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# Inventory Replenishment Policies With Linear Deterioration Under Inflation Are Optimal

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## ABSTRACT

In this paper, we create a stock-keeping system for perishable goods within a constrained time frame for future planning. We have done our research in an inflationary setting to make it more applicable to the current market. Understanding inflation has provided researchers with a tool that can be used in a wider variety of settings without raising eyebrows. It is important to note that the chosen environment is specific to the original setting in which the model was created. Most researchers have ignored the decay factor or assumed a constant decay rate in the inventory model, which is not practical. We consider an inventory where the deterioration rate increases linearly with time, which is more realistic. The problem is formulated in a mathematical model and used to find the best solution. The robustness of the model and its responsiveness to parameter changes are tested in a number of ways. At last, a simulation is executed to verify the results.

**Keywords:** Inventory, Supply chain, Deterioration, Inflation

## INTRODUCTION

A generalised dynamic scheduling model for inventory items with Weibull distributed deterioration was proposed by **J. M. Chen in 1998**. With this system, products are put into specific warehouses depending on how long they are expected to last and how quickly they degrade. The model can dynamically modify stockpiling patterns in response to shifts in supply and demand. As a result, we can better serve our customers and make better use of our resources.By taking into account defective quality items when applying EPQ/EOQ formulas and by considering that poor quality items may be singled out at the end of a 100% screening process, **M. K. Salameh and M. Y. Jaber (2000)** expanded the conventional EPQ/EOQ model where individual units available for purchase.**H. L. Yang et al (2002)**make up for shortfalls; they increased the theoretical size of their inventory. Within a constrained time horizon, it is desirable to minimise the total cost by determining the minimum acceptable number of reworks and the best possible timing for when these reworks should be performed. As a mathematical model, the inventory size model helps businesses plan for how much stock they'll need on hand to meet customer demands. The model assumes that the company will have a finite number of production cycles and that demand will remain stable. In the event of a supply crunch, the model predicts that the company will have to modify some of its offerings. Authors **L. Moon et al. 2005** predicting future commodity demand while factoring in inflation and exchange rates is one of the company's specialties.**S. T. Lo et al 2007**developed a model that takes into account the needs of both manufacturers and retailers in terms of stock management. Understanding how products are made and distributed between retailers and manufacturers is made easier with this model, which accounts for the competing priorities of these two groups. Model developed by **H. L. Yang et al(2010)** is an overarching framework that includes many submodels. This model details the interplay between a system's constituent parts and the mutual impact of various systems.If the permitted delay in payment if the overdue amount is compensated in the given credit, **D. Chakraborty et al. (2018)** developed two warehouse inventory models with ramp-type demand rate and three-parameter Weibull distribution deterioration (ThPWD) under inflation conditions. The purpose of the research by **X. Huang et al. (2021)** is to examine the impact of inflation on the pricing and replenishment strategy of perishable food, and to compare these results to those obtained in a scenario where inflation is not taken into account. They will also analyse how factors like inflation, quality decline, and the time value of money influence choices made along the food supply chain.Using a simulation method, **Y. Ekren et al. 2022**explored the optimum levels of reordering and reordering for multiple e-grocers within a common network. They determined that the most productive level of cooperation between online supermarkets.According to research by **M. Akhtar et al. (2023),** there is a sweet spot for the product's selling price, an optimum number of completion cycles, and a maximum shortage level that maximizes the retailer's total profit over a finite time horizon, and these numbers are 3, 9, and 10, respectively.

**ASSUMPTIONS AND NOTATIONS**

1. Both the rate of restocking and the amount of time needed to do so are fixed at zero.
2. In an hour, we review just one thing.
3. The annual demand rate for unit $\alpha$is fixed.
4. The time horizon H contains a number of periods denoted by the letter m.
5. Over the entirety of the time horizon H, it is possible that certain goods will be in short supply. On the other hand, the products are filled to their utmost capacity at time$t = H$, which is when the final filling of the products occurs. This indicates that certain products will be discontinued at some point in the near future.
6. The amount of time that has passed is directly proportional to the rate at which something deteriorates.
7. The first model prohibits withdrawal, limiting $C_2$ production. In the second model, full withdrawal is allowed, but units are limited in $C_2$ production.
8. Cost per order is the cost of producing one order in economics measured by C. Order type, unit cost, and inventory holding cost determine order cost.
9. R, inflation-adjusted discount rate.
10. $T_m = H$ &$T_0 = 0, T_j$is the amount of time that has elapsed since the beginning of the system and up to the jth replenishment cycle $j = 1,2, \dots, m$.
11. In the jth replenishment cycle $j = 1,2, \dots, m$, the stock level is measured at time $t_j$.

## A MODEL AND ITS SOLUTION

Here we discuss two models:

### MODEL I: No Shortage Permitted

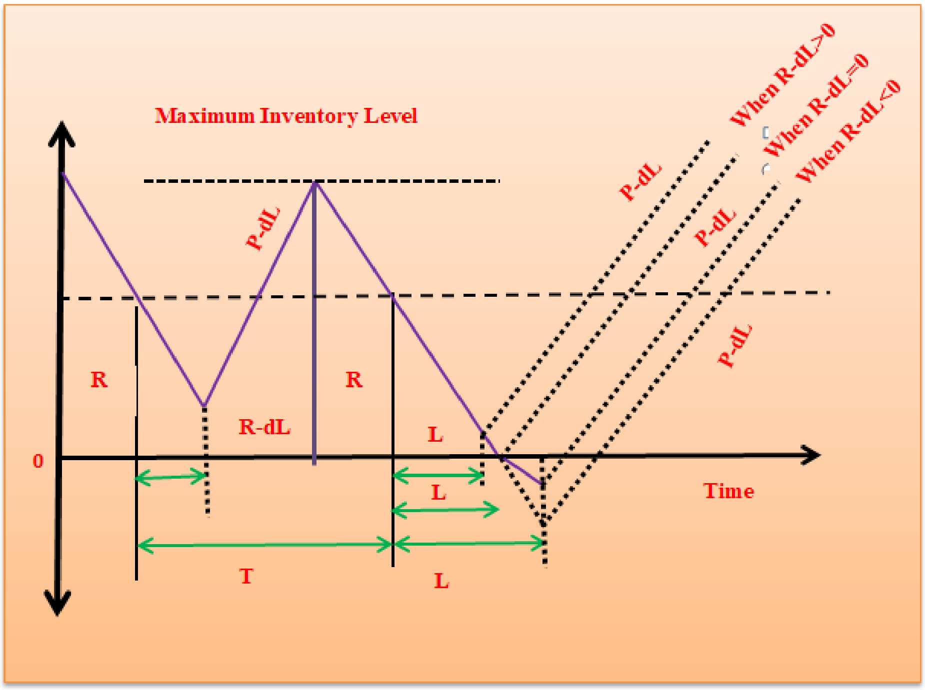


**Fig 1: No shortage permitted**

Over the entire time horizon H, there are m refills, the current values of total refills based on:

$$C(R) = A\,\frac{(1-e^{-RH})}{\left(1-e^{-\frac{RH}{m}}\right)}$$

Total purchasing costs is given by:

$$C(P) = \alpha C\left[T + \frac{\theta T^2}{2} + (b+\theta^2)\frac{T^3}{6}\right]\frac{(1-e^{-RH})}{\left(1-e^{-\frac{RH}{m}}\right)}$$

