IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

TOTAL COLORING OF SOME SPECIAL CLASSES OF SPLITTING GRAPHS

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Abstract: Over the one fifty years, various work have been on the coloring of graphs such as vertex coloring and edge coloring. The last fifty years total coloring of the graphs has been considered by various authors. In recent era, the total coloring have been extensively studied in different families of graphs. The splitting graph S(G) of a graph G is obtained from G, by taking a new vertex v' for every vertex v ϵ V (G) and joining v' to all vertices of G adjacent to v is called a splitting graph of G. In this paper, we discuss about the total coloring of splitting graphs of bistar, splitting graph of double crown graph and splitting graph of n-pan graph.

Index Terms - Graph coloring, total coloring, bistar graph, double crown graph, n-pan graph, splitting graph.

I. INTRODUCTION:

In this paper, we have considered finite, simple and undirected graphs. Let G=(V(G),E(G)) be a graph with the vertex set V(G) and the edge set E(G) respectively. In 1965, the concept of total coloring was introduced by M.Behzad (1) and V.G.Vizing (6). The total coloring(7),(,8) of a graph is a coloring to the elements of the graph G, for which any adjacent vertices or edges and incident elements are colored differently. The total chromatic number χ_T (G) of a graph G is the minimum cardinality k such that G may have a total coloring by k- colors. In 1967, M.Behzad (2) came out new ideology that, the total chromatic number of complete graph and complete bipartite graph. He studied the total chromatic number of several classes of graphs in his Ph.D thesis called " Graphs and their chromatic numbers" . In 1980, the splitting graph was introduced by E.Sampath Kumar and H.B.Walikar (5). In this paper, we investigate the total chromatic number of splitting graph of bistar , splitting graph of double crown and splitting graph of n-pan graph families. For all standard graph theoretic terminology and notations, we refer to (3) and (4).

II. PRELIMINARIES

DEFINITION 2.1:

A vertex coloring or coloring of a graph in an assignment of colors to its vertices, so that no two adjacent vertices have same color. A k- coloring of a graph G is coloring of G which uses exactly k- colors. The chromatic number χ_v (G) is defined as the minimum k for which G has the k- coloring.

DEFINITION 2.2:

An edge coloring or line coloring of a graph G is an assignment of color to its edges, so that no two adjacent edges are assigned the same color. A k-edge coloring of a graph G is edge coloring of G which uses exactly k- colors. The edge chromatic number χ_E (G) is the minimum k for which G has the k-edge coloring.

DEFINITION 2.3:

A total coloring of G, is a function f : S ⟶ C where S = V(G) U E (G) and C is a set of colors to satisfies the given conditions.

(i) No two adjacent vertices receive the same color.
(ii) No two adjacent edges receive the same color.
(iii) No edge and its end vertices receive the same color.

The total chromatic number χ_T (G) is the minimum k for which G has the k– total coloring.

DEFINITION 2.4:

A bistar graph $B_{m,n}$ is the graph obtained from K_2 by joining m pendent edges to one end and n pendent edges to the other end of K_2 .

DEFINITION 2.5:

The double crown graph C_n^{++} is the graph obtained from the cycle C_n by attaching two pendent edges at each vertex of C_n.

DEFINITION 2.6:

The n- pan P_n^* is the graph obtained by joining a cycle graph C_n to a singleton graph K_1 with a bridge.

DEFINITION 2.7:

The splitting graph S(G) of a graph G is obtained from G, by taking a new vertex v' for every vertex v϶V(G) and joining v' to all vertices of G adjacent to v is called a splitting graph of G.

RESULTS 2.8:

(i) The total chromatic number of P_n is three.
(ii) The total chromatic number of C_n is $\chi_T(C_n) = \begin{cases} 3, & n \equiv 0 \text{ (mode 3)}, \\ 4, & \text{otherwise} \end{cases}$
(iii) The total chromatic number of throny graph G^* of G is $\chi_T(G^*) \geq \Delta(G^*) + 1$

III. MAIN RESULTS:

In this paper we investigate, the total chromatic number of three classes of splitting graphs.

