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AN UPDATED SURVEY OF PENDANT DOMINATION PARAMETERS IN GRAPHS

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Abstract: Let G be any graph. A dominating set S in G is called pendant dominating set if $\langle S \rangle$ contains atleast one pendant vertex. The least cardinality of a pendant dominating set in G is called pendant domination number of G , denoted by $\gamma_{pe}(G)$. In this survey, we present recent results on pendant dominating sets of graphs.
Keywords : Dominating set , Pendant dominating set .

1 Introduction

Let G be any graph. The concept of paired domination is an interesting concept introduced by Teresa W. Haynes in with the following application in mind. If we think of each vertex υ as the possible location for a guard capable of protecting each vertex in its closed neighborhood, then *domination* requires every vertex to be protected. For total domination, each guard must, in turn, be protected by other guard. But for paired-domination, each guard is assigned another adjacent one, and they are designated as backups for each other. The authors in [9] introduce pendant domination for which at least one guard is assigned a backup.

2 Basic Definitions

Let G =(V,E) be any graph with |V(G)| = n and |E(G)| = m edges. Then n, m are respectively called the order and the size of the graph G. For each vertex υ ∊ V, the open neighborhood of υ is the set N(υ) containing all the vertices u adjacent to υ and the closed neighborhood of υ is the set N[υ] containing υ and all the vertices u adjacent to υ. Let S be any subset of V, then the open neighborhood of S is N(S)=$\cup_{v \in S} N(v)$ and the closed neighborhood of S is N[S] = N(S)∪ S.

The minimum and maximum of the degree among the vertices of G is denoted by δ(G) and Δ(G) respectively. A graph G is said to be regular if δ(G) = Δ(G). A vertex υ of a graph G is called *a cut vertex* if its removal increases the number of components. A *bridge* or *cut edge* of a graph is an edge whose removal increases the number of components. A vertex of degree zero is called an isolated vertex and a vertex of a degree one is called a pendant vertex. An edge incident to a pendant vertex is called a pendant edge. The graph containing no cycle is called a tree. A complete bi-partite graph $K_{1,3}$ is a tree called as *claw*. Any graph containing no subgraph isomorphic to $K_{1,3}$ is called a claw-free graph.

A set M ⊆ E(G) is called a matching of G if no two edges in M are incident in G. The two ends of an edge are said to be matched under M. If every vertex of G is matched under M, then M is called a perfect matching. The cardinality of the maximum matching is called the matching number of G, denoted by m(G).

A subset S of V(G) is a dominating set of G if each vertex u ∊ V – S is adjacent to a vertex in S. The least cardinality of a dominating set in G is called the domination number of G and is usually denoted by γ(G).

A dominating set S of a graph G is said to be paired dominating set of G if the induced subgraph $\langle S \rangle$ contains at least one perfect matching. Any paired dominating set with minimum cardinality is called a minimum paired dominating set. The cardinality of the minimum paired dominating is called the paired domination number of G and is denoted by $\gamma_{pd}(G)$. A paired dominating set with cardinality $\gamma_{pd}(G)$ is referred as γ_{pd}-set. A dominating set S is called a total dominating set if ⟨S⟩ contains no isolated vertex. The cardinality of the minimum total dominating set is called the total domination number of G and is denoted by $\gamma_t(G)$. A total dominating set with cardinality $\gamma_t(G)$ is called as γ_t-set.

The set S of vertices in a graph G is called an independent set if no two vertices in S are adjacent. A dominating set S of a graph G is an independent dominating set if ⟨S⟩ has no edges. The minimum cardinality of an independent dominating set is called the independent domination number, denoted by i(G) and the independence number $\beta_0(G)$ is the maximum cardinality of an independent set of G.

The corona of two disjoint graphs G_1 and G_2 is defined to be the graph G = G_1 o G_2 formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where the *i*th vertex of G_1 is adjacent to every vertex in the *i*th copy of G_2. If G and H are disjoint graphs, then the join of G and H denoted by G + H is the graph such that V(G+H) = V(G)∪V(H) and E(G + H) = E(G)∪E(H)∪*uv* : *u* ∈ V(G), υ ∈ V (H). The line graph L(G) of a graph G is the graph whose vertex set corresponds to the edges of G such that two vertices of L(G) are adjacent if and only if the corresponding edges of G are adjacent.