Total holding costs is given by:

$$C(H) = \alpha C_1\left[\left\{\frac{H}{m} + \frac{\theta\,H^2}{2\,m^2} + \frac{(b+\theta^2)H^3}{6\,m^3}\right\}\left(\frac{e^{-\frac{RH}{m}}}{-R} + \frac{1}{R}\right) + \left\{1 + \frac{\theta\,H}{m} + \frac{\theta^2 H^2}{2\,m^2} + \frac{(b+\theta^2)\theta^3 H^3}{6\,m^3}\right\}\left\{\frac{H\,e^{-RH}}{R\,m} + \frac{e^{-\frac{RH}{m}}}{R^2} - \frac{1}{R^2}\right\}\right]\frac{(1-e^{-RH})}{\left(1-e^{-\frac{RH}{m}}\right)}$$

Total variable cost is given by

$$TC(m) = C(R) + C(P) + C(H)$$

$$\Rightarrow TC(m) = \left[A + \alpha C\left[T + \frac{\theta\, T^2}{2\, m^2} + \frac{(b+\theta^2)T^3}{6\, m^3}\right] + \alpha C_1\left\{\frac{H}{m} + \frac{\theta\, H^2}{2\, m^2} + \frac{(b+\theta^2)H^3}{6\, m^3}\right\}\left\{\frac{e^{-\frac{RH}{m}}}{-R} + \frac{1}{R}\right\}\right.$$
$$\left. + \alpha C_1\left\{1 + \frac{\theta\, H}{m} + \frac{\theta^2 H^2}{2\, m^2} + \frac{(b+\theta^2)\theta^3 H^3}{6\, m^3}\right\}\left\{\frac{H\, e^{-RH}}{R\, m} + \frac{e^{-\frac{RH}{m}}}{R^2} - \frac{1}{R^2}\right\}\right]\frac{(1-e^{-RH})}{\left(1-e^{-\frac{RH}{m}}\right)}$$

**BEST SOLUTION**

K and m are the inputs to the total cost function, denoted by $TC(m,k)$. In the case where m is a discrete variable, minimising $TC(m,k)$requires satisfying the following inequality: $\frac{dTC(m,k)}{dK} = 0$. And it is also demonstrated that $\frac{d^2TC(m,k)}{dK^2} > 0$.

$$I(t) = \alpha\left[(T-t) + \frac{\theta}{2}(T^2 - t^2) + \frac{(b+\theta^2)}{6}(T^3 - t^3)\right]\left[1 - \left\{\theta t + (b-\theta^2)\frac{t^2}{2}\right\}\right]$$

**NUMERICAL**

Several numerical examples are solved to demonstrate the efficacy of the proposed method. Tables and graphs illustrate the outcomes in each instance:

$$\alpha = 550\, units, \theta = 155, A = 270, C_1 = 1.52\, per\ unit\ per\ year, C = 5\, per\ unit, R = 0.155,$$
$$H = 20\, yrs.$$

Our approach to finding a solution yields the following values for the total variable cost and the order quantity to fulfill:

$$m^* = 30, Q^* = 267.345\ \&TC^*(m) = 16654.64$$

## COST-BENEFIT ANALYSIS

Any decision-making scenario has the potential for parameter value changes due to uncertainty, making sensitivity analysis a valuable tool.

| **Parameters** | **Variation of the different Parameters** | | | | |
|---|---|---|---|---|---|
| | **Percentage** | **−50** | **−25** | **25** | **50** |
| $\alpha$ | Q | 0.873 | 0.934 | 1.133 | 1.220 |
| | TC | 0.689 | 0.831 | 1.182 | 1.352 |
| $\theta$ | Q | 1.034 | 1.018 | 0.956 | 0.889 |
| | TC | 0.922 | 0.983 | 1.014 | 1.028 |
| A | Q | 0.916 | 0.932 | 1.136 | 1.140 |
| | TC | 0.831 | 0.873 | 1.138 | 1.157 |
| C | Q | 1.146 | 1.094 | 0.947 | 0.893 |
| | TC | 0.689 | 0.892 | 1.139 | 1.164 |
| $C_1$ | Q | 1.054 | 1.042 | 0.996 | 0.964 |
| | TC | 0.769 | 0.885 | 1.022 | 1.033 |
| R | Q | 1.062 | 1.057 | 0.990 | 0.988 |
| | TC | 1.218 | 1.132 | 0.898 | 0.756 |