THEOREM 3.1:

The splitting graph of bistar graph S($B_{m,n}$) is 2(max(m,n)) + 3, (m,n ≥ 2), total colorable.

PROOF:

From the definition of splitting graph, the splitting graph of bistar obtained from the bistar by taking a new vertex v' from every vertex $v \in V(B_{m,n})$ and joining v' to all vertices of $B_{m,n}$ adjacent to v. The order and size of splitting graph of bistar is 2(m+n) +4 and 3(m+n) +3. The vertex set and the edge set of $S(B_{m,n})$ are given below.

$V(S(B_{m,n})) = \{ u, v, u', v' \} \cup \{ u_i, u_i' / 1 \le i \le m \} \cup \{ v_j, v_j' / 1 \le j \le n \}$

$E(S(B_{m,n})) = \{ uv, uv', vu' \} \cup \{ uu_i, uu_i', u'u_i / 1 \le i \le m \} \cup \{ vv_j, vv_j', v'v_j / 1 \le j \le n \}$

Now we construct the total coloring, we define,

$f: V((S(B_{m,n})) \cup E(S(B_{m,n})) \longrightarrow \{ 1,2,....2(\max(m+n)) + 3 \}$

We shall assign the colors to the vertices and the edges of $S(B_{m,n})$ as follows.

STEP 1:

First, we shall assign colors to the vertices.

$f(u) = 1, f(u') = 1, f(v) = 3$ and $f(v') = 3$

$f(u_i) = 2 \ (1 \le i \le m)$

$f(u_i') = 2 \ (1 \le i \le m)$

$f(v_j) = 2 \ (1 \le j \le n)$

$f(v_j') = 2 \ (1 \le j \le n)$

STEP 2:

Next, we shall assign the colors to the edges.

$f(uv) = 2$, $f(vv') = 1$ and $f(v_1 v') = 1$

$f(u u_i') = i + 2, \quad (1 \le i \le m)$

$f(u'u_i) = i + 2, \quad (1 \le i \le m)$

$f(uu_i) = i + m + 2, (1 \le i \le m)$

In similar manner,

$f(v v_j') = j + 2, \quad (2 \le j \le n)$

$f(v'v_j) = j + 2, \quad (2 \le j \le n)$

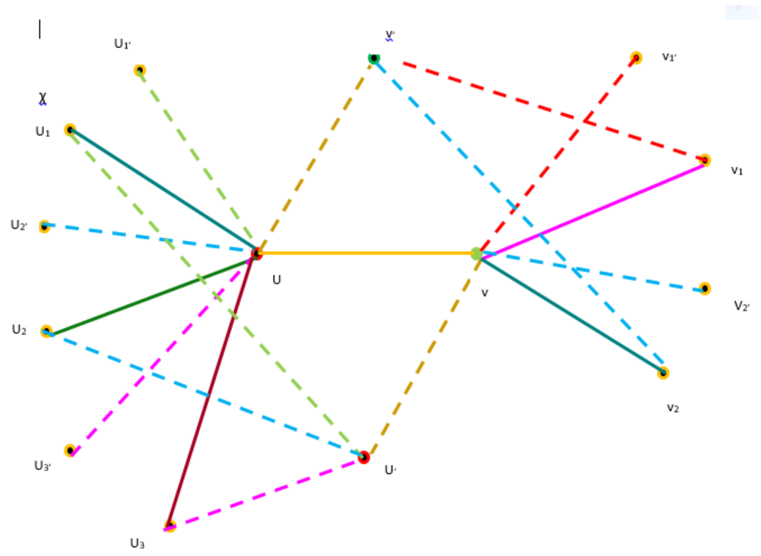
$f(vv_j) = j + n + 2, (1 \le j \le n)$

finally f(uv') and f(vu') receive the same new color.

By applying the above method of coloring pattern, the splitting graph of bistar is properly total colored with 2(max(m,n)) +3 colors.