Any graph G with at least one bridge is called a bridged graph. The *n*-Barbell graph is the simple graph obtained by connecting two copies of a complete graph K_n by a bridge. The *n* Pan graph is the graph obtained by joining a cycle graph C_n to a singleton graph K_1 with a bridge. The ladder graph is a Cartesian product of P_2 and P_n where P_n is a path graph.

Theorem 2.1. [9] *Let P_n be a path with $n \geq 2$ vertices and C_n be a cycle with $n \geq 3$ vertices. Then*

$$\gamma_{pe}(P_n or C_n) = \begin{cases} \frac{n}{3}+1, & \text{if } n \equiv 0 \pmod 3; \\ \frac{n}{3} & \text{if } n \equiv 1 \pmod 3; \\ \frac{n}{3}+1 & \text{if } n \equiv 2 \pmod 3; \end{cases}$$

Theorem 2.2. [9] *A dominating set S is a minimal pendant dominating set if and only if for each vertex u ∈ S one of the following condition holds.*

1. *u is either an isolate or a pendant vertex of S.*
2. *each vertex of S − {u} lies in a cycle.*
3. *there exists a vertex v ∈ V − S for which N (v) ∩ S = {u}.*

Theorem 2.3. [9] *Let T be any Tree. Then $\gamma_{pe}(T) = \gamma(T)$ if and only if there is a γ−set which is not independent in T.*

Theorem 2.4. [9] *Let G be any graph. Then $\gamma_{pe}(G) = \gamma(G)$ if and only if G contains a γ set which is either an independent set in G or each vertex of S belongs to some cycle in S.*

Proposition 2.1. [9] *Let G be any graph with $n \geq 3$ vertices. Then $n-m \leq \gamma_{pe}(G) \leq n-1$.*

Let G be the collection of graphs of following types. A cycle, path, star, wheel and a complete graph each of order 4 and a path, cycle of order 5.

Theorem 2.5. [9] *Let G be a connected graph of order n. Then $\gamma_{pe}(G) = n - 2$ if and only if G ∈ G.*

Theorem 2.6. [9] *Let G be any graph. Then $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \leq \gamma_{pe}(G) \leq n - \Delta(G) + 1$. Further if G is a tree, then $\gamma_{pe}(G) = n - \Delta(G) + 1$ if and only if G is a wounded spider obtained by subdividing even number of edges of a star.*

Proposition 2.2. [9] *Let G be an acyclic graph. Then $\gamma_{pe}(G) \leq \gamma_t(G) \leq \gamma_{pr}(G)$. Equality holds if G is either a cycle or a path of order 4k.*

Theorem 2.7. [9] *For any graph G, $\gamma_{pe}(G) \leq i(G) + 1$. Equality holds if G is a claw-free graph. Further, for any positive integer k, there exists a graph H such that $i(H) - \gamma_{pe}(H) = k$.*

Theorem 2.8. [9] *Let G be a graph connected with n vertices and H be any graph. Then*

$$\gamma_{pe}(G \text{ o } H) = \begin{cases} n+1, & \text{if G is a cycle and } \gamma(H) \geq 2; \\ n, & \text{otherwise} \end{cases}$$

Proof. For any connected graph G and any graph H, we have $\gamma(G \circ H) = n$ and hence $\gamma_{pe}(G \circ H) \leq n + 1$. First, suppose G is not a cycle, then clearly V(G) itself a pendant dominating set in G. Assume G is a cycle. If $\gamma(H) = 1$ then for any vertex v ∈ G, the set S = (V − {v})∪{u} is a pendant dominating set in G ∘ H, where {u} is a γ−set of H. Therefore, $\gamma_{pe}(G \circ H) = n$. Suppose $\gamma(H) \geq 2$, since V (G) contains no pendant vertex, we must have that $\gamma_{pe}(G \circ H) \geq n + 1$. On the other hand, for any vertex v of H, the set $V(G) \cup \{v\}$ is a pendant dominating set of size n + 1. Therefore $\gamma_{pe}(G) = n + 1$.

The Cartesian product of two graphs G_1 and G_2 is the graph, denoted by $G_1 \times G_2$, with $G_1 \times G_2 = V(G_1) \times V(G_2)$ (where × denotes the Cartesian product of sets) and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V(G_1 \times G_2)$ whenever $[u_1 = v_1$ and $(u_2, v_2) \in E(G_2)]$ or $[u_2 = v_2$ and $(u_1, v_1) \in E(G_1)]$. If each G_1 and G_2 is a path P_m and P_n (respectively), then we will call $P_m \times P_n$, a m × n grid graph. For notational convenience, we denote $P_m \times P_n$ by $P_{m,n}$.

