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NUMERICAL APPROCH OF FIRST ORDER DIFFERENTIAL EQUATIONS ARISING IN THE TIME CONSTANT OF A SENSOR

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Abstract: Many processes and phenomena in chemistry, and generally in sciences, can be described by first-order differential equations. These equations are the most important and most repeatedly used to illustrate natural laws. Even though the math is the same in all cases, the student may not always easily understand the similarities because the relevant equations appear in different topics and contain different quantities and units. This text was written to present a unified view on various examples; all of them can be mathematically described by first-order differential equations. The following example is discussed like time constants of sensors.

Index Terms - **Differential equations , Time constants of sensor**

THE TIME CONSTANT OF A SENSOR

Sensors measure a physical or chemical quantity and transduce it to an output signal which is read, monitored or stored. Possible physical quantities are temperature, pressure, radiative flux, magnetic field strength, etc. Chemical quantities are mainly concentrations and activities of molecules, atoms and ions. The recorded signals are usually voltages or currents. The most typical feature of a signal is that the results are one dimensional, e.g. the output signal is a single quantity, i.e. one measures only that signal and not a dependence of that signal on another given quantity. Most devices for chemical analysis produce two-dimensional read-outs, e.g. optical spectra in which the absorbance is displayed as a function of wavelength ($E = f(\lambda)$), voltammograms in which currents are displayed as function of electrode potential or X-ray diffractograms, in which the intensity of diffracted rays is displayed as function of diffraction angle, etc. In modern Instrumentation, one has even expanded the

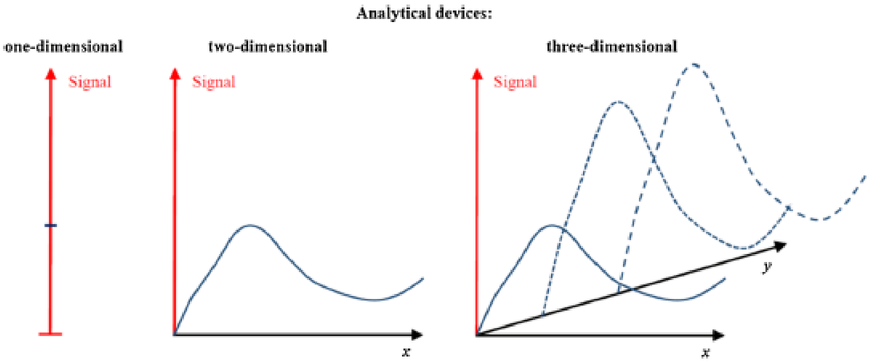
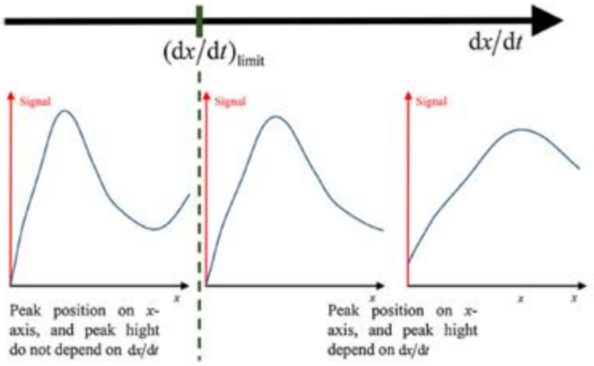


Fig.1 A comparison of the three common dimensionalities of analytical devices

Fig.2 Possible distortion of a spectrum when the rate of changing x with time above a limiting value $(dx/dt)_{limit}$



dimensionality to three, when, as an example, optical spectra (E= $f(\lambda)$) (or mass spectra, i.e. ion intensities versus the mass-to-charge ratio of ions) are displayed as a function of elution time of a chromatogram. Figure 1 gives a comparison of the common dimensionalities of analytical measurements.

Since any measurement needs time, there is nothing like an instantaneous establishment of a signal. This is easy to see when using a sensor, e.g. a pH electrode: There is always a certain time period in which the reading changes until we finally have the impression that a constant end value is reached. The same is true also for two- or three dimensional measurements, but we cannot easily detect it because the variation of the measured signal (e.g. the absorbance) anyway changes as a function of the varied quantities (e.g. the wavelength) and thus with time. Normally, the wavelength is changed with the so-called scan rate $\frac{d\lambda}{dt}$ (rate of recording the spectrum), and generally (see Fig.2), the quantity x is varied with a scan rate $\frac{dx}{dt}$ (which may be also zero). Whether we measure at each wavelength really the end value of the absorbance can be only seen if we decrease the rate at which the wavelength is changed (in the extreme even keeping the wavelength constant). Referring to Fig. 2, this means in general terms, that a variation of the scan rate $\frac{dx}{dt}$ may give a reproducible and identical response only below a certain limiting rate $(\frac{dx}{dt})_{limit}$. If that rate is exceeded, the signal cannot establish its true value and the spectra are distorted (the signal lags behind) (cf. Fig. 2).

