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An Introduction To Micro Vague Topological Space

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Abstract: The purpose of this paper is to define and study the new hybrid topological space called the Micro Vague Topological Space. Also, Micro Vague Pre-Open, Micro Vague Pre-Closed, Micro Vague Semi Open, Micro Vague Semi Closed sets are introduced and their properties are investigated.

Index Terms **-** Micro Vague Topological Space, Micro Vague Pre-Open set, Micro Vague Pre-Closed set, Micro Vague Semi Open set, Micro Vague Semi Closed set

I. Introduction

Nano Topology was first introduced by M. Lellis Thivagar [1] in the year 2013. Nano Topology is based on the concept of lower approximation, upper approximation and boundary region. Nano Topology contains maximum 5 Nano open sets or minimum 3 Nano open sets including the universal set and the empty set. Later in 2019 S. Chandrasekar [3] defined the Micro Topology by applying the simple extension concept in Nano Topology.

Gau and Buehrer [2] in the year 1993 presented Vague set as a set of objects each of which has a grade of membership whose value is a continuous subinterval of [0, 1]. Such a set is characterized by a truth membership function and a false membership function in the interval [0,1]. By combining Nano Topology and Vague Topology, N. Gayathri and M. Helen [4] in 2020 established the concept of Nano Vague Topological Space.

Now by applying the simple extension concept on Nano Vague Topology, we present the new hybrid topology called the Micro Vague Topology. Also, the Micro Vague Pre-Open, Micro Vague Pre-Closed, Micro Vague Semi Open, Micro Vague Semi Closed sets are defined and their properties are investigated.

II. Preliminaries

Definition 2.1 [5]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U.

1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is,
   $L_R(X)= \cup_{x\in U}\{R(x) : R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by $x \in U$.
2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classifieds as X with respect to R and it is denoted by $UR(X)$. That is,
   $U_R(X) = \cup_{x\in U}\{R(x) : R(x) \cap X = \varphi\}$

3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as $B_R(X)$ denotes not-X with respect to R and it. That is,
$B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 [5]

Let (U, R) be the approximation space and let $X, Y \subseteq U$. Then the following conditions hold:

i. $L_R(X) \subseteq X \subseteq U_R(X)$
ii. $L_R(\phi) = U_R(\phi) = \phi$
iii. $L_R(U) = U_R(U) = U$
iv. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
v. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
vi. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
vii. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
viii. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
ix. $U_R(X^C) = [L_R(X)]^C$
x. $L_R(X^C) = [U_R(X)]^C$
xi. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
xii. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.3 [1]

Let U be an universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$ satisfies the following axioms

1. U, $\phi \in \tau_R(X)$
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is called the Nano topology on U with respect to X. The space $(U, \tau_R(X))$ is the Nano topological space. The elements of $\tau_R(X)$ are called Nano open sets.

Definition 2.4 [3]

$(U, \tau_R(X))$ is a Nano topological space here $\mu_R(X) = \{N \cup (N' \cap \mu)\}$: N, $N' \in \tau_R(X)$ and is called Micro topology of $\tau_R(X)$ by μ where $\mu \notin \tau_R(X)$. The Micro topology $\mu_R(X)$ satisfies the following axioms

(1) U, $\phi \in \mu_R(X)$.
(2) The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.
(3) The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet (U, $\tau_R(X), \mu_R(X)$) is called Micro topological spaces and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.5 [3]

- Mic-int(A) = ∪{G / G is a Mic-OS in X and G⊆A}.
- Mic-cl(A) = ∩{K / K is a Mic-CS in X and A⊆K}.

Definition 2.6 [3]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro Topological Space and A ⊆ U, then,

- Micro-Pre-open if A ⊆ Mic-int (Mic-cl(A)) and
- Micro-Pre-closed set if Mic-cl (Mic-int(A)) ⊆ A.

Definition 2.7 [3]

Let $(U, \tau_R(X), \mu_R(X))$ be a Micro-topological space. and A ⊆ U. Then A is said to be

- Micro-semi open, if A ⊆ Mic-cl (Mic-int(A)).
- Micro-semi closed, if Mic-int (Mic-cl(A)) ⊆ A.

Definition 2.8 [9]

Let X be a non-empty set in the universe of discourse. A fuzzy set AF on X is defined as follows:

$$A_F = \{< \mathbf{x}, \boldsymbol{\delta}_\mathbf{A}(\mathbf{x}) > \mathbf{x} \in \mathbf{X}$$

Where $\boldsymbol{\delta}_\mathbf{A}(\mathbf{x}): \mathbf{X} \to [\mathbf{0}, \mathbf{1}]$ is represented as grade of membership function of the fuzzy set A_F.

Definition 2.9 [7]

Let X be a non-empty set in the universe of discourse. An Intuitionistic Fuzzy set A_{IF} on X is defined as follows:

$$A_{IF} = \{< \mathbf{x}, \boldsymbol{\delta}_\mathbf{A}(\mathbf{x}), \boldsymbol{\rho}_\mathbf{A}(\mathbf{x}) > \mathbf{x} \in \mathbf{X}$$

Where $\boldsymbol{\delta}_\mathbf{A}(\mathbf{x}): \mathbf{X} \to [\mathbf{0}, \mathbf{1}]$ and $\boldsymbol{\rho}_\mathbf{A}(\mathbf{x}): \mathbf{X} \to [\mathbf{0}, \mathbf{1}]$ represents the degree of membership function and degree of non-membership function of the element $\mathbf{x} \in \mathbf{X}$ to the set AIF, which is a subset of X. and also for every element $\mathbf{x} \in \mathbf{X}$, the functions $\boldsymbol{\delta}_\mathbf{A}(\mathbf{x})$ and $\boldsymbol{\rho}_\mathbf{A}(\mathbf{x})$ are ranges $\mathbf{0} \leq \boldsymbol{\delta}_\mathbf{A}(\mathbf{x}) + \boldsymbol{\rho}_\mathbf{A}(\mathbf{x}) \leq \mathbf{1}$.