Theorem 2.9. [7] *For all* $n \geq 2$, $\gamma_{pe}(P_{2,n}) = \left\lceil \frac{2n}{3} \right\rceil$.

Theorem 2.10 [7] *For all* $n \geq 4$, $\gamma_{pe}(P_{3,n}) = n$.

Theorem 2.11. [7] *For all* $n \geq 5$, $\gamma_{pe}(P_{4,n}) = \left\lceil \frac{4n}{3} \right\rceil$.

Theorem 2.12. [7] *For all* $n \geq 6$.

$$\gamma_{pe}(P_{5,n}) = \begin{cases} \frac{5n}{3}, & \text{if } n \equiv 0, 3 \pmod 6; \\ \left\lceil \frac{5n+1}{3} \right\rceil, & \text{otherwise.} \end{cases}$$

Theorem 2.13. [8] *Let G be a path with n vertices. Then*

$$\gamma_{pe}(\Im(G)) = \begin{cases} 3, & \text{if } n = 5; \\ 2\left\lfloor \frac{n}{3} \right\rfloor, & \text{otherwise} \end{cases}$$

Theorem 2.14. [8] *Let* $G \cong S_n$ *be a crown graph with 2n vertices. Then* $\gamma_{pe}(\Im(G)) = \gamma(\Im(G))$.

Theorem 2.15 [8] *Let* $G \cong H_n$ *be a helm graph. Then* $\gamma_{pe}(\Im(G)) = 2n - 2$.

Definition 2.1. [8] For m ≥ 3, Jahangir graph $J_{n,m}$ is a graph of order nm + 1, consisting of a cycle of order nm with one vertex adjacent to exactly m vertices of C_{nm} at a distance n to each other. Jahangir graph $J_{2,8}$ is shown in Fig.2.

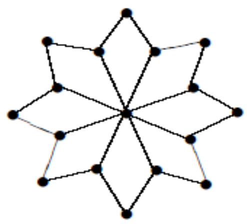

Fig.2. Jahangir graph $J_{2,8}$

Theorem 2.16. [8] *Let* $G \cong J_{n,m}$ *be a Jahangir graph with* $m \geq 3$ *and* $n \cong 0$ *or* $1 (\mathrm{mod}\ 3)$*, then*

$$\gamma_{pe}(\Im(J_{n,m})) = \begin{cases} 2\left\lceil \frac{nm}{3} \right\rceil, & \text{if } m = 3; \\ 2\left\lceil \frac{nm}{3} \right\rceil - 2, & \text{otherwise} \end{cases}$$

Theorem 2.17. [8] *Let* $G \cong J_{n,m}$ *be a Jahangir graph with* $m \geq 3$ *and* $n \cong 2 (\mathrm{mod}\ 3)$*, then*

$$\gamma_{pe}(\Im(G)) = \begin{cases} 2\left\lceil \frac{nm}{3} \right\rceil, & \text{if } m \cong 0 \text{ or } 1 (\mathrm{mod}\ 3); \\ 2\left\lceil \frac{nm}{3} \right\rceil, & \text{if } m \cong 2 (\mathrm{mod}\ 3) \end{cases}.$$

Definition 2.2. [8] The gear graph is a wheel graph with a vertex added between each pair adjacent graph vertices of the outer cycle. The gear graph G has 2n + 1 vertices and 3n edges. In Fig.3. we display G_8.

Theorem 2.18. [8] Let G_n *be a gear graph with* $n \geq 3$*. Then*

$$\gamma(\Im(G_n)) = \begin{cases} \left\lceil \frac{4n}{3} \right\rceil, & \text{if } n \cong 0 \text{ or } 1 \pmod 3 \\ \left\lceil \frac{4n}{3} \right\rceil + 1, & \text{if } n \cong 2 \pmod 3 \end{cases}$$

Proposition 2.3. [8] *Let G be any connected graph of order n. Then* $1 \le \gamma(G) \le \gamma_{pe}(G) \le \gamma(\Im(G)) \le \gamma_{pe}(\Im(G)) \le 2n$. *Further,* $\gamma_{pe}(\Im(G)) = 2$ *if and only if G contains an edge of degree atleast* $n - 2$.