Figure 2 shows impressively that it is important to know the rate at which the signal is established for a given x value. In case of a sensor, i.e. a one-dimensional device where no parameters like x or y are changed, the time change of the signal can be studied following a concentration step. The introduction of the sensor into a solution can be regarded as a concentration step. Figure 3 depicts two different kinds of response of a sensor on a concentration step. Figure 3 depicts two basic types of time responses of sensors. The different sensor behaviours shown in B and C can be modelled with the help of different differential equations. Whereas the response curve shown in B can be modelled with a first-order differential equation; the curve shown in C needs higher-order differential equations. At this point, it is necessary to note that it is impossible to realize a concentration step with infinite rate of concentration rise, as shown in Fig. 3a. This means, when the temporal response properties of a sensor are studied, this concentration rise has to be much quicker than the response of the sensor. Further, also the response shown in Fig. 3b is to some extend an idealization, and in reality there may be always a sluggish response at the start, but it may be on such short time scale that it escapes our recognition.

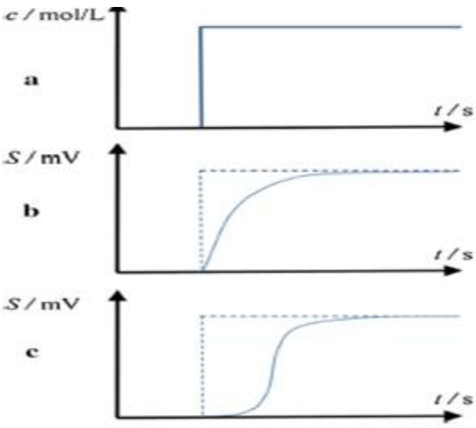


Fig 3 **a** The concentration is stepped from zero to a constant value. **b** The signal of the detector starts to respond immediately when the step is made, and the slope of the response over time continuously decreases until it finally approaches a constant final value. The response curve has no turning point. **c** The sensor starts slowly to respond (but with *increasing rate/slope*), and after a turning point the response slows down (*decreasing slope*) until it approaches the final value

The response curve shown in fig.3b can be modeled as follows

$$S = S_{max} - w(t) \tag{1}$$

w is a time dependent quantity for which we write the first order differential equation

$$p_1 \frac{dw}{dt} + p_2 w(t) = 0 \tag{2}$$

Integrating, we get

$$\int_{w_0}^{w} \frac{dw}{w(t)} = -\frac{p_2}{p_1} \int_0^t dt \tag{3}$$

$$ln \frac{w(t)}{w} = -\frac{p_2}{p_1} t \tag{4}$$

$$\frac{w(t)}{w_0} = e^{-\frac{p_2}{p_1} t} \tag{5}$$

$$w(t) = we^{-\frac{p_2}{p_1} t} \tag{6}$$

So eqn.(1) follows that

$$S = S_{max} - we^{-\frac{p_2}{p_1} t} \tag{7}$$

Obviously , w_0 should be equal to S_{max}

So eqn.(7) becomes $S = S_{max} - S_{max} e^{-\frac{p_2}{p_1} t} = S_{max}(1 - e^{-\frac{p_2}{p_1} t})$ (8)

Since the term $(1 - e^{-\frac{p_2}{p_1} t})$ should not have a unit, it follows that the ratio $\frac{p_2}{p_1}$ must be a reciprocal time and we may write eqn.(8) using the definition $\frac{p_1}{p_2} = \tau$ i.e $\frac{p_2}{p_1} = \frac{1}{\tau}$

$$S = S_{max}(1 - e^{-\frac{1}{\tau}}) \tag{9}$$

The quantity τ is called the time constant of the sensor.

At $t = \tau$ the signal has the value

$$S = S_{max}(1 - e^{-1}) \approx 0.6325$$

In other words , after elapse of τ, the signal has reached 63.2% of its final value.

Conclusion:

In this paper, we discussed the importance of first order differential equations in many areas such as physics, chemistry and engineering.It should be mentioned that there are many other processes in science, which are based on first-order differential equations, e.g.the time constant of sensor

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