Definition 2.10 [2]

A Vague set V in the universe of discourse X is characterized by a truth membership function t_v, and a falsity membership function f_v as follows:

$$t_v: X \to [0,1]; \; f_v: X \to [0,1] \text{ and } t_v + f_v \leq 1,$$

where $t_v(x)$ is a lower bound on the grade of membership of x derived from the evidence for x and $f_v(x)$ is a lower bound on the grade of membership of the negation of x derived from the evidence against x. The Vague set A is written as

$$A = \{\langle x, t_A(x), 1 - f_A(x)\rangle | x \in X\}.$$

III. Micro Vague Topological Space

Definition 3.1

Let U be a non-empty finite set of objects called the Universe and R be an equivalence relation on U. Let A be a Vague set in U with truth membership function μ_A and false membership function σ_A (for the sake of simplicity, we use the notation $\gamma_A(x) = 1 - \sigma_A(x)$). The Vague lower approximation, Vague upper approximation and the Vague Boundary region of A in the vague approximation space (U, R) denoted by $\underline{V(A)}$, $\overline{V(A)}$ and $B_V(A)$ respectively are defined as follows:

i. $\underline{V(A)} = \{\langle x, (\mu_{\underline{A}}(x), \gamma_{\underline{A}}(x))\rangle / y \in [x]_R, x \in U\}$

ii. $\overline{V(A)} = \{\langle x, (\mu_{\overline{A}}(x), \gamma_{\overline{A}}(x))\rangle / y \in [x]_R, x \in U\}$

iii. $B_V(A) = \overline{V(A)} - \underline{V(A)}$

Where, $\mu_{\underline{A}}(x) = \wedge_{y \in [x]_R} \mu_A(y)$, $\gamma_{\underline{A}}(x) = \wedge_{y \in [x]_R} \gamma_A(y)$

$\mu_{\overline{A}}(x) = \vee_{y \in [x]_R} \mu_A(y)$, $\gamma_{\overline{A}}(x) = \vee_{y \in [x]_R} \gamma_A(y)$

Definition 3.2

Let U be a Universe and R be an equivalence relation on U. Let A be a Vague set in U and if the collection $\tau_R(A) = \{0_V, 1_V, \underline{V(A)}, \overline{V(A)}, B_V(A)\}$ forms a topology then it is said to be a Nano Vague Topology. We call $(U, \tau_R(A))$ as the Nano Vague Topological Space. The elements of $\tau_R(A)$ are called Nano Vague Open sets.

Definition 3.3

Let $(U, \tau_R(A))$ be a Nano Vague Topological Space. Let $\eta_R(A) = \{S \cup (S' \cap \eta): S, S' \in \tau_R(A)\ and\ \eta \notin \tau_R(A)\}$. Then $\eta_R(A)$ is called the Micro Vague Topology of $\tau_R(A)$ by η if it satisfies the following axioms:

1. $0_{\mathcal{MV}}, 1_{\mathcal{MV}} \in \eta_R(A)$
2. The union of the elements of any sub collection of $\eta_R(A)$ is in $\eta_R(A)$
3. The intersection of the elements of any finite sub collection of $\eta_R(A)$ is in $\eta_R(A)$.

Now, $\eta_R(A)$ is called the Micro Vague Topology (Shortly $\mathcal{MV}$ Topology) on U with respect to A. The triplet $(U, \tau_R(A), \eta_R(A))$ is called the Micro Vague Topological Space. The elements of $\eta_R(A)$ are called Micro Vague open sets and the complement of a Micro Vague Open set is called a Micro Vague Closed set.

Example

Let $U = \{\rho, \lambda, \theta\}$ be the universe of discourse. Let $U/R = \{\{\rho\}, \{\lambda, \theta\}\}$ be an equivalence relation on U. Let $A = \{< (\rho, 0.3, 0.7) >, < \lambda, (0.4, 0.8) >, < \theta, (0.5, 0.5) >\}$ be a subset of U. Then $\tau_R(A) = \{0_{NV}, 1_{NV}, \{< \rho, (0.3,0.7) >, < \lambda, (0.4,0.5) >, < \theta, (0.4,0.5) >\}, \{< \rho, (0.3,0.7) >, < \lambda, (0.5,0.8) >, < \theta, (0.5,0.8) >\}, \{< \rho, (0.3,0.7) >, < \lambda, (0.5,0.6) >, < \theta, (0.5,0.6) >\}\}$ is a Nano Vague Topology on U. Let $\eta = \{< \rho, (0.5,0.8) >, < \lambda, (0.3,0.5) >, < \theta, (0.2,0.4) >\}$. Then, $\eta_R(A) = \{0_{\mathcal{MV}}, 1_{\mathcal{MV}}, \{< \rho, (0.3,0.7) >, < \lambda, (0.4,0.5) >, < \theta, (0.4,0.5) >\}, \{< \rho, (0.3,0.7) >, < \lambda, (0.5,0.8) >, < \theta, (0.5,0.8) >\}, \{\rho, (0.3,0.7) >, < \lambda, (0.5,0.6) >, < \theta, (0.5,0.6) >\}, \{< \rho, (0.5,0.8) >, < \lambda, (0.3,0.5) >, < \theta, (0.2,0.4) >\}, \{< \rho, (0.3,0.7) >, < \lambda, (0.3,0.5) >, < \theta, (0.2,0.4) >\}, \{< \rho, (0.5,0.8) >, < \lambda, (0.4,0.5) >, < \theta, (0.4,0.5) >\}, \{< \rho, (0.5,0.8) >, < \lambda, (0.5,0.8) >, < \theta, (0.5,0.8) >\}, \{< \rho, (0.5,0.8) >, < \lambda, (0.5,0.6) >, < \theta, (0.5,0.6) >\}\}$ is called the Micro Vague Topology on U with respect to A and the elements in the Micro Vague Topology are called Micro Vague Open sets. The triplet $(U, \tau_R(A), \eta_R(A))$ is called as the Micro Vague Topological Space.