Hence the theorem.

EXAMPLE3.2



$X_T(S(B_{3,2})) = 9$

THEOREM 3.3:

The splitting graph of double crown graph $S(C_n^{++})$ is ten total colorable.

PROOF:

From the definition of splitting graph, the splitting graph of double crown graph $S(C_n^{++})$ obtained from the C_n^{++} by taking a new vertex v' for every vertex $v \in V(C_n^{++})$ and joining v' to all vertices of C_n^{++} adjacent to v. The order and size of $S(C_n^{++})$ is 6n and 9n respectively.

Hence, the vertex and the edge set are given by,

$V(S(C_n^{++})) = \{ v_i, v_i' / 1 \leq i \leq n\} \cup \{ v_{ij}, v_{ij}' / 1 \leq i \leq n, j = 1,2\}$

$E(S(C_n^{++})) = \{ (v_i v_{i+1}) / 1 \leq i \leq n-1\} \cup \{ v_n v_1 \} \cup \{(v_i v_{ij}), (v_i' v_{ij}), (v_i v_{ij}') / 1 \leq i \leq n, j = 1,2 \} \cup \{ (v_i v_{i+1}') / 1 \leq i \leq n-1\} \cup \{(v_i' v_{i-1}) / 2 \leq i \leq n\} \cup \{(v_n' v_1), (v_n v_1')\}$

Now we construct the total coloring, define

$f: V(S(C_n^{++})) \cup E(S(C_n^{++})) \longrightarrow \{ 1,2,.........10\}$,

We assign the colors to the vertices and the edges as given below. We consider two cases as follows.

CASE (I): when 'n' is even

STEP 1: We shall assign colors to the vertices.

$$f(v_i) = \begin{cases} 1 & \text{if } i \text{ is odd}, \quad 1 \le i \le n \\ 2 & \text{if } i \text{ is even}, \quad 1 \le i \le n \end{cases}$$

$$f(v_i') = \begin{cases} 1 & \text{if } i \text{ is odd}, \quad 1 \le i \le n \\ 2 & \text{if } i \text{ is even}, \quad 1 \le i \le n \end{cases}$$

$$f(v_{ij}) = \begin{cases} 1 & \text{if } i \text{ is even}, \quad 1 \le i \le n, j = 1, 2 \\ 2 & \text{if } i \text{ is odd}, \quad 1 \le i \le n, j = 1, 2 \end{cases}$$

$$f(v_{ij}') = \begin{cases} 1 & \text{if } i \text{ is even}, \quad 1 \le i \le n, j = 1, 2 \\ 2 & \text{if } i \text{ is odd}, \quad 1 \le i \le n, j = 1, 2 \end{cases}$$

STEP 2: We shall assign colors to the edges.

$$f(v_i v_{i+1}) = \begin{cases} 5 & \text{if } i \text{ is even}, \quad 1 \le i \le n-1 \\ 6 & \text{if } i \text{ is odd}, \quad 1 \le i \le n-1 \end{cases}$$

$$f(v_n v_1) = 5$$

$$f(v_i v_{ij}) = \begin{cases} 3, & 1 \le i \le n, \ j = 1 \\ 4, & 1 \le i \le n, \ j = 2 \end{cases}$$

$$f(v_i' v_{ij}) = \begin{cases} 3, & 1 \le i \le n, \ j = 2 \\ 4, & 1 \le i \le n, \ j = 1 \end{cases}$$

$$f(v_i v_{ij}') = \begin{cases} 7, & 1 \le i \le n, \ j = 1 \\ 8, & 1 \le i \le n, \ j = 2 \end{cases}$$

$f(v_i v_{i+1}') = 9, (1 \le i \le n-1)$, $f(v_i v_{i-1}') = 10, (2 \le i \le n)$, $f(v_n v_1') = 9$ and $f(v_1 v_n') = 10$

CASE (II): when 'n' is odd

STEP 1: We shall assign colors to the vertices.

