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Gourava Indices Of Product Graphs

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ABSTRACT: First Gourava index is defined as $GO_1(G) = \sum_{uv \in E(G)} [d_u + d_v + d_u d_v]$, where d_u is degree of vertex u. In this paper first Gourava index, second Gourava index, product connectivity Gourava index, sum connectivity Gourava index, first hyper Gourava index and second hyper Gourava index of tensor product, corona product and cartesian product of graphs are studied.

KEYWORDS: Cartesian product, corona product, Gourava index, first hyper Gourava index, second hyper Gourava index, tensor product.

I.INTRODUCTION

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Let G = (V,E) be a graph with order |V(G)| = n and size |E(G)| = m. The degree of a vertex is denoted by d_u and defined as the number of vertices adjacent to $u \in V(G)$. The edge connecting the vertices u and v is denoted by uv. All graphs considered here are finite, undirected and simple. In the field of graph theory, the graph operations produce new graph from initial ones. Binary operations create a graph from two initial graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ such as graph union, graph intersection, graph join and graph products etc. Different versions of degree-based topological indices of molecular graphs are studied in [1-2]. Zagreb indices are widely studied in the literature for example [3-4]. There are several molecular graphs that can be realised as a product of graphs, for example nanotorus as $C_n \square C_m$, nanotubes as $P_n \Box C_m$, grid as $P_n \Box P_m$ [5]. R.P.Kumar et al. derived some topological indices of mesh, grid, torus and cylinder in [6]. W.Gao et al. studied multiple ABC index and multiple GA index of square grid [7].In [8] topological indices of grid are computed. Topological indices of graph product are studied in many papers [9-15]. Topological indices of graph operations are obtained in [16-18]. Figure for cartesian product of P₄ and P₆ is taken from [19] to study topological indices of family of Gourava index. Cartesian product of a cycle C_n with a path P_m is $P_{(n,m)}$ generalized prism graph with |V|=mn and |E|=n(2m-1)[20-21]. Weiner index and Hosaya polynomial of tubes and tori was studied by M.V.Diudea in 2005 [22]. Tensor product of two graphs G₁ and G₂ is the graph denoted by $G_1 \otimes G_2$, with vertex set $V(G_1 \otimes G_2) = V(G_1) \times V(G_2)$, and any two of its vertices (u_1, v_1) and (u_2, v_2) are adjacent whenever u_1 is adjacent to u_2 in G_1 and v_1 is adjacent to v_2 in G_2 [23-24]. The corona product of two graphs G and H is defined as the graph obtained by taking one copy of G and |V(G)| copies of H and joining the i-th vertex of G to every vertex in the i-th copy of H. The corona product is denoted by G⊙H [25-28]. Cartesian product of two graphs $G_1 = (V_1, E_1)$ and $G_2(V_2, E_2)$ denoted by $G_1 \times G_2$ or $G_1 \square G_2$ containing vertex set $V_1 \times V_2$ where (u_1, u_2) is adjacent with (v_1,v_2) iff where $[u_1=u_2]$ and $v_1,v_2 \in E_2$ or $[v_1=v_2]$ and $u_1u_2 \in E_1$ [29]. Gourava indices are degree-based indices defined in [30-35] as:

- 1) First Gourava index = $GO_1(G) = \sum_{uv \in E(G)} [d_u + d_v + d_u d_v]$.
- 2) Second Gourava index = $GO_2(G) = \sum_{uv \in E(G)} [(d_u + d_v)d_ud_v].$
- 3) Product connectivity Gourava index = $PGO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_u + d_v)d_u d_v}}$.

Sum connectivity Gourava index = $SGO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_u + d_v) + d_u d_v}}$.

- 5) First hyper Gourava index = $HGO_1(G) = \sum_{uv \in E(G)} [d_u + d_v + d_u d_v]^2$.
- 6) Second hyper Gourava index = $HGO_2(G) = \sum_{uv \in E(G)} [(d_u + d_v)(d_u d_v)]^2$.

All the symbols and notations used in this paper are standard and mainly taken from books of graph theory [36-38]. In this paper first Gourava index, second Gourava index, product connectivity Gourava index, sum connectivity Gourava index, first hyper Gourava index and second hyper Gourava index of tensor product, corona product and cartesian product of graphs are studied.