Definition 3.4

Let U be the Universe and $X \subseteq U$. Let $\mathcal{G}$ and $\mathcal{H}$ be two $\mathcal{MV}$ sets in the $\mathcal{MV}$ Topological Space $(U, \tau_R(X), \eta_R(X))$ of the form $\mathcal{G} = \{\langle x, [\mu_{\mathcal{G}}(x), \gamma_{\mathcal{G}}(x)]\rangle / x \in X\}$ and $\mathcal{H} = \{\langle x, [\mu_{\mathcal{H}}(x), \gamma_{\mathcal{H}}(x)]\rangle / x \in X\}$ respectively. Then the following conditions holds:

(i) $\mathcal{G} \subseteq \mathcal{H}\ iff\ \mu_{\mathcal{G}}(x) \leq \mu_{\mathcal{H}}(x), \gamma_{\mathcal{G}}(x) \leq \gamma_{\mathcal{H}}(x)\ \ \forall x \in U$

(ii) $\mathcal{G} = \mathcal{H}\ iff\ \mathcal{G} \subseteq \mathcal{H}\ and\ \mathcal{H} \subseteq \mathcal{G}$

(iii)$\mathcal{G}^C = \{\langle x, 1 - \gamma_{\mathcal{G}}(x), 1 - \mu_{\mathcal{G}}(x)\rangle / \forall x \in U\}$

(iv)$\mathcal{G} \cup \mathcal{H} = \{\langle x, (\mu_{\mathcal{G}}(x) \vee \mu_{\mathcal{H}}(x),\ \gamma_{\mathcal{G}}(x) \vee \gamma_{\mathcal{H}}(x))\rangle / \forall x \in U\}$

(v) $\mathcal{G} \cap \mathcal{H} = \{\langle x, (\mu_{\mathcal{G}}(x) \wedge \mu_{\mathcal{H}}(x),\ \gamma_{\mathcal{G}}(x) \wedge \gamma_{\mathcal{H}}(x))\rangle / \forall x \in U\}$

(vi)$0_{\mathcal{MV}} = \langle x, (0,0)\rangle$ and $1_{\mathcal{MV}} = \langle x, (1,1)\rangle\ \forall x \in U$

Remarks

1. In Micro Vague Topological Space, the boundary region cannot be empty.
2. Let $\{\eta_i | i \in \mathbb{I}\}$ be the family of Micro Vague topologies on X_i, then $\cap_{i \in \mathbb{I}} \eta_i$ is a Micro vague topology in X.
3. Let $(U, \tau_R(X), \eta_R(X))$ and $(U, \tau_R(X), \eta'_R(X))$ be two Micro Vague topological spaces over X. Then $(U, \tau_R(X), \eta_R(X) \cup \eta'_R(X))$ need not to be a Micro Vague Topological space.

Proposition 3.1

Let $\mathcal{J}, \mathcal{K}$ and $\mathcal{L}$ be the Micro Vague sets in the Micro Vague Topological Space $(U, \tau_R(X), \eta_R(X))$. Then the following properties hold:

1. $\mathcal{J} \cup 0_{\mathcal{MV}} = \mathcal{J}$
2. $\mathcal{J} \cup 1_{\mathcal{MV}} = 1_{\mathcal{MV}}$
3. $\mathcal{J} \cap 0_{\mathcal{MV}} = 0_{\mathcal{MV}}$
4. $\mathcal{J} \cap 1_{\mathcal{MV}} = \mathcal{J}$

5. Commutative: $\mathcal{J} \cup \mathcal{K} = \mathcal{K} \cup \mathcal{J}$
6. Associative: (i) $\mathcal{J} \cup (\mathcal{K} \cup \mathcal{L}) = (\mathcal{J} \cup \mathcal{K}) \cup \mathcal{L}$
   (ii) $\mathcal{J} \cap (\mathcal{K} \cap \mathcal{L}) = (\mathcal{J} \cap \mathcal{K}) \cap \mathcal{L}$
7. Distributive: (i) $\mathcal{J} \cup (\mathcal{K} \cap \mathcal{L}) = (\mathcal{J} \cup \mathcal{K}) \cap (\mathcal{J} \cup \mathcal{L})$
   (ii) $\mathcal{J} \cap (\mathcal{K} \cup \mathcal{L}) = (\mathcal{J} \cap \mathcal{K}) \cup (\mathcal{J} \cap \mathcal{L})$

Proof: The results are straightforward

Proposition 3.2

Let $(U, \tau_R(X), \eta_R(X))$ be a Micro Vague Topological Space. The elements in $\eta_R(X)$ are called Micro Vague Open sets and the complement of the Micro Vague Open sets are called Micro Vague Closed sets. Then,

a. $0_{\mathcal{MV}}$ and $1_{\mathcal{MV}}$ are Micro Vague Closed sets

b. The arbitrary Union of Micro Vague Closed sets on η over U with respect to X is also a Micro Vague closed set over U with respect to X

c. The intersection of any Micro Vague Closed sets on η over U with respect to X is a Micro Vague Closed set

Proof: The proof is obvious from the definition itself

Definition 3.5

Let $(U, \tau_R(X), \eta_R(X))$ be a Micro Vague Topological Space over X. Let $\mathcal{P}$ be a $\mathcal{MV}$ set, then the Micro Vague interior of $\mathcal{P}$ is defined as the union of all Micro Vague Open sets contained in $\mathcal{P}$.

(i.e) $MVint(\mathcal{P}) = \vee\{\text{G: G is a } \mathcal{MV} \text{ open set and G} \subseteq \mathcal{P}\}$.

Clearly, $\mathcal{MV}$int($\mathcal{P}$) is the largest $\mathcal{MV}$ open set that is contained in $\mathcal{P}$.

