mu_R(a_{33}) = 6.5 \\
\end{array}
\]

Step 3: The opportunity loss table for each alternative with the states of nature is depicted below

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Expected level of Sale (in Rupees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>A</td>
<td>8.75</td>
</tr>
<tr>
<td>B</td>
<td>8.71</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
</tr>
<tr>
<td>Column Maximum</td>
<td>14.25</td>
</tr>
</tbody>
</table>
Step 4: The opportunity loss table and the maximum loss in each row is entered and shown in the below table

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Expected level of Sale (in Rupees)</th>
<th>Decision Column (Maximum Loss)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>A</td>
<td>8.75</td>
<td>15.5</td>
</tr>
<tr>
<td>B</td>
<td>8.71</td>
<td>11.5</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Result: Since the minimum of maximum loss is in alternative C = 7.8 rupees, this alternative must be selected.

Conclusion: In this article, we have described and solved fuzzy decision making problem and its pay off matrix whose elements are Icosikaitetragonal fuzzy number. We have illustrated the alternative selection of the fuzzy valued decision making problem converting to crisp valued decision making problem using ranking techniques. The Crisp valued decision making problem is solved by savage minimax regret criterion.

References:


