IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

Intuitionistic Fuzzy Soft Modules

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Abstract

Molodtsov (1999) initiated the concept of soft sets in [1]. Maji et al. (2003) defined some operations on soft sets in [22]. Aktas and Cagman defined the concept of generalized soft groups in [16]. Sun et al. (2008) has given the concept of soft modules in [21]. In this paper, intuitionistic fuzzy soft module concept is introduced along with some operations on intuitionistic fuzzy soft sets given. Finally some basic properties were discussed.

Keywords

Soft sets, Fuzzy soft set.
Intuitionistic fuzzy soft set, Intuitionistic fuzzy soft module.

1. Introduction

Molodtsov [1] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. After Molodtsov's work, some different applications of soft sets were studied in [2]. Maji et al. [3] presented the concept of fuzzy soft sets. The theory of fuzzy sets, first developed by Zadeh in [4], is the most appropriate framework for dealing with uncertainties. The concept of intuitionistic fuzzy sets which is a generalization of fuzzy sets was introduced by Atanassov in [13]. Davvaz [14] defined the concept of intuitionistic fuzzy module by using intuitionistic fuzzy sets. Gunduz (Aras) and Davvaz in [15] did some investigations on intuitionistic fuzzy modules. The definition of fuzzy soft groups was given by some authors [19,20]. Qiu-Mei Sun et al. [21] defined soft modules and investigated their basic properties.

The main purpose of this paper is to introduce a basic version of intuitionistic fuzzy soft module theory, which extends the notion of modules by including some algebraic structures in soft sets. Finally, we investigate some basic properties of intuitionistic fuzzy soft modules.

2. Preliminaries

In this section, we recall some basic concepts of fuzzy soft set theory. Let E be a convenient parameter set for the universe X.

Definition 2.1 ([22]). Let X be an universal set and E be the set of parameters. A pair (F, E) is called a soft set over X if and only if F is a mapping from E into the set of all subsets of the set

X, i.e., $F : E \to P(X)$, where $P(X)$ is the power set of X.

In this manner, a soft set (F, E) consist of a collection of approximations:

$(F, E) = \{F(e) : e \in E\}$.

Definition 2.2 ([3]). Let I^X denote the set of all fuzzy sets on X and $A \subset E$. A pair (f, A) is called a fuzzy soft set over X, where f is a mapping from A into I^X.

That is, for each $a \in A$, $f(a) = f_a : X \rightarrow I$, is a fuzzy set on X.

Definition 2.3 ([3]). For two fuzzy soft sets (f, A) and (g, B) over an universal set X, we say that (f, A) is a fuzzy soft subset of (g, B) and write $(f, A) \subseteq (g, B)$ if

(i) $A \subset B$, and

(ii) For each $a \in A$, $f_a \leq g_a$, that is, f_a is fuzzy subset of g_a.

Definition 2.4 ([3]). Two fuzzy soft sets (f, A) and (g, B) over a common universe X are said tobe equal if $(f, A) \subseteq (g, B)$ and $(g, B) \subseteq (f, A)$.

Definition 2.5 ([3]). If (f, A) and (g, B) are two soft sets, then (f, A) and (g, B) is denoted as $(f, A) \wedge (g, B)$.

$(f, A) \wedge (g, B)$ is defined as $(h, A \times B)$ where $h(a, b) = h_{a,b} = f_a \wedge g_b, \forall (a, b) \in A \times B$.

Now, let M be a left R-module A be any nonempty set. Let $F: A\rightarrow$ (M) refers to a set-valued function and the pair (F, A) is a soft set over M.

Definition 2.6 ([21]). Let (F, A) be a soft set over M. (F, A) is said to be a soft module over M if and only if $F(x) < M$ for all $x \in A$.

Definition 2.7 ([21]). Let (F, A) and (G, B) be two soft modules over M and N respectively. Then $(F, A)\times(G, B) = (H, A \times B)$ is defined as $H(x, y) = F(x) \times G(y)$ for all $(x, y) \in A \times B$.

Proposition 2.8([21]). Let (F, A) and (G, B) be two soft modules over M and N respectively.

Then $(F, A)\times(G, B)$ is soft module over $M \times N$.

Definition 2.9 ([19]). Let (F, A) be a fuzzy soft set over G. Then (F, A) is said to be a fuzzy soft group over G if and only if $F(x)$ is a fuzzy subgroup of G, for all $x \in A$.