Definition 3.6

Let $(U, \tau_R(X), \eta_R(X))$ be a Micro Vague Topological Space over X. Let $\mathcal{Q}$ be a $\mathcal{MV}$ set, then the Micro Vague closure of $\mathcal{Q}$ is defined as the intersection of all Micro Vague Closed sets containing $\mathcal{Q}$.

(i.e) $MVcl(\mathcal{Q}) = \wedge\{\text{K: K is } \mathcal{MV} \text{ closed set and } \mathcal{Q} \subseteq \text{K}\}$.

Clearly, $\mathcal{MV}$cl($\mathcal{Q}$) is the smallest $\mathcal{MV}$ closed set that contains $\mathcal{Q}$.

Proposition 3.3

For any two $\mathcal{MV}$ sets $\mathcal{K}$ and $\mathcal{L}$ in $\mathcal{MV}$ Topological space $(U, \tau_R(X), \eta_R(X))$, the following statements hold:

1. $\mathcal{K}$ is $\mathcal{MV}$ Closed set if and only if $\mathcal{MV}$-cl($\mathcal{K}$) = $\mathcal{K}$
2. $\mathcal{K}$ is $\mathcal{MV}$ Open set if and only if $\mathcal{MV}$-int($\mathcal{K}$) = $\mathcal{K}$
3. $\mathcal{K} \subseteq \mathcal{L}$ implies $\mathcal{MV}$-int($\mathcal{K}$) $\subseteq$ $\mathcal{MV}$-int($\mathcal{L}$) and $\mathcal{MV}$-cl($\mathcal{K}$) $\subseteq$ $\mathcal{MV}$-cl($\mathcal{L}$)
4. $\mathcal{MV}$-cl($\mathcal{MV}$-cl($\mathcal{K}$)) = $\mathcal{MV}$-cl($\mathcal{K}$) and $\mathcal{MV}$-int($\mathcal{MV}$-int($\mathcal{K}$)) = $\mathcal{MV}$-int($\mathcal{K}$)

Proof

1. If $\mathcal{K}$ is a $\mathcal{MV}$ closed set, then $\mathcal{K}$ is the smallest $\mathcal{MV}$ closed set containing itself and hence $\mathcal{MV}$-cl($\mathcal{K}$) = $\mathcal{K}$. Conversely if $\mathcal{MV}$-cl($\mathcal{K}$) = $\mathcal{K}$, then $\mathcal{K}$ is the smallest $\mathcal{MV}$ closed set containing itself and hence $\mathcal{K}$ is $\mathcal{MV}$ closed set.
2. Let $\mathcal{K}$ be a Micro Vague Open set in the Micro Vague Topological space $(U, \tau_R(A), \eta_R(A))$. We know that $\mathcal{MV}$-int($\mathcal{K}$) of any set is a subset of the set $\mathcal{K}$. So, $\mathcal{MV}$-int($\mathcal{K}$) $\subseteq \mathcal{K}$. Since, $\mathcal{K}$ is a Micro Vague open set, we have $\mathcal{K} \subseteq$ $\mathcal{MV}$-int($\mathcal{K}$). Therefore, $\mathcal{MV}$-int($\mathcal{K}$)= $\mathcal{K}$. Conversely suppose if $\mathcal{MV}$-int($\mathcal{K}$) = $\mathcal{K}$, then since $\mathcal{MV}$-int($\mathcal{K}$) is a $\mathcal{MV}$ open set, clearly $\mathcal{K}$ is also a $\mathcal{MV}$ open set.
3. Let $\mathcal{K} \subseteq \mathcal{L}$, then $1 - \mathcal{K} \subseteq 1 - \mathcal{L}$, this implies that $\mathcal{MV}$-cl $(1 - \mathcal{K}) \subseteq$ $\mathcal{MV}$-cl $(1 - \mathcal{L}) \Rightarrow$ $\mathcal{MV}$-int($\mathcal{K}$) $\subseteq$ $\mathcal{MV}$-int($\mathcal{L}$). Similarly, it is proved that $\mathcal{MV}$-cl($\mathcal{K}$) $\subseteq$ $\mathcal{MV}$-cl($\mathcal{L}$).

4. Since $\mathcal{MV}$-int($\mathcal{K}$) is a $\mathcal{MV}$ open set, $\mathcal{MV}$-int ($\mathcal{MV}$-int($\mathcal{K}$)) = $\mathcal{MV}$-int($\mathcal{K}$). Similarly, since $\mathcal{MV}$-int($\mathcal{K}$) is a $\mathcal{MV}$ closed set, then $\mathcal{MV}$-cl ($\mathcal{MV}$-cl($\mathcal{K}$)) = $\mathcal{MV}$-cl($\mathcal{K}$).

Proposition 3.4

For any $\mathcal{MV}$ set $\mathcal{K}$ in $\mathcal{MV}$ Topological space $(U, \tau_R(X), \eta_R(X))$, the following statements hold:

(i) $\mathcal{MV}$-cl ($\mathcal{K}^C$) = [$\mathcal{MV}$-int($\mathcal{K}$)]^C

(ii) $\mathcal{MV}$-int ($\mathcal{K}^C$) = [$\mathcal{MV}$-cl($\mathcal{K}$)]^C

Proof: The proof is obvious from the definitions.