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II.MATERIALS AND METHODS

A molecular graph is a simple graph related to the structure of a chemical compound. A molecular graph is constructed by representing each atom of a molecule by vertex and bonds between atoms by edges. In tensor product of two graphs we have, the vertex set $|V(G)| = n^2$ and edge set $|E(G)| = 2(n-1)^2$ for $G=P_4 \otimes P_4$. The corona product of two complete is denoted by $K_n \odot K_m$. The tensor product between P_4 and P_4 , corona product between K_4 and K_3 and cartesian product of P_4 and P_4 are shown in figure 1,2 and 3 respectively. The edge partition represented for different product graph are in given table (1-3).

III.RESULTS AND DISCUSSION

Let G be a graph with n vertices and m edges. Different versions of Gourava index of product graphs are computed for path graphs and complete graphs. The edge partition of product graphs is considered for degree of end vertices. By graph operation a new graph is obtained. For these graphs the degree of end vertices is decided by observation and used in the computation of Gourava indices.

Tensor product

Theorem 1.1: Let P_4 and P_4 be two path graphs.

Then first Gourava index of tensor product $P_4 \otimes P_4$ is $GO_1(G) = 68+112(n-3)+48(n-3)^2$. **Proof.** Partition the edge set E(G) in four sets E_1 , E_2 , E_3 and E_4 as $|E_{14}| = 4$, $|E_{22}| = 4$, $|E_{24}| = 8(n-3)$ and $|E_{44}| = 2(n-3)^2$ and using table (1). Therefore $GO_1(G) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=$

Therefore $GO_1(G) = \sum_{uv \in E(G)} [d_u + d_v + d_u d_v]$

 $=\sum_{14\in E_1} [1+4+1*4] + \sum_{22\in E_2} [2+2+2*2] + \sum_{24\in E_3} [2+4+2*4] + \sum_{44\in E_4} [4+4+4*4]$

$$=9*4+8*4+14*8(n-3)+\frac{48(n-3)^2}{48(n-3)^2}$$

 $= 68 + 112(n - 3) + 48(n - 3)^{2}$. Theorem 1.2: Let P₄ and P₄ be two path graphs.

Then second Gourava index of tensor product $P_4 \otimes P_4$ is $GO_2(G) = 144 + 384(n-3) + 256(n-3)^2$.

Proof. Partition the edge set E(G) in four sets E_1 , E_2 , E_3 and E_4 as $|E_{14}| = 4$, $|E_{22}| = 4$, $|E_{24}| = 8(n-3)$ and $|E_{44}| = 2(n-3)^2$ and using table (1). Therefore $GO_2(G) = \sum_{uv \in E(G)} [(d_u + d_v)d_ud_v]$

 $=\sum_{14\in E_1}[(1+4)1*4]+\sum_{22\in E_2}[(2+2)2*2]+\sum_{24\in E_3}[(2+4)2*4]+\sum_{44\in E_4}[(4+4)4*4]$

 $= 20 * 4 + 16 * 4 + 48 * 8(n - 3) + 128 * 2(n - 3)^{2}$

 $= 144 + 384(n - 3) + 256(n - 3)^2$. **Theorem 1.3:** Let P₄ and P₄ be two path graphs.

Then product connectivity Gourava index of tensor product $P_4 \otimes P_4$ is $PGO(G) = 1.8944 + \frac{8(n-3)}{\sqrt{48}} + \frac{2(n-3)^2}{\sqrt{128}}$. **Proof.** Partition the edge set E(G) in four sets E_1 , E_2 , E_3 and E_4 as $|E_{14}| = 4$, $|E_{22}| = 4$, $|E_{24}| = 8(n-3)$ and $|E_{44}| = 2(n-3)^2$ and using table (1).