Proposition 2.4. [8] *Let G be any graph. If* $diam(G) = 2$ *then* $\gamma_{pe}(\Im(G)) \le \delta(G) + 1$. *Equality holds if G is a path.*

Proposition 2.5. [8] *Let G be a connected graph of order* $n \ge 2$. *Then* $\gamma_{pe}(\Im(G)) \le \gamma(\Im(G)) + \delta(\Im(G))$

3 Vertex Removal

We observe that the pendant domination parameter value of a graph G may be increases or decreases or remains same when a point is removal from G. For an example in a complete graph K_m ($m > 2$) or complete bipartite graph $K_{m,n}$ removal of any one point it does not affect the number of γ_{pe}. In a sunlet graph the removal of a vertex of degree one it decreases the value of γ_{pe} by one. In barbell graph v_1, v_2 are the adjacent vertices connected two copies of complete graphs. If we removal of the vertex v_1 in barbell graph increases the value of γ_{pe} by 2. Hence we can define the point set V (G) of G into three subsets

$$V_{pe}^0 = \{(u, v) \in V : \gamma_{pe}(G - v) = \gamma_{pe}(G)\}$$

$$V_{pe}^- = \{(u, v) \in V : \gamma_{pe}(G - v) < \gamma_{pe}(G)\}$$

$$V_{pe}^+ = \{(u, v) \in V : \gamma_{pe}(G - v) > \gamma_{pe}(G)\}$$

Theorem 3.1. [6] *If* $G \cong P_n$ *and* $n \ge 3$, *then we have*

(i) If $n \equiv 0 \pmod 3$ *or* $n \equiv 1 \pmod 3$ *then*

$$v_i \in \begin{cases} V_{pe}^0, & \text{if } i = 1 \text{ or } n; \\ V_{pe}^+, & \text{if } i \equiv 1 \text{ or } 2 \text{ or } 3 \pmod 3 \end{cases}$$

(ii) If $n \equiv 2 \pmod 3$, *then*

$$v_i \in \begin{cases} V_{pe}^-, & \text{if } i = 1 \text{ or } n; \\ V_{pe}^+, & i \equiv 0 \pmod 3; \\ V_{pe}^0, & i \equiv 1 \text{ or } 2 \pmod 3. \end{cases}$$

Theorem 3.2. *[6] If* C_n *is a cycle with* $n \ge 4$ *vertices, then*

$$V(C_n) \in \begin{cases} V_{pe}^-, & \text{if } n \equiv 2 \pmod 3; \\ V_{pe}^0, & \text{Otherwise} \end{cases}$$

4 Edge Removal

In this section, we analyse the effect of edge removal in the pendant domination number $\gamma_{pe}(G)$ of graph G. As in the case of vertex removal, we can observe that the pendant domination number $\gamma_{pe}(G)$ of a graph G may increase or decrease remain same when an edge is removed from G. Hence we can partition the edge set E(G) of G into 3 subsets as E_{pe}^+, E_{pe}^- *and* E_{pe}^o below.

$$E_{pe}^- = \{(u, v) \in E : \gamma_{pe}(G - uv) \le \gamma_{pe}(G)\}$$

$$E_{pe}^0 = \{(u, v) \in E : \gamma_{pe}(G - uv) = \gamma_{pe}(G)\}$$

$$E_{pe}^+ = \{(u, v) \in E : \gamma_{pe}(G - uv) \ge \gamma_{pe}(G)\}$$

Theorem 4.1. [6] *Let* P_n *be a path with* $n \ge 3$ *vertices, then we have*

(i)If $n \equiv 0 \pmod 3$, *then*

$$(v_i, v_{i+1}) \in \begin{cases} E_{pe}^{-}, & i = 1 \text{ or } n-1; \\ E_{pe}^{+}, & \text{if } i \equiv 0 \pmod 3; \\ E_{pe}^{0}, & \text{if } i \equiv 1 \pmod 3. \end{cases}$$

(ii)If n ≡ 1(mod 3), then

$$(v_i, v_{i+1}) \in \begin{cases} E_{pe}^{0}, & \text{if } i \equiv 2 \pmod 3 \text{ or } i = 1 \text{ or } n-1; \\ E_{pe}^{+}, & i \equiv 0 \text{ or } 1 \pmod 3 \end{cases}$$

(iii) if n ≡ 2 (mod3), then

$$(v_i, v_{i+1}) \in \begin{cases} E_{pe}^{-}, & \text{if } i = 1 \text{ or } n-1; \\ E_{pe}^{0}, & \textit{Otherwise} \end{cases}$$

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