Proposition 3.5

For any two $\mathcal{MV}$ sets $\mathcal{K}$ and $\mathcal{L}$ in $\mathcal{MV}$ Topological space $(U, \tau_R(X), \eta_R(X))$, the following statements hold:

1. $\mathcal{MV}$-cl ($\mathcal{K} \cup \mathcal{L}$) = $\mathcal{MV}$-cl ($\mathcal{K}$) ∪ $\mathcal{MV}$-cl ($\mathcal{L}$)
2. $\mathcal{MV}$-cl ($\mathcal{K} \cap \mathcal{L}$) ⊆ $\mathcal{MV}$-cl($\mathcal{K}$) ∩ $\mathcal{MV}$-cl ($\mathcal{L}$)
3. $\mathcal{MV}$-int ($\mathcal{K} \cup \mathcal{L}$) ⊇ $\mathcal{MV}$-int($\mathcal{K}$) ∪ $\mathcal{MV}$-int ($\mathcal{L}$)
4. $\mathcal{MV}$-int ($\mathcal{K} \cap \mathcal{L}$) = $\mathcal{MV}$-int($\mathcal{K}$) ∩ $\mathcal{MV}$-int ($\mathcal{L}$)

Proof

1. Since $\mathcal{K} \subseteq \mathcal{K} \cup \mathcal{L}$ and $\mathcal{L} \subseteq \mathcal{K} \cup \mathcal{L}$, then $\mathcal{MV}$-cl($\mathcal{K}$) ⊆ $\mathcal{MV}$-cl($\mathcal{K} \cup \mathcal{L}$) and $\mathcal{MV}$-cl($\mathcal{L}$) ⊆ $\mathcal{MV}$-cl($\mathcal{K} \cup \mathcal{L}$). Therefore, $\mathcal{MV}$-cl($\mathcal{K}$) ∪ $\mathcal{MV}$-cl($\mathcal{L}$) ⊆ $\mathcal{MV}$-cl($\mathcal{K} \cup \mathcal{L}$).
   Conversely since $\mathcal{K} \subseteq$ $\mathcal{MV}$-cl($\mathcal{K}$) and $\mathcal{L} \subseteq$ $\mathcal{MV}$-cl($\mathcal{L}$), then $\mathcal{K} \cup \mathcal{L} \subseteq$ $\mathcal{MV}$-cl($\mathcal{K}$) ∪ $\mathcal{MV}$-cl($\mathcal{L}$). Besides $\mathcal{MV}$-cl($\mathcal{K} \cup \mathcal{L}$) is the smallest $\mathcal{MV}$ closed set that containing $\mathcal{K} \cup \mathcal{L}$. Therefore, $\mathcal{MV}$-cl ($\mathcal{K} \cup \mathcal{L}$) ⊆ $\mathcal{MV}$-cl($\mathcal{K}$) ∪ $\mathcal{MV}$-cl($\mathcal{L}$). Thus, $\mathcal{MV}$-cl($\mathcal{K} \cup \mathcal{L}$) = $\mathcal{MV}$-cl($\mathcal{K}$) ∪ $\mathcal{MV}$-cl($\mathcal{L}$).
2. Since, $\mathcal{K} \cap \mathcal{L} \subseteq$ $\mathcal{MV}$-cl($\mathcal{K}$) ∩ $\mathcal{MV}$-cl($\mathcal{L}$) and $\mathcal{MV}$-cl($\mathcal{K} \cap \mathcal{L}$) is the smallest $\mathcal{MV}$ closed set that containing $\mathcal{K} \cap \mathcal{L}$, then $\mathcal{MV}$-cl($\mathcal{K} \cap \mathcal{L}$) ⊆ $\mathcal{MV}$-cl($\mathcal{K}$) ∩ $\mathcal{MV}$-cl($\mathcal{L}$)
3. Since $\mathcal{K} \subseteq \mathcal{K} \cup \mathcal{L}$ and $\mathcal{L} \subseteq \mathcal{K} \cup \mathcal{L}$, then $\mathcal{MV}$-int($\mathcal{K}$) ⊆ $\mathcal{MV}$-int($\mathcal{K} \cup \mathcal{L}$) and $\mathcal{MV}$-int($\mathcal{L}$) ⊆ $\mathcal{MV}$-int($\mathcal{K} \cup \mathcal{L}$). Therefore, $\mathcal{MV}$-int($\mathcal{K}$) ∪ $\mathcal{MV}$-int($\mathcal{L}$) ⊆ $\mathcal{MV}$-int ($\mathcal{K} \cup \mathcal{L}$).
   Conversely since $\mathcal{K} \subseteq$ $\mathcal{MV}$-int($\mathcal{K}$) and $\mathcal{L} \subseteq$ $\mathcal{MV}$-int($\mathcal{L}$), then $\mathcal{K} \cup \mathcal{L} \subseteq$ $\mathcal{MV}$-int($\mathcal{K}$) ∪ $\mathcal{MV}$-int($\mathcal{L}$). Besides $\mathcal{MV}$-int($\mathcal{K} \cup \mathcal{L}$) is the largest $\mathcal{MV}$ open set that contained in $\mathcal{K} \cup \mathcal{L}$. Therefore, $\mathcal{MV}$-int($\mathcal{K} \cup \mathcal{L}$) ⊆ $\mathcal{MV}$-int($\mathcal{K}$) ∪ $\mathcal{MV}$-int($\mathcal{L}$). Thus, $\mathcal{MV}$-int($\mathcal{K} \cup \mathcal{L}$) ⊇ $\mathcal{MV}$-int($\mathcal{K}$) ∪ $\mathcal{MV}$-int($\mathcal{L}$).
4. Since $\mathcal{K} \cap \mathcal{L} \subseteq \mathcal{K}$ and $\mathcal{K} \cap \mathcal{L} \subseteq \mathcal{L}$, then $\mathcal{MV}$-int($\mathcal{K} \cap \mathcal{L}$) ⊆ $\mathcal{MV}$-int($\mathcal{K}$) and $\mathcal{MV}$-int($\mathcal{K} \cap \mathcal{L}$) ⊆ $\mathcal{MV}$-int($\mathcal{L}$). So, $\mathcal{MV}$-int($\mathcal{K} \cap \mathcal{L}$) ⊆ $\mathcal{MV}$-int($\mathcal{K}$) ∩ $\mathcal{MV}$-int($\mathcal{L}$).
   On the other hand, since $\mathcal{MV}$-int($\mathcal{K}$) ⊆ $\mathcal{K}$ and $\mathcal{MV}$-int($\mathcal{L}$) ⊆ $\mathcal{L}$, then $\mathcal{MV}$-int($\mathcal{K}$) ∩ $\mathcal{MV}$-int($\mathcal{L}$) ⊆ $\mathcal{K} \cap \mathcal{L}$. Besides $\mathcal{MV}$-int($\mathcal{K} \cap \mathcal{L}$) ⊆ $\mathcal{K} \cap \mathcal{L}$ and it is the biggest $\mathcal{MV}$ open set that contained in $\mathcal{K} \cap \mathcal{L}$. Therefore, $\mathcal{MV}$-int($\mathcal{K}$) ∩ $\mathcal{MV}$-int($\mathcal{L}$) ⊆ $\mathcal{MV}$-int($\mathcal{K} \cap \mathcal{L}$). Thus $\mathcal{MV}$-int($\mathcal{K} \cap \mathcal{L}$) = $\mathcal{MV}$-int($\mathcal{K}$) ∩ $\mathcal{MV}$-int($\mathcal{L}$).