Therefore
$$PGO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_u + d_v)d_u d_v}}$$

 $= \sum_{14 \in E_1} \left[\frac{1}{\sqrt{[(1+4)1*4]}} \right] + \sum_{22 \in E_2} \left[\frac{1}{\sqrt{[(2+2)2*2]}} \right] + \sum_{24 \in E_3} \left[\frac{1}{\sqrt{[(2+4)2*4]}} \right] + \sum_{44 \in E_4} \left[\left(\frac{1}{\sqrt{[(4+4)4*4]}} \right] \right]$
 $= \frac{4}{\sqrt{[5*4]}} + \frac{4}{\sqrt{[4*4]}} + \frac{8(n-3)}{\sqrt{[6*8]}} + \frac{2(n-3)^2}{\sqrt{[(1+4)1*4]}}$
 $= 1.8944 + \frac{8(n-3)}{\sqrt{48}} + \frac{2(n-3)^2}{\sqrt{128}}.$

Theorem 1.4: Sum connectivity Gourava index of tensor product $P_4 \otimes P_4 = SGO(G) = 4.1614 + \frac{16(n-3)}{\sqrt{14}} + \frac{8(n-3)^2}{\sqrt{24}}$ **Theorem 1.5:** First hyper Gourava index of tensor product $P_4 \otimes P_4 = HGO_1(G) = 580 + 1568(n-3) + 1152(n-3)$ $(3)^2$.

Theorem 1.6: Second hyper Gourava index of tensor product $P_4 \otimes P_4 = HGO_2(G) = 2624 + 18432(n-3) + 1843$ $32768(n-3)^2$. **Corona product**

Theorem 2.1: Let K_n and K_m be two complete graphs with order n and m.

Then first Gourava index of corona product $(K_n \odot K_m)$ is $GO_1(G) = m(2+m)n^mC_2 + nm(m^2 + nm + m + n - m)$ 1) + $[2(n+m-1) + (n+m-1)^2]^n C_2$.

Proof. By using table (2), the edges for the corona product of complete graphs of order n and m on degree are (m,m),(m,n+m-1) and (n+m-1,n+m-1).

Therefore $GO_1(G) = \sum_{uv \in E(G)} [d_u + d_v + d_u d_v]$

 $=\sum_{mm\in E_1}[m+m+m*m]+\sum_{m(n+m-1)\in E_2}[m+n+m-1+m(n+m-1)]+\sum_{(n+m-1)(n+m-1)\in E_3}[2(n+m-1)+m(n+m-1)]+\sum_{m(n+m-1)\in E_3}[2$ 1) + $(n + m - 1)^2$]

 $= m(2+m) n^{m}C_{2} + nm(m^{2}+nm+m+n-1) + [2(n+m-1)+(n+m-1)^{2}]^{n}C_{2}.$

Theorem 2.2: Let K_n and K_m be two complete graphs with order n and m.

Then second Gourava index of corona product $(K_n \odot K_m)$ is

 $GO_2(G) = (2m^3) n^m C_2 + (2m + n - 1)(m^2 + nm - m)nm + 2(n + m - 1)^3 ({}^nC_2).$ **Proof.** By using table (2), the edges for the corona product of complete graphs of order n and m on degree are (m,m),(m,n+m-1) and (n+m-1,n+m-1).

Therefore $GO_2(G) = \sum_{uv \in E(G)} [(d_u + d_v)d_ud_v]$

 $= \sum_{mm \in E_1} [(m+m)m^2] + \sum_{m(n+m-1) \in E_2} [(m+n+m-1)m(n+m-1)] + \sum_{(n+m-1)(n+m-1) \in E_3} [2(n+m-1)m(n+m-1)] + \sum_{m(n+m-1) \in E_3} [2(n+m-1)m(n+m-1)m(n+m-1)] + \sum_{m(n+m-1) \in E_3} [2(n+m-1)m(n+m$ $1)(n + m - 1)^2$ JCR

 $= (2m^3) n^m C_2 + (2m + n - 1)(m^2 + nm - m)nm + 2(n + m - 1)^3 (n^2 C_2).$ Theorem 2.3: Let K_n and K_m be two complete graphs with order n and m.

Then product connectivity Gourava index of corona product $(K_n \odot K_m)$ is

$$PGO(G) = \frac{1}{\sqrt{2m^3}} n^m C_2 + \frac{nm}{\sqrt{(2m+n-1)(m^2+nm-m)}} + \frac{1}{\sqrt{2(n+m-1)^3}} ({}^n C_2).$$

Proof. By using table (2), the edges for the corona product of complete graphs of order n and m on degree are (m,m),(m,n+m-1) and (n+m-1,n+m-1).