**Table 1**

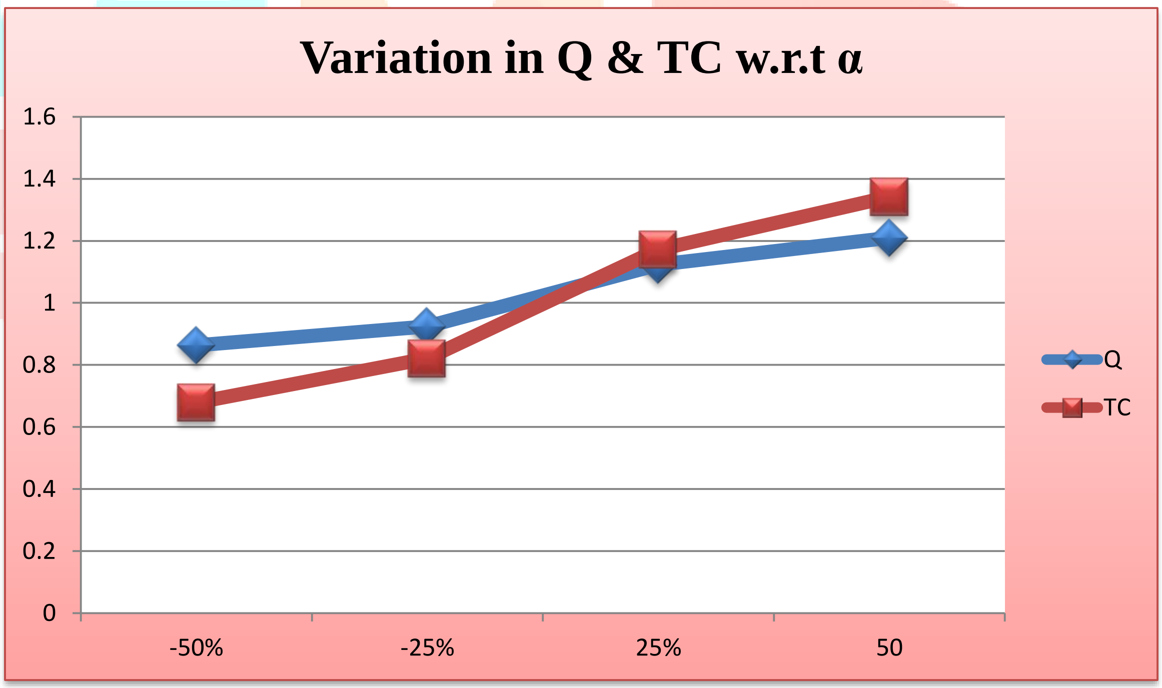


**Fig 2: Variation in Q & TC w.r.t α**

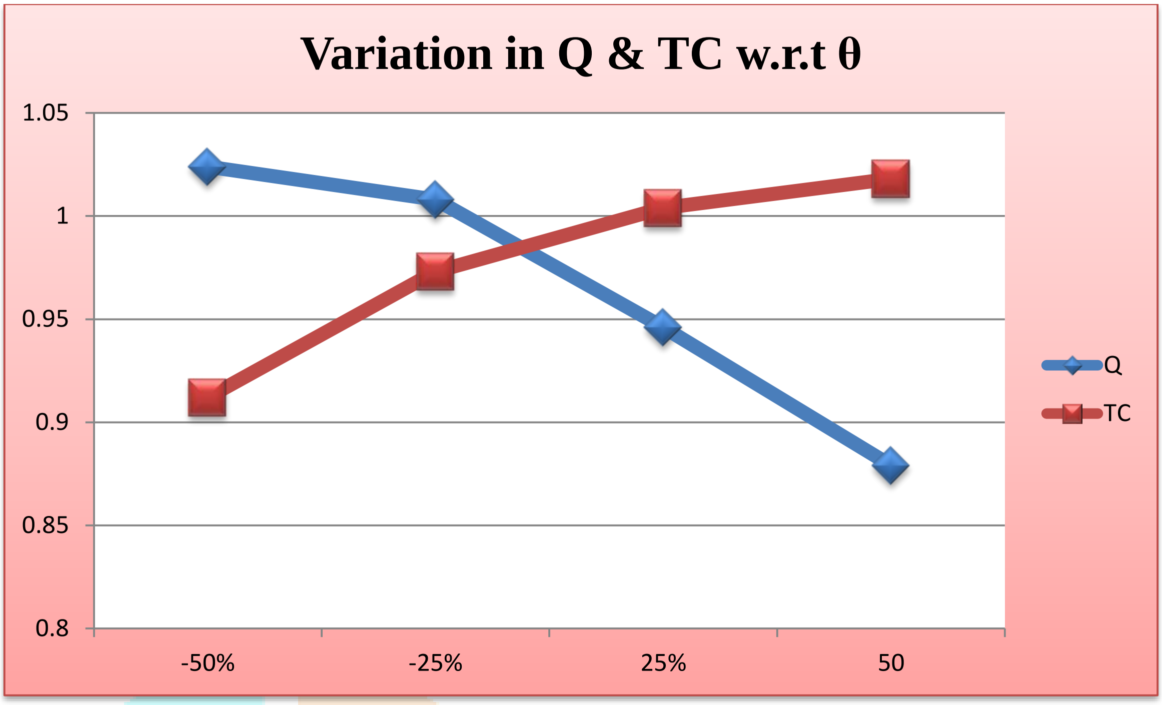


**Fig 3: Variation in Q & TC w.r.t θ**

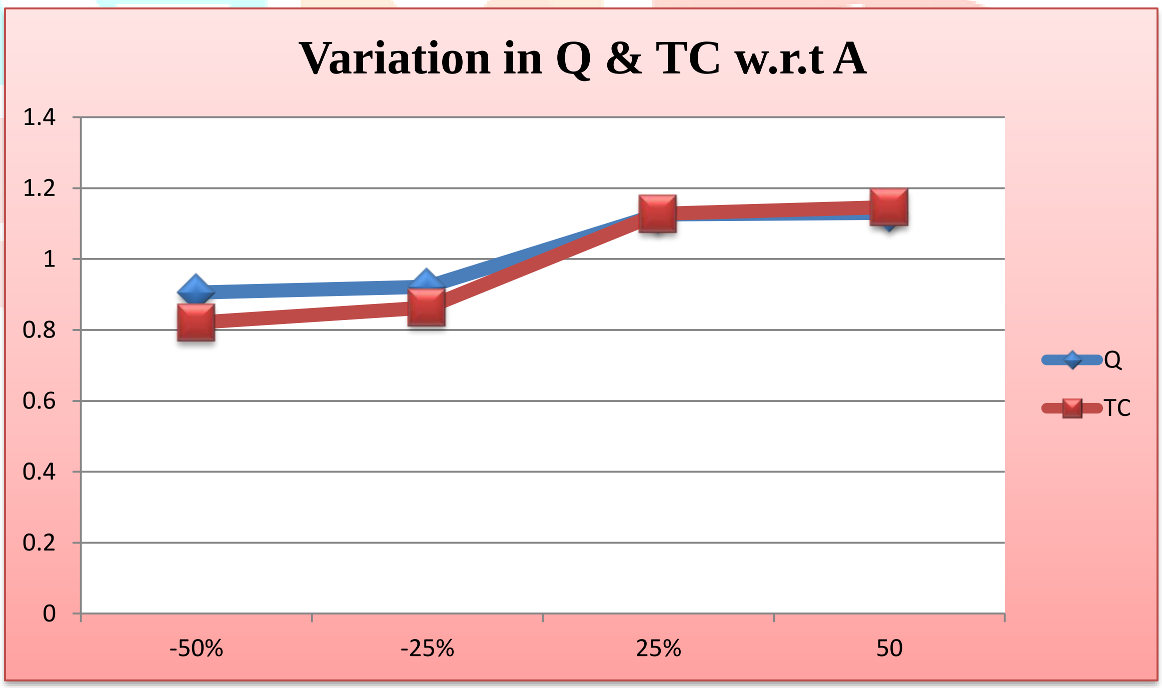


**Fig 4: Variation in Q & TC w.r.t A**

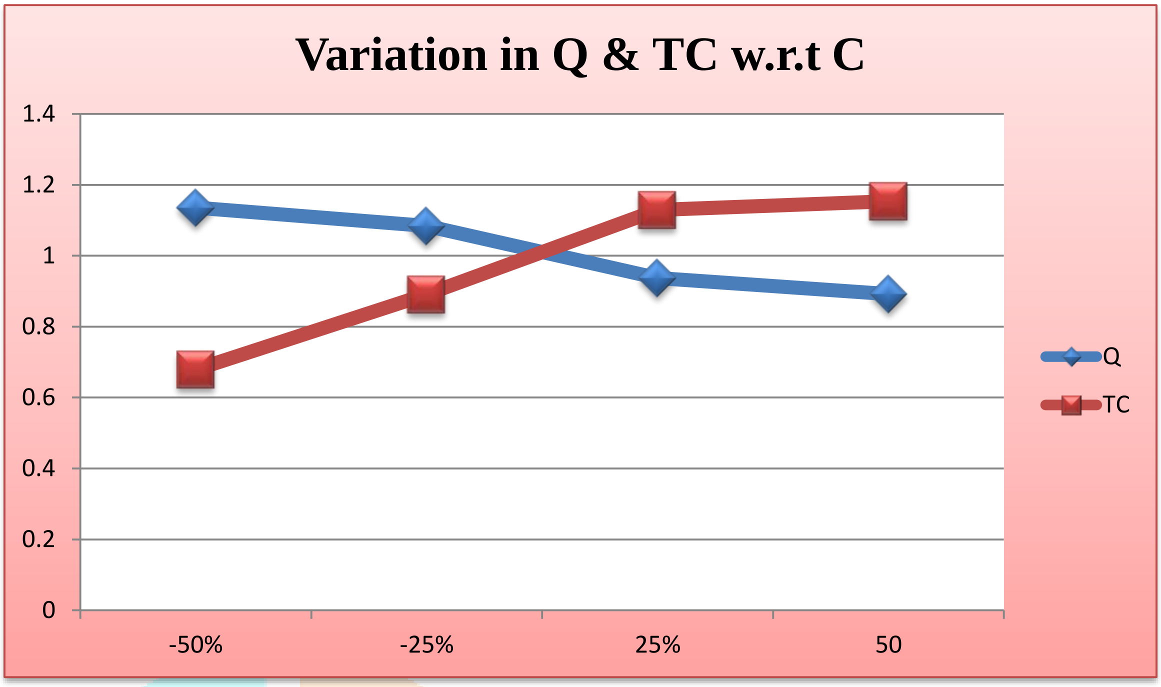


**Fig 5: Variation in Q & TC w.r.t C**

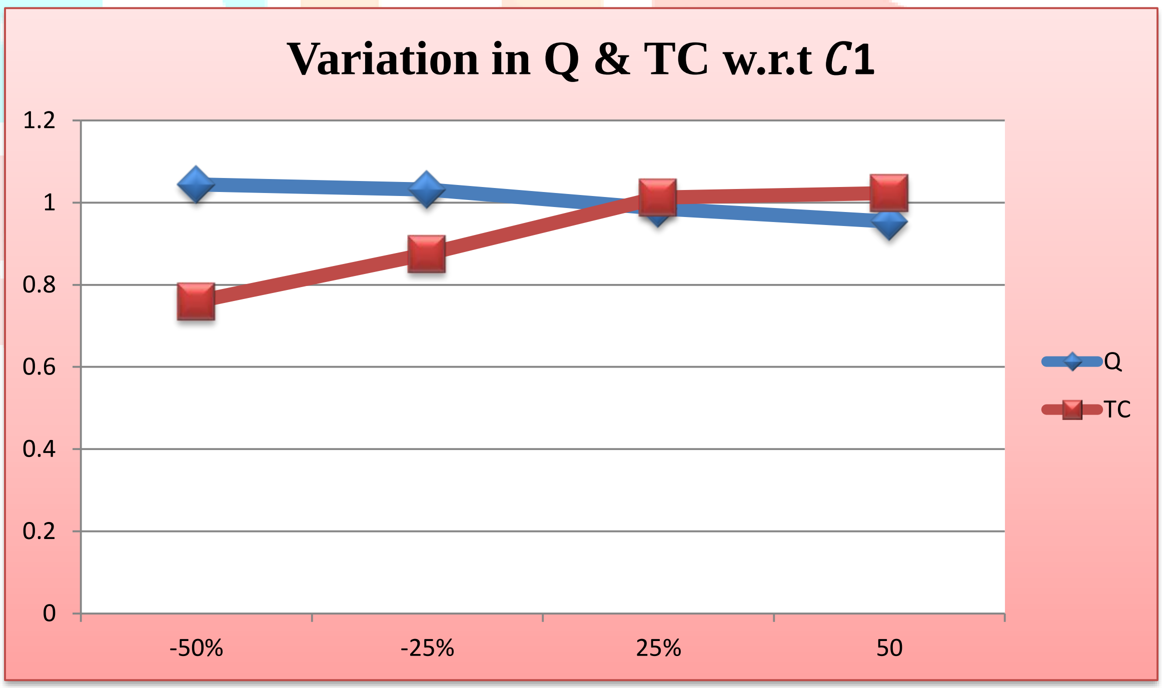


**Fig 6: Variation in Q & TC w.r.t $C_1$**

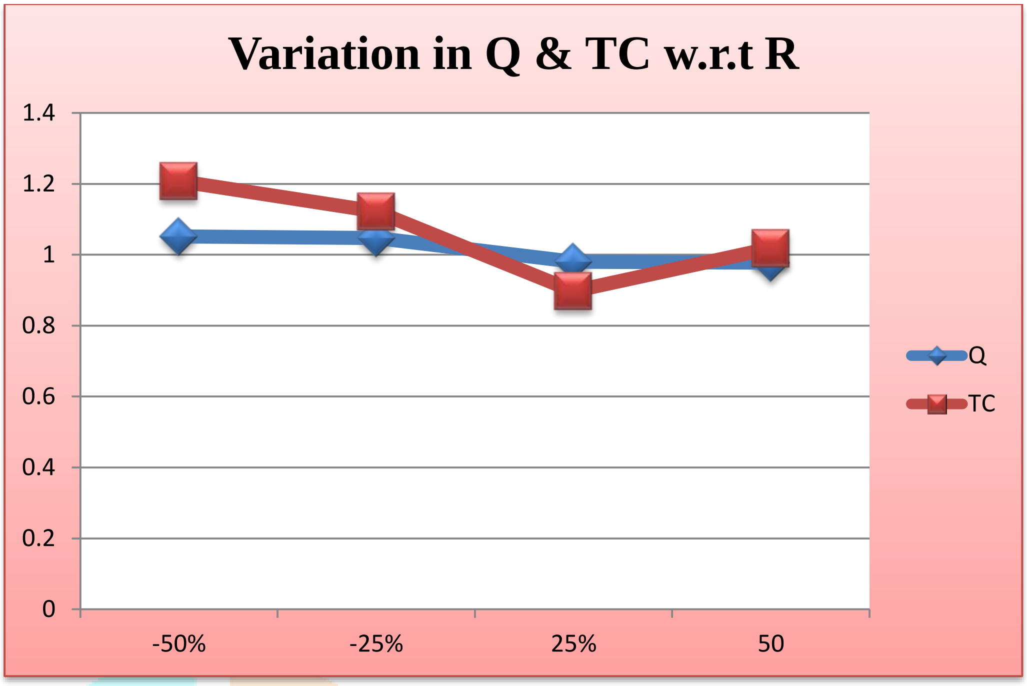


**Fig 7: Variation in Q & TC w.r.t R**

## MODEL II: Shortages permitted with complete backlogging

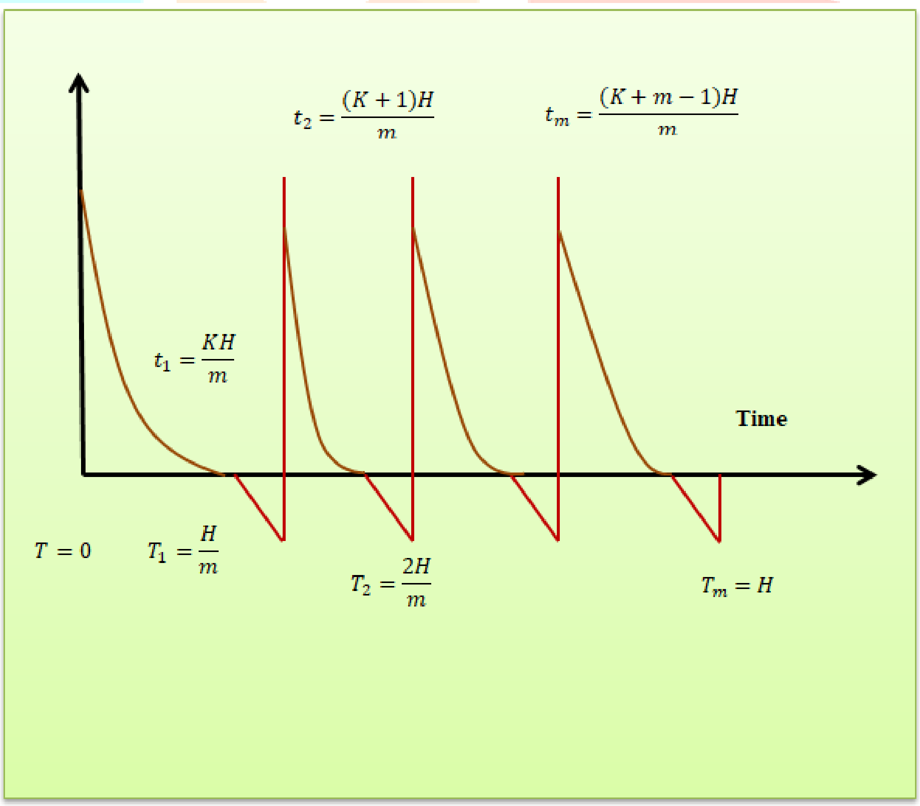


**Fig 8: Shortages permitted with complete backlogging**

Total replenishment costs is given by

$$C(R) = A\,\frac{\left(e^{\frac{RH}{m}} - e^{-RH}\right)}{\left(e^{\frac{RH}{m}} - 1\right)}$$

Total purchasing cost is given by:

$$C(P) = C\left[\alpha\left\{1+\frac{\theta}{2}\frac{KH}{m}+(b+\theta^2)\frac{K^2H^2}{6}\right\}\frac{KH}{m}\frac{(1-e^{-RH})}{\left(1-e^{-\frac{RH}{m}}\right)}+\alpha(1-K)\frac{H}{m}\frac{(1-e^{-RH})}{\left(e^{\frac{RH}{m}}-1\right)}\right]$$

Total holding costs is given as:

$$C(H) = \alpha C_1\left[\left\{\frac{H}{m}+\frac{\theta\, H^2}{2\,m^2}+\frac{(b+\theta^2)H^3}{6\,m^3}\right\}\left(\frac{e^{-\frac{RH}{m}}}{-R}+\frac{1}{R}\right)\right.$$
$$\left.+\left\{1+\frac{\theta\, H}{m}+\frac{\theta^2H^2}{2\,m^2}+\frac{(b+\theta^2)\theta^3H^3}{6\,m^3}\right\}\left\{\frac{H\,e^{-RH}}{R\,m}+\frac{e^{-\frac{RH}{m}}}{R^2}-\frac{1}{R^2}\right\}\right]\frac{(1-e^{-RH})}{\left(1-e^{-\frac{RH}{m}}\right)}$$

Total shortages costs are:

$$C(S) = \frac{\alpha\, C_2}{R^2}\left[(K-1)\frac{HR}{m}+\left\{e^{\frac{R(1-K)H}{m}}-1\right\}\right]e^{-\frac{RH}{m}}\frac{(1-e^{-RH})}{\left(e^{\frac{RH}{m}}-1\right)}$$

Total variable cost of the system is:

$$TC(m,k) = C(R)+C(P)+C(H)+C(S)$$

$$\Rightarrow TC(m,k) = AD + C\alpha\left[1+\frac{\theta}{2}\frac{KH}{m}+(b+\theta^2)\frac{K^2H^2}{6}\right]\frac{KHE}{m}+C\alpha(1-K)\frac{H}{m}F$$