IV. MICRO VAGUE PRE-OPEN SETS

Definition 4.1

Let $(U, \tau_R(X), \eta_R(X))$ be a $\mathcal{MV}$ Topological Space and $Q \subseteq U$. Then, Q is said to be $\mathcal{MV}$ Pre-Open if $Q \subseteq$ $\mathcal{MV}$-int ($\mathcal{MV}$-cl(Q)) and $\mathcal{MV}$ Pre closed if $\mathcal{MV}$-cl ($\mathcal{MV}$-int(Q)) ⊆ Q.

Definition 4.2

Let $(U, \tau_R(X), \eta_R(X))$ be a $\mathcal{MV}$ Topological Space and $Q \subseteq U$. The Union of all $\mathcal{MV}$-Pre-open sets contained in Q is called $\mathcal{MV}$-pre-interior of Q and it is denoted by $\mathcal{MV}$-pre-int (Q). (i.e) $\mathcal{MV}$-pre-int (Q) = ∪ {B / B is $\mathcal{MV}$-pre-open and B ⊆ Q}.

Definition 4.3

Let $(U, \tau_R(X), \eta_R(X))$ be a $\mathscr{MV}$ Topological Space and $Q \subseteq U$. The intersection of all $\mathscr{MV}$-pre closed sets containing Q is called $\mathscr{MV}$-pre-closure of Q and it is denoted by $\mathscr{MV}$-pre-cl(Q). (i.e) $\mathscr{MV}$-pre-cl(Q) = ∩ {B / B is $\mathscr{MV}$-pre closed and $Q \subseteq$ B}.

Theorem 4.4

1. Arbitrary union of $\mathscr{MV}$-Pre-open sets is $\mathscr{MV}$-Pre-open.
2. Arbitrary intersection of $\mathscr{MV}$-pre closed sets is $\mathscr{MV}$-Pre closed.

Proof: 1. Let $\{\mathcal{P}_\alpha \mid \alpha \in I\}$ be the family of $\mathscr{MV}$-Pre-Open sets in X. By definition, for each α, $\mathcal{P}_\alpha \subseteq \mathscr{MV}$-int ($\mathscr{MV}$-cl($\mathcal{P}_\alpha$)), this implies that $\cup \mathcal{P}_\alpha \subseteq \cup\{\mathscr{MV}$-int ($\mathscr{MV}$-cl ($\mathcal{P}_\alpha$))}. Since ∪($\mathscr{MV}$-int($\mathscr{MV}$-cl($\mathcal{P}_\alpha$))) ⊆ $\mathscr{MV}$-int(∪{$\mathscr{MV}$-cl($\mathcal{P}_\alpha$)}) and $\mathscr{MV}$-int(∪{$\mathscr{MV}$-cl($\mathcal{P}_\alpha$)}) = $\mathscr{MV}$-int($\mathscr{MV}$-cl($\cup \mathcal{P}_\alpha$)), this implies that $\cup \mathcal{P}_\alpha \subseteq \mathscr{MV}$-int($\mathscr{MV}$-cl($\cup \mathcal{P}_\alpha$)). Hence, $\cup \mathcal{P}_\alpha$ is $\mathscr{MV}$-Pre-Open.

2. Let $\{Q_\alpha \mid \alpha \in I\}$ be the family of $\mathscr{MV}$-Pre-Closed sets in X. Let $\mathcal{P}_\alpha = (Q_\alpha)^C$, then $\{\mathcal{P}_\alpha \mid \alpha \in I\}$ is a family of $\mathscr{MV}$-Pre-Open Sets. By (1), $\cup \mathcal{P}_\alpha = (\cup Q_\alpha)^C$ is $\mathscr{MV}$-Pre-open. Consequently $\cap (Q_\alpha)^C$ is $\mathscr{MV}$-Pre-Open. Hence, $\cap Q_\alpha$ is $\mathscr{MV}$-Pre-Closed.

Theorem 4.5

1. Let $\mathcal{P} \subseteq (U, \tau_R(X), \eta_R(X))$. Then $\mathscr{MV}$-Pre int($\mathcal{P}$) is equal to the union of all $\mathscr{MV}$-Pre-Open set contained in $\mathcal{P}$.
2. If $\mathcal{P}$ is $\mathscr{MV}$-Pre-open set, then $\mathcal{P}$ = $\mathscr{MV}$-Pre int($\mathcal{P}$).