Therefore $PGO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_u + d_v)d_u d_v}}$

$$=\sum_{mm\in E_1}\frac{1}{\sqrt{(m+m)m^2}}+\sum_{m(n+m-1)\in E_2}\frac{1}{\sqrt{(m+n+m-1)m(n+m-1)}}+\sum_{(n+m-1)(n+m-1)\in E_3}\frac{1}{\sqrt{2(n+m-1)(n+m-1)^2}}$$

$$=\frac{1}{\sqrt{2m^3}} n^m C_2 + \frac{nm}{\sqrt{(2m+n-1)(m^2+nm-m)}} + \frac{1}{\sqrt{2(n+m-1)^3}} ({}^n C_2)$$

Theorem 2.4: Sum connectivity Gourava index of corona product $(K_n \odot K_m)$ is $\frac{1}{\sqrt{2m+m^2}} n^m C_2 + \frac{1}{\sqrt{2m+m^2}} n^m C_2$ $\frac{nm}{\sqrt{(2m+n+m-1)+(m^2+nm-m)}} + \frac{1}{\sqrt{2(n+m-1)+(n+m-1)^2}} \left({}^n C_2 \right).$ Theorem 2.7. Figure 1

Theorem 2.5: First hyper Gourava index of corona product $(K_n \odot K_m)$ is $(2m + m^2)^2 n^m C_2 + [(2m + n - 1) + m^2)^2 n^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2 n^2 C_2 + [(2m + n - 1) + m^2)^2$ $(m^{2} + nm - m)^{2}nm + [2(n + m - 1) + (n + m - 1)^{2}]^{2} ({}^{n}C_{2}).$

Theorem 2.6: Second hyper Gourava index of corona product $(K_n \odot K_m)$ is $2m^3 n^m C_2 + [(2m + n - 1)(m^2 + m^2)]$ $(nm - m)^{2}nm + [2(n + m - 1)^{3}]^{2} ({}^{n}C_{2}).$

Cartesian product

Theorem 3.1: Let P₄ and P₄ be two path graphs.

Then first Gourava index of cartesian product $P_4 \square P_4$ is $GO_1(G) = 88 + 15(2m + 2n - 12) + 19(2m + 2n - 8n) + 19(2m + 2n$ 24(2nm - 5m - 5n + 12).**Proof.** Partition the edge set E(G) in four sets E_1 , E_2 , E_3 and E_4 as $|E_{23}| = 8$, $|E_{33}| = 2m + 2n - 12$, $|E_{34}| = 2m + 2n - 8$ and $|E_{44}| = 2mn - 5m - 5n + 12$, table (3). Therefore $GO_1(G) = \sum_{uv \in E(G)} [d_u + d_v + d_u d_v]$

 $=\sum_{23\in E_1}[2+3+2*3]+\sum_{33\in E_2}[3+3+3*3]+\sum_{34\in E_2}[3+4+3*4]+\sum_{44\in E_4}[4+4+4*4]$

= 88 + 15(2m + 2n - 12) + 19(2m + 2n - 8n) + 24(2nm - 5m - 5n + 12).**Theorem 3.2:** Let P₄ and P₄ be two path graphs.

Then second Gourava index of cartesian product $P_4 \square P_4$ is $GO_2(G) = 240 + 54(2m + 2n - 12) + 84(2m + 2n - 12)$ 8) + 128(2nm - 5m - 5n + 12).

Proof. Partition the edge set E(G) in four sets E_1 , E_2 , E_3 and E_4 as $|E_{23}| = 8$, $|E_{33}| = 2m + 2n - 12$, $|E_{34}| = 2m + 2n - 8$ and $|E_{44}| = 2mn - 5m - 5n + 12$, table (3).

Therefore $GO_2(G) = \sum_{uv \in E(G)} [(d_u + d_v)d_u d_v]$ $=\sum_{23\in E_1} [(2+3)2*3] + \sum_{33\in E_2} [(3+3)3*3] + \sum_{34\in E_3} [(3+4)3*4] + \sum_{44\in E_4} [(4+4)4*4]$

= 240 + 54(2m + 2n - 12) + 84(2m + 2n - 8) + 128(2nm - 5m - 5n + 12).**Theorem 3.3:** Let P₄ and P₄ be two path graphs.