$$+\alpha C_1\left[\left\{1+\frac{\theta HK}{2m}+\frac{1}{6}(b+\theta^2)\frac{K^2H^2}{m^2}\right\}\frac{KH}{m\,R}\left(1-e^{-\frac{RKH}{m}}\right)\right.$$
$$\left.+\left\{1+\frac{\theta HK}{m}+\frac{\theta^2H^2K^2}{2\,m^2}+\frac{(b+\theta^2)\theta^3H^3K^3}{6\,m^3}\right\}\left\{\frac{H}{mR}e^{-\frac{RKH}{m}}+\frac{e^{-\frac{RH}{m^2}}}{R^2}-\frac{1}{R^2}\right\}\right]E$$
$$+\frac{C_2\alpha}{R^2}\left[\frac{(K-1)HR}{m}+\left\{e^{-\frac{R(K-1)H}{m}}-1\right\}\right]F$$

Where

$$D = \frac{e^{\frac{RH}{m}}-e^{-RH}}{e^{\frac{RH}{m}}-1}$$

$$E = \frac{1-e^{-RH}}{1-e^{-\frac{RH}{m}}}$$

$$F = \frac{1-e^{-RH}}{e^{\frac{RH}{m}}-1}$$

## BEST SOLUTION

K and m are the inputs to the total cost function, denoted by $TC(m,k)$. In the case where m is a discrete variable, minimising $TC(m,k)$requires satisfying the following inequality: $\frac{dTC(m,k)}{dK}=0$. And it is also demonstrated that $\frac{d^2TC(m,k)}{dK^2}>0$.