Proof: 1. We need to prove that $\mathscr{MV}$-Pre-int($\mathcal{P}$) = ∪{ $\mathcal{B}$ / $\mathcal{B}$ is $\mathscr{MV}$-Pre-open set and $\mathcal{B} \subseteq \mathcal{P}$}. Let $x \in \mathscr{MV}$-Pre-int($\mathcal{P}$). Then there exist a $\mathscr{MV}$-Pre-open set $\mathcal{B}$ such that $x \in \mathcal{B} \subseteq \mathcal{P}$. Hence, $x \in$ ∪{ $\mathcal{B}$ / $\mathcal{B}$ is $\mathscr{MV}$-Pre-open set and $\mathcal{B} \subseteq \mathcal{P}$}. Conversely suppose $x \in$ ∪{ $\mathcal{B}$ / $\mathcal{B}$ is $\mathscr{MV}$-Pre-open set and $\mathcal{B} \subseteq \mathcal{P}$} then there exist a set $\mathcal{B}_o \subseteq \mathcal{P}$ such that $x \in \mathcal{B}_o$, where $\mathcal{B}_o$ is $\mathscr{MV}$-Pre-open set. (i.e) $x \in \mathscr{MV}$-Pre-int($\mathcal{P}$). Hence, ∪{ $\mathcal{B}$ / $\mathcal{B}$ is $\mathscr{MV}$-Pre-open set and $\mathcal{B} \subseteq \mathcal{P}$} ⊆ $\mathscr{MV}$-Pre-int($\mathcal{P}$). So, $\mathscr{MV}$-Pre-int($\mathcal{P}$) = ∪{$\mathcal{B}$ / $\mathcal{B}$ is $\mathscr{MV}$-Pre-open set and $\mathcal{B} \subseteq \mathcal{P}$}.

2. Assume $\mathcal{P}$ is a $\mathscr{MV}$-Pre-open set then $\mathcal{P} \subseteq$ ∪{ $\mathcal{B}$ / $\mathcal{B}$ is $\mathscr{MV}$-Pre-open set and $\mathcal{B} \subseteq \mathcal{P}$} and every other element in this collection is subset of $\mathcal{P}$. Hence by (1) $\mathscr{MV}$-Pre-int($\mathcal{P}$) = $\mathcal{P}$.

Theorem 4.6

1. $\mathscr{MV}$-Pre-Int($\mathcal{P} \cup Q$) ⊇ $\mathscr{MV}$-Pre-int($\mathcal{P}$) ∪ $\mathscr{MV}$-Pre-int(Q)
2. $\mathscr{MV}$-Pre-int($\mathcal{P} \cap Q$) = $\mathscr{MV}$-Pre-int($\mathcal{P}$) ∩ $\mathscr{MV}$-Pre-int(Q)

Proof: 1. The fact that $\mathscr{MV}$-Pre-int($\mathcal{P}$) ⊆ $\mathcal{P}$ and $\mathscr{MV}$-Pre-int(Q) ⊆ Q implies that $\mathscr{MV}$-Pre-int($\mathcal{P}$) ∪ $\mathscr{MV}$-Pre-int(Q) ⊆ $\mathcal{P} \cup Q$. Since, $\mathscr{MV}$-Pre-interior of a set is $\mathscr{MV}$-Pre-open, $\mathscr{MV}$-Pre-int($\mathcal{P}$) and $\mathscr{MV}$-Pre-int(Q) are pre-open. Hence, by theorem (4.5), $\mathscr{MV}$-Pre-int($\mathcal{P}$) ∪ $\mathscr{MV}$-Pre-int(Q) is $\mathscr{MV}$-Pre-open and contained in $\mathcal{P} \cup Q$. Since, $\mathscr{MV}$-Pre-int($\mathcal{P} \cup Q$) is the largest $\mathscr{MV}$-Pre-Open set contained in $\mathcal{P} \cup Q$, it follows that $\mathscr{MV}$-Pre-int($\mathcal{P}$) ∪ $\mathscr{MV}$-Pre-int(Q) ⊆ $\mathscr{MV}$-Pre-int($\mathcal{P} \cup Q$).

2. Let $x \in \mathscr{MV}$-Pre-int($\mathcal{P} \cap Q$). Then there exist a $\mathscr{MV}$-Pre-open set $\mathcal{H}$, such that $x \in \mathcal{H} \subseteq (\mathcal{P} \cap Q)$. That is there exist a $\mathscr{MV}$-Pre-open set, such that $x \in \mathcal{H} \subseteq \mathcal{P}$ and $x \in \mathcal{H} \subseteq Q$. Hence $x \in \mathscr{MV}$-Pre-int($\mathcal{P}$) and x ∈ $\mathscr{MV}$-Pre-int(Q). That is $x \in \mathscr{MV}$-Pre-int($\mathcal{P}$) ∩ $\mathscr{MV}$-Pre-int(Q). Thus $\mathscr{MV}$-Pre-int($\mathcal{P} \cap Q$) ⊆ $\mathscr{MV}$-Pre-int($\mathcal{P}$) ∩ $\mathscr{MV}$-Pre-int(Q). Retracing the above steps, we get the converse.

Theorem 4.7

Every $\mathscr{MV}$-Closed set is $\mathscr{MV}$-Pre-Closed.

Proof: Let $\mathcal{P}$ be a $\mathscr{MV}$-Closed, then we have $\mathscr{MV}$-cl($\mathscr{MV}$-cl($\mathcal{P}$)) ⊆ $\mathcal{P}$. Since $\mathscr{MV}$-cl($\mathscr{MV}$-int($\mathcal{P}$)) ⊆ $\mathscr{MV}$-cl($\mathscr{MV}$-cl($\mathcal{P}$)) ⊆ $\mathcal{P}$, $\mathcal{P}$ is $\mathscr{MV}$-Pre Closed.

Theorem 4.8

$\mathcal{P}$ is $\mathcal{MV}$-Pre closed iff $\mathcal{P}$ = $\mathcal{MV}$-Pre-cl($\mathcal{P}$).

Proof: $\mathcal{MV}$-Pre-cl($\mathcal{P}$) = ∩ {$\mathcal{B}$ / $\mathcal{B}$ is $\mathcal{MV}$-Pre-Closed Set and $\mathcal{P} \subseteq \mathcal{B}$}. If $\mathcal{P}$ is $\mathcal{MV}$-Pre-Closed set, then $\mathcal{P}$ is a member of the above collection and each member contains $\mathcal{P}$. Hence their intersection is $\mathcal{P}$ and $\mathcal{MV}$-Pre-cl($\mathcal{P}$) = $\mathcal{P}$. Conversely, if $\mathcal{P}$ = $\mathcal{MV}$-Pre-cl($\mathcal{P}$), then $\mathcal{P}$ is $\mathcal{MV}$-Pre-Closed is obvious.