Then product connectivity Gourava index of cartesian product $P_4 \square P_4$ is $PGO(G) = 1.46 + \frac{2m+2n-12}{\sqrt{54}} + \frac{2m+2n-8}{\sqrt{84}} + \frac{2m+2n-8}{$ 2nm - 5m - 5n + 12

 $\sqrt{128}$ **Proof.** Partition the edge set E(G) in four sets E_1 , E_2 , E_3 and E_4 as $|E_{23}| = 8$, $|E_{33}| = 2m + 2n - 12$, $|E_{34}| = 2m + 2n - 8$ and $|E_{44}| = 2mn-5m-5n+12$, table (3).

Therefore $PGO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_u + d_v)d_u d_v}}$

$$\begin{split} &= \sum_{23 \in E_1} \left[\frac{1}{\sqrt{[(2+3)2*3]}} \right] + \sum_{33 \in E_2} \left[\frac{1}{\sqrt{[(3+3)3*3]}} \right] + \sum_{34 \in E_3} \left[\frac{1}{\sqrt{[(3+4)3*4]}} \right] + \sum_{44 \in E_4} \left[\left(\frac{1}{\sqrt{[(4+4)4*4]}} \right] \\ &= 1.46 + \frac{2m + 2n - 12}{\sqrt{54}} + \frac{2m + 2n - 8}{\sqrt{84}} + \frac{2nm - 5m - 5n + 12}{\sqrt{128}}. \end{split}$$

 $\sqrt{128}$

Theorem 3.4: Sum connectivity Gourava index of cartesian product $P_4 \square P_4 = SGO(G) = 2.421 + \frac{(2m+2n-12)}{\sqrt{15}} + \frac{1}{\sqrt{15}}$

 $\frac{(2m+2n-8)}{\sqrt{10}} + \frac{(2nm-5m-5n+12)}{\sqrt{24}}$ $\sqrt{19}$ $\sqrt{24}$

 $\sqrt{54}$

Theorem 3.5: First hyper Gourava index of cartesian product $P_4 \square P_4 = HGO_1(G) = 968 + 225(2m + 2n - 12) + 120(2m + 2n -$ 361(2m + 2n - 8) + 16384(2nm - 5m - 5n + 12).

Theorem 3.6: Second hyper Gourava index of cartesian product $P_4 \square P_4 = HGO_2(G) = 7200 + 2916(2m + 2n - 2m + 2m))$ 12) + 7056(2m + 2n - 8) + 16384(2nm - 5m - 5n + 12).

du, dv	1,4	2,2	2,4	4,4
Number of edges	4	4	8(n-3)	$2(n-3)^2$

Table 1. Edge partition of tensor product $P_4 \otimes P_4$.

 $\sqrt{84}$

du, dv	(m,m)	(m, n+m-1)	(n+m-1, n+m-1)
Number of edges	n ^m C ₂	nm	ⁿ C ₂

Table 2. Edge partition of corona product K_n⊙K_m.

$m_{2,3}$	$m_{3,3}$	$m_{3,4}$	$m_{4,4}$
8	$2(n_1+n_2-6)$	$2(n_1+n_2-4)$	$2n_1n_2 - 5n_1 - 5n_2 + 12$
	<u> </u>	-	

Table 3. Edge partition of cartesian product $Pn_1 \Box Pn_2$.

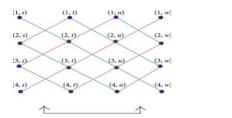


Fig. 1. Tensor product of P₄ and P₄

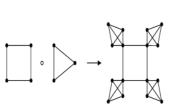


Fig.2. Corona product K4 and K3

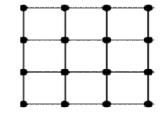


Fig.3. Cartesian product of P_4 and P_4 .

4. Conclusion

Tensor product, corona product and cartesian product of different versions of Gourava index are obtained. If partition of edge set E(G) is known then the degree based topological indices and graph operations of any molecular graph can be computed.

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