## NUMERICAL

The following numerical examples are solved using the proposed method to demonstrate its efficacy. In each case, we display both the modernized and conventional outcomes of a given calculation:

$$\alpha = 550\,units, \theta = 0.45, A = 270, C_1 = 1.52\,per\ unit\ per\ year, C = 5\,per\ unit, R = 0.155, C_2 = 3.56\,per\ unit\ per\ year, H = 20\,yrs.$$

The optimal order quantity and total variable cost are found using the solution method.

$$m^* = 30, Q^* = 185.673\ \&TC^*(m) = 15832.728$$

## COST-BENEFIT ANALYSIS

The results of a sensitivity analysis can help you see what effect these modifications may have. Such information is useful for determining the best next steps to take.

| **Parameters** | **Variation of the different Parameters** | | | | |
|---|---|---|---|---|---|
| | **Percentage** | **−50** | **−25** | **25** | **50** |
| $\boldsymbol{\alpha}$ | Q | 0.847 | 0.946 | 1.125 | 1.220 |
| | TC | 0.749 | 0.821 | 1.129 | 1.226 |
| $\boldsymbol{\theta}$ | Q | 1.026 | 1.020 | 0.989 | 0.872 |
| | TC | 0.945 | 0.999 | 1.015 | 1.021 |
| **A** | Q | 0.920 | 0.982 | 1.122 | 1.137 |
| | TC | 0.851 | 0.899 | 1.125 | 1.142 |
| **C** | Q | 1.244 | 1.023 | 0.987 | 0.911 |
| | TC | 0.899 | 0.971 | 1.134 | 1.153 |
| $\boldsymbol{C_1}$ | Q | 1.134 | 1.035 | 0.966 | 0.821 |
| | TC | 0.714 | 0.860 | 1.026 | 1.063 |
| **R** | Q | 1.055 | 1.038 | 0.974 | 0.960 |
| | TC | 1.231 | 1.200 | 0.898 | 0.723 |
| $\boldsymbol{C_2}$ | Q | 0.975 | 0.988 | 1.124 | 1.227 |
| | TC | 0.852 | 0.990 | 1.038 | 1.055 |