Theorem 2.13 ([19]). Let (F, A) and (H, A) be two fuzzy soft groups over G. Then their intersection $(F, A) \,\tilde{}\, (H, A)$ is a fuzzy soft group over G.

Theorem 2.14 ([19]). Let (F, A) and (H, B) be two fuzzy soft groups over G. If $A\cap B=\emptyset$, then $(F, A) \,\tilde{}\, (H, B)$ is a fuzzy soft group over G.

Definition 2.16 ([13]). An intuitionistic fuzzy set A in a non-empty set X is an object having the form $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$,where the functions $\mu_A : X \rightarrow [0, 1]$ and $\lambda_A : X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \lambda_A)$ for the intuitionistic fuzzy set $\{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$.

3.Intuitionistic fuzzy soft modules

In this section, we firstly define an intuitionistic fuzzy soft module and give some operations on this set. Let X be an universal set and IFS(X) denote the family of intuitionistic fuzzy sets on X.

Definition 3.1. Let (F, A) be an intuitionistic fuzzy soft set over M. Then (F, A) is said to be an intuitionistic fuzzy soft module over M if and only if $\forall a \in A, F(a) = (F_a, F^a)$ is an intuitionistic fuzzy submodule of M.

Theorem 3.1. Let (F, A) and (H, B) be two intuitionistic fuzzy soft modules over M. Then their intersection (F, A) ∩ (H, B) is an intuitionistic fuzzy soft module over M.

Proof. Let (F, A) ∩ (H, B) = (G, C), where C = A ∩ B. Since the intuitionistic fuzzy soft set $(G_c, G^c) = (F_c \wedge H_c, F^c \vee H^c)$ is an intuitionistic fuzzy submodule, for $\forall c \in C$, (G, C) is an intuitionistic fuzzy soft module over M.

Theorem 3.2. Let (F, A) and (H, B) be two intuitionistic fuzzy soft modules over M. Then (F, A) ∧ (H, B) is an intuitionistic fuzzy soft module over M.

Proof. By definition 2.7, (F, A)∧(H, B) = (G, A x B). Since (F^a, F_a) and (H^b, H_b) are intuitionistic fuzzy submodules of M, $(F_a \wedge H^b, F^a \vee H_b)$ is an intuitionistic fuzzy submodule of M. Thus G(a, b) = $F_a \wedge H_b$, $F^a \vee H^b$ is an intuitionistic fuzzy submodule of M for all (a, b) ∈ A x B. Hence we find that (F, A)∧(H, B) is an intuitionistic fuzzy soft module over M.

Theorem 3.3. Let (F, A) and (H, B) be two intuitionistic fuzzy soft modules over M. If A∩B = Ø, then (F, A) ∪ (H, B) is an intuitionistic fuzzy soft module over M.

Proof. By Definition 2.5, we can write (F, A)∪(H, B) = (G, C). Since A∩B = Ø, it follows that either c ∈ A−B or c ∈ B−A for all c ∈ C . If c ∈ A − B, then G(b) = (F_b, F^b) is an intuitionistic fuzzy submodule of M, and if c ∈ B − A, then G(b) = $(H_b, H^b)$is an intuitionistic fuzzy submodule of M. Hence, (F, A) ∪ (H, B) is an intuitionistic fuzzy soft module over M.

Definition 3.2. Let (F, A) and (H, B) be two intuitionistic fuzzy soft modules over M. Then (F, A) is called an intuitionistic fuzzy soft submodule of (H, B) if

(1) A ⊂ B.
(2) For all a ∈ A, $(F_a, F\ )$ is an intuitionistic fuzzy submodule of $(H_a, H\ )$.

Theorem 3.4. Let (F, A) and (H, A) be two intuitionistic fuzzy soft modules over M.

If F(a) ≤ H(a) for all a ∈ A, then (F,A) is an intuitionistic fuzzy submodule of (H,A).

Proof. The proof of the theorem is straightforward.

4.Conclusion

This paper summarized the basic concepts of intuitionistic fuzzy soft sets and intuitionistic fuzzy soft modules. By using these concepts, we studied the algebraic properties of intuitionistic fuzzy soft sets in module structure. This work focused on intuitionistic fuzzy soft modules, intuitionistic fuzzy soft submodules, and intuitionistic fuzzy soft homomorphisms. To extend this work, one could study the properties of intuitionistic fuzzy soft sets in other algebraic structures such as rings and fields.

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