V. Micro Vague Semi Open Sets

Definition 5.1

Let $(U, \tau_R(X), \eta_R(X))$ be a $\mathcal{MV}$ Topological Space and $\mathcal{P} \subseteq U$. Then, $\mathcal{P}$ is said to be $\mathcal{MV}$-Semi Open if $\mathcal{P} \subseteq$ $\mathcal{MV}$-cl($\mathcal{MV}$-int($\mathcal{P}$)) and $\mathcal{MV}$-Semi Closed if $\mathcal{MV}$-int($\mathcal{MV}$-cl($\mathcal{P}$)) $\subseteq \mathcal{P}$.

Definition 5.2

Let $(U, \tau_R(X), \eta_R(X))$ be a $\mathcal{MV}$ Topological Space and $\mathcal{P} \subseteq U$. The union of all $\mathcal{MV}$-Semi open sets contained in $\mathcal{P}$ is called $\mathcal{MV}$-Semi-interior of $\mathcal{P}$ and it is denoted by $\mathcal{MV}$-Semi-int($\mathcal{P}$). (i. e) $\mathcal{MV}$-Semi-int($\mathcal{P}$) = ∪ {$\mathcal{B}$ / $\mathcal{B}$ is $\mathcal{MV}$-semi open and $\mathcal{B} \subseteq \mathcal{P}$}. The set of all $\mathcal{MV}$-semi-interior points of $\mathcal{P}$ is called the $\mathcal{MV}$-semi-interior of $\mathcal{P}$ and it is denoted by $\mathcal{MV}$-semi-int($\mathcal{P}$).

Definition 5.3

Let $(U, \tau_R(X), \eta_R(X))$ be a $\mathcal{MV}$ Topological Space and $\mathcal{P} \subseteq U$. The intersection of all $\mathcal{MV}$-Semi closed sets containing $\mathcal{P}$ is called $\mathcal{MV}$-Semi-closure of $\mathcal{P}$ and it is denoted by $\mathcal{MV}$-Semi-cl($\mathcal{P}$). (i.e) $\mathcal{MV}$-Semi-cl($\mathcal{P}$) = ∩ {$\mathcal{B}$ / $\mathcal{B}$ is $\mathcal{MV}$-semi closed and $\mathcal{P} \subseteq \mathcal{B}$}.

Theorem 5.4

1. Every $\mathcal{MV}$-Open set is $\mathcal{MV}$-Semi open.
2. Every $\mathcal{MV}$-Closed set is $\mathcal{MV}$-Semi Closed.

Proof

1. If $\mathcal{P}$ is $\mathcal{MV}$-Open set, then $\mathcal{P} \subseteq$ $\mathcal{MV}$-int($\mathcal{MV}$-int($\mathcal{P}$)). Since $\mathcal{MV}$-int($\mathcal{MV}$-int($\mathcal{P}$)) $\subseteq$ $\mathcal{MV}$-cl($\mathcal{MV}$-int($\mathcal{P}$)), $\mathcal{P} \subseteq$ $\mathcal{MV}$-cl($\mathcal{MV}$-int($\mathcal{P}$)). Hence $\mathcal{P}$ is $\mathcal{MV}$-Semi Open.
2. If $\mathcal{P}$ is $\mathcal{MV}$-Closed set, then we have $\mathcal{MV}$-cl($\mathcal{MV}$-cl($\mathcal{P}$)) $\subseteq \mathcal{P}$. Since $\mathcal{MV}$-int($\mathcal{MV}$-cl($\mathcal{P}$)) $\subseteq$ $\mathcal{MV}$-cl($\mathcal{MV}$-cl($\mathcal{P}$)), $\mathcal{MV}$-int($\mathcal{MV}$-cl($\mathcal{P}$)) $\subseteq \mathcal{P}$. Hence $\mathcal{P}$ is $\mathcal{MV}$-Semi Closed.

VI. Conclusion

In this Paper, we have introduced the new hybrid topology called the Micro Vague Topological Space and the properties of Micro Vague open sets, Micro Vague Closed sets, Micro Vague Pre-open set, Micro Vague Pre closed set, Micro Vague Semi open set, Micro Vague Semi closed sets are discussed. This shall be extended in the future research with some applications.

References

[1]. M.L. Thivagar and C. Richard, On Nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, 1/1 (2013), 31-37.

[2]. W.L. Gau and D.J. Buehrer, Vague sets, IEEE Transactions on Systems Man and Cybernetics, 1993, 23(2), pp 610-614.

[3]. S. Chandrasekar, On Micro Topological Spaces, Journal of New Theory, 26(2019), 23-31.

[4]. N. Gayathri and M. Helen, New Topological Structure of Nano Topology Via Vague Topology, International Journal of Mathematical Archive, 11(6), 2020, 67-70.

[5]. Z. Pawlak, Rough Sets, International Journal of Computer and Information Sciences, 11, 341-356, 1982.

[6]. F. Smarandache, A unifying field in logics Neutrosophic probability, set and logic, Rehoboth American Research Press,1999.

[7]. Atanassov K.T., Intuitionistic fuzzy sets, Fuzzy sets and systems, vol 20, p.87-96, 1986.

[8]. Chang C.L., Fuzzy Topological spaces, J.Math.Anal.Appl., vol24, pl82-190,1968.
[9]. Zadeh L.A., Fuzzy Sets, Information and Control, p.338-335, 1965.
[10]. Biswas R, Vague Groups, Internat. J Comput. Cognition. 4(2) (2006), 20 – 23.
[11]. Bustince. H, Burillo. P, vague sets are intuitionistic fuzzy sets, Fuzzy sets and systems, 79 (1996), 403 – 405.