**Table 2**

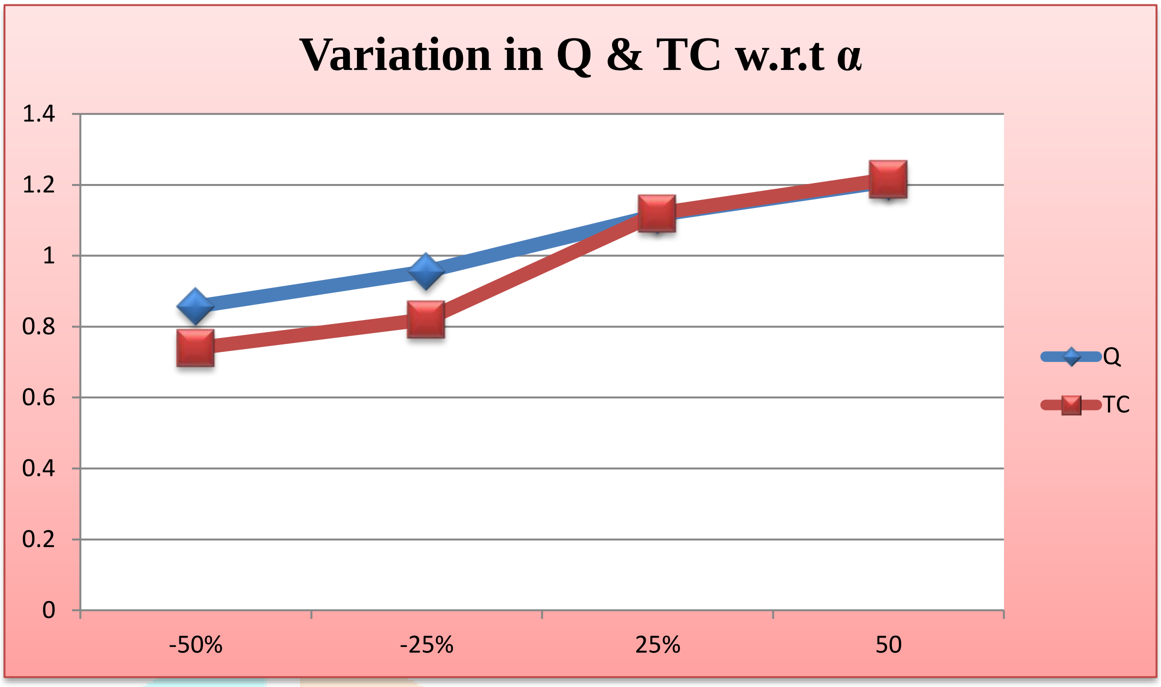


Fig 9: Variation in Q & TC w.r.t α

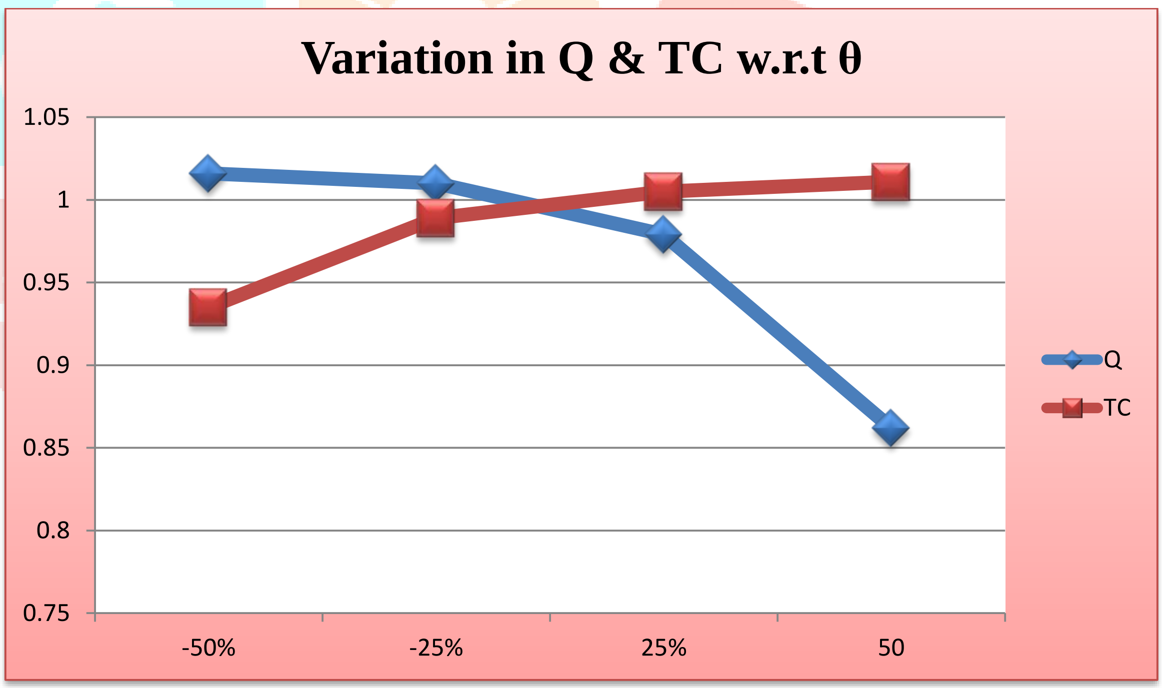


Fig 10: Variation in Q & TC w.r.t θ

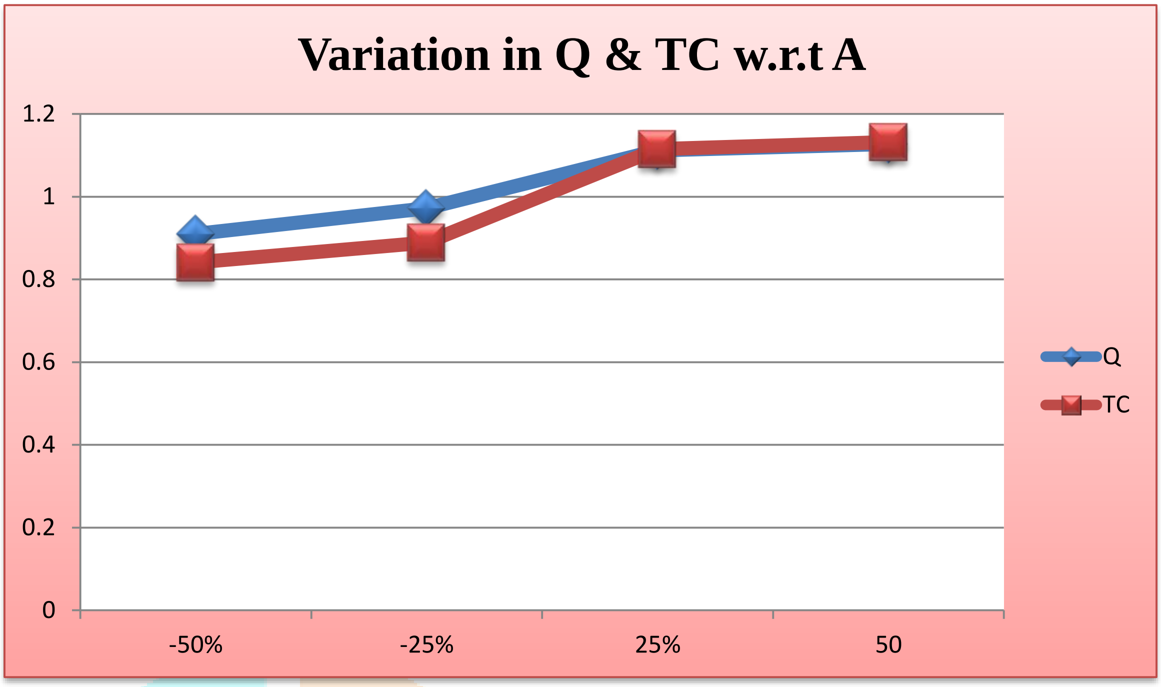


Fig 11: Variation in Q & TC w.r.t A

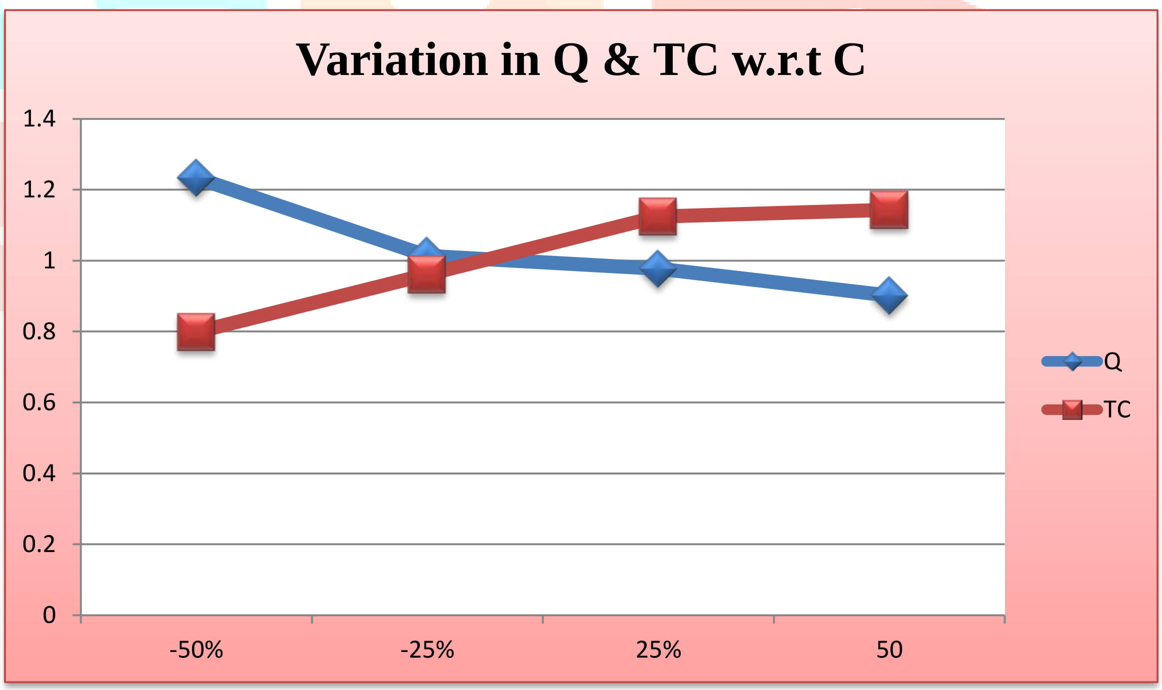


Fig 12: Variation in Q & TC w.r.t C

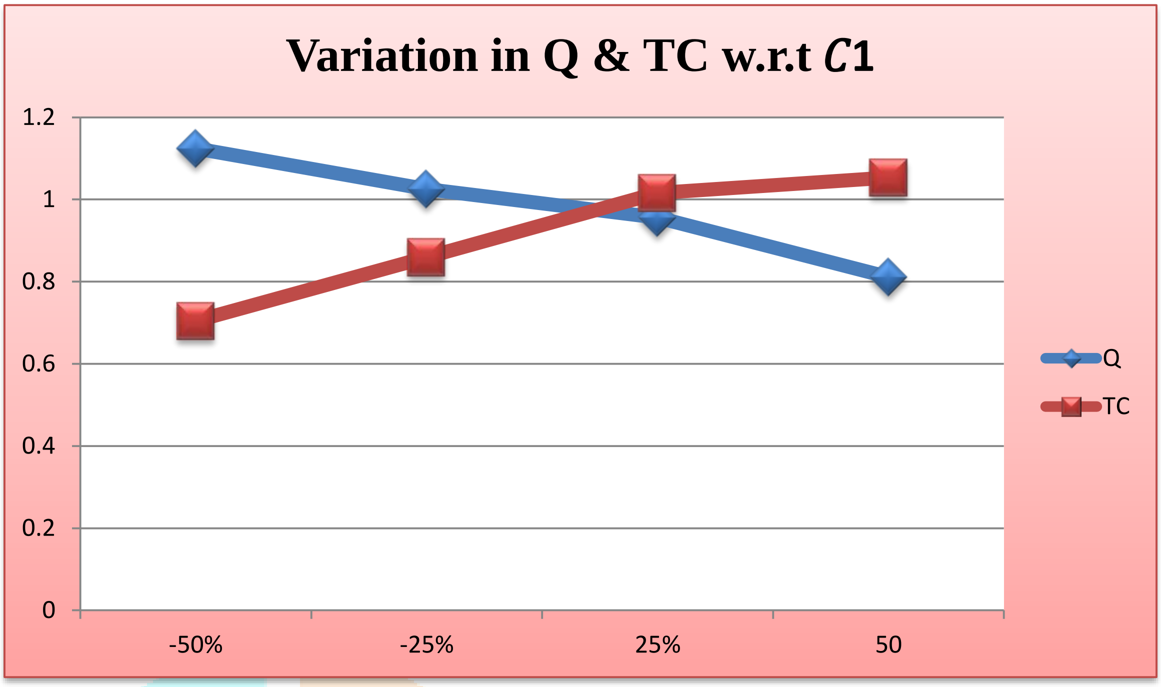


Fig 13: Variation in Q & TC w.r.t $C_1$

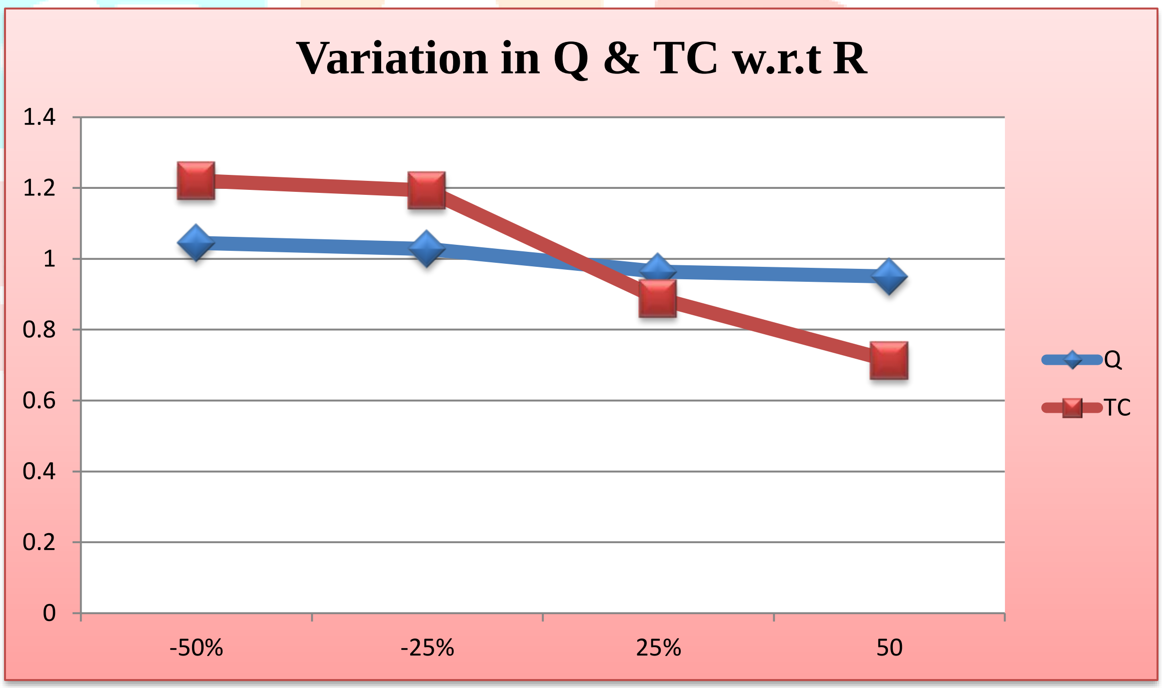


Fig 14: Variation in Q & TC w.r.t R

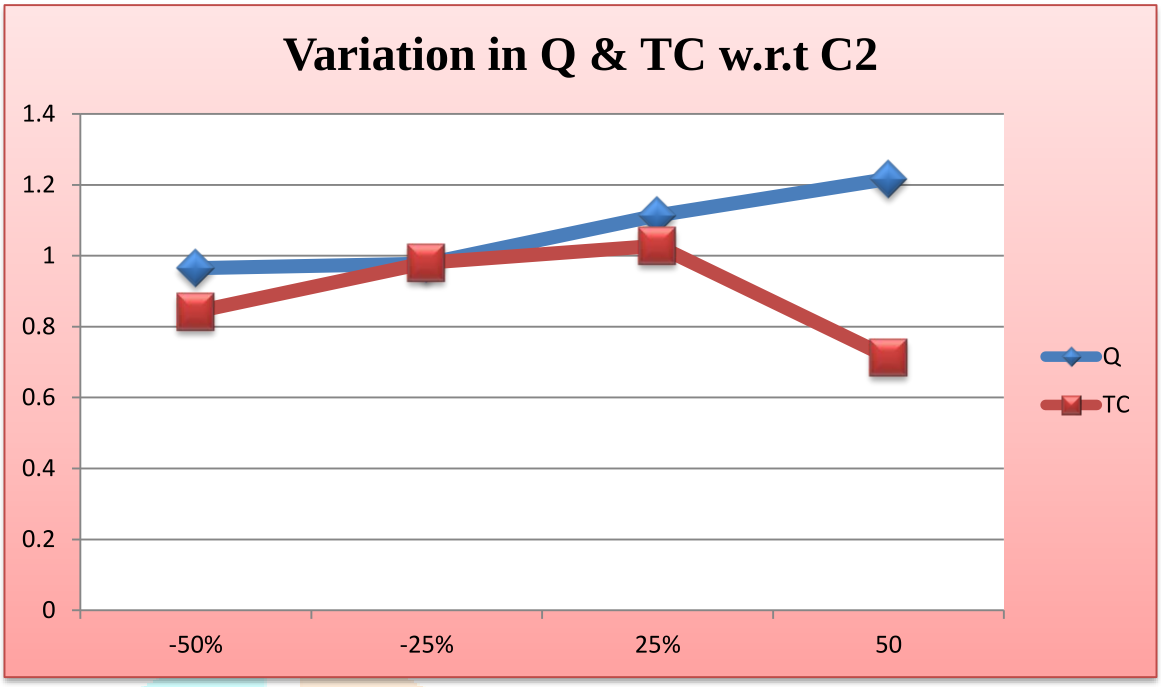


**Fig 15: Variation in Q & TC w.r.t $C_2$**

## CONCLUSION

This paper presents an inventory system designed for a short-term planning horizon in an inflationary economy. This provides a study that is more relevant and acceptable in the current market.There is nothing generic about the inventory model used in this investigation. However, the decay factor has been largely disregarded by researchers, or a constant decay rate has been assumed within the inventory model, neither of which is realistic. We think about a stock whose depreciation is linear with time.A mathematical model is constructed to describe the problem and then used to find the best answer. To illustrate how the model would shift in response to varying assumptions, a sensitivity analysis is conducted. From a monetary viewpoint, this model can be used to better regulate commercial inventory as things stand.There is a lot of room for growth and improvement in this model. The scope of this research can be broadened to include additional factors like deterioration rate and random demand assumption.

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