$$f(v_i) = \begin{cases} 1 & \text{if } i \text{ is odd}, \ 1 \le i \le n-1 \\ 2 & \text{if } i \text{ is even}, \ 1 \le i \le n-1 \end{cases}$$

$$f(v_i') = \begin{cases} 1 & \text{if } i \text{ is odd}, \ 1 \le i \le n-1 \\ 2 & \text{if } i \text{ is even}, \ 1 \le i \le n-1 \end{cases}$$

$$f(v_{ij}) = \begin{cases} 1 & \text{if } i \text{ is even}, \ 1 \le i \le n, j = 1, 2 \\ 2 & \text{if } i \text{ is odd}, \ 1 \le i \le n, j = 1, 2 \end{cases}$$

$$f(v_{ij}') = \begin{cases} 1 & \text{if } i \text{ is even}, \ 1 \le i \le n, j = 1, 2 \\ 2 & \text{if } i \text{ is odd}, \ 1 \le i \le n, j = 1, 2 \end{cases}$$

$f(v_n) = 3$ and $f(v_n') = 3$

STEP 2:

We shall assign colors to the edges.

$$f(v_i v_{i+1}) = \begin{cases} 5 & \text{if } i \text{ is even}, \ 1 \le i \le n-2 \\ 6 & \text{if } i \text{ is odd}, \ 1 \le i \le n-2 \end{cases}$$

$f(v_{n-1} v_n) = 1$and $f(v_n v_1) = 5$

$$f(v_i v_{ij}) = \begin{cases} 3, & 1 \le i \le n-1, \ j = 1 \\ 4, & 1 \le i \le n, \ j = 2 \end{cases}$$

$f(v_n v_{n1}) = 2$

$$f(v_i' v_{ij}) = \begin{cases} 3, & 1 \le i \le n-1, \ j = 2 \\ 4, & 1 \le i \le n-1, \ j = 1 \end{cases}$$

$$f(v_i v_{ij}') = \begin{cases} 7, & 1 \le i \le n, \ j = 1 \\ 8, & 1 \le i \le n, \ j = 2 \end{cases}$$

$f(v_n' v_{n1}) = 4$ and $f(v_n' v_{n2}) = 5$.

$f(v_i v_{i+1}') = 9, (1 \le i \le n-1)$,

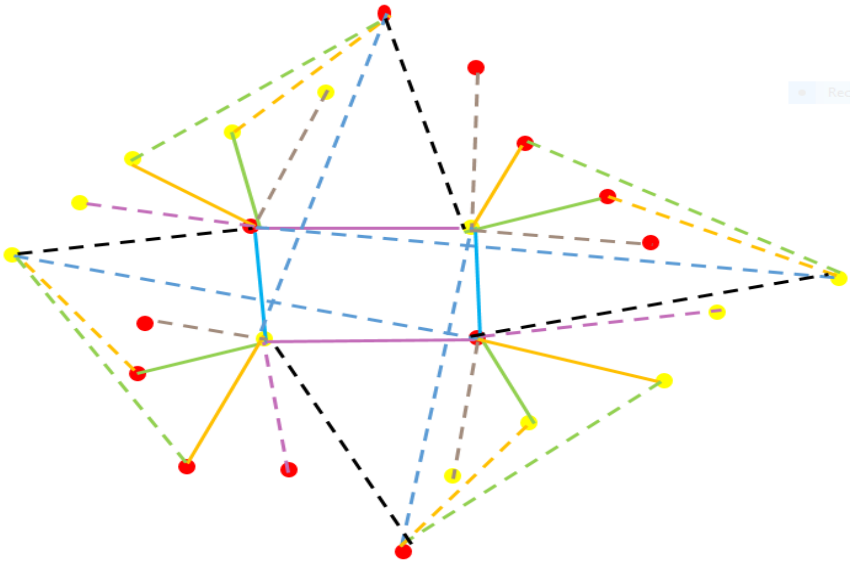
$f(v_n v_1') = 9$,

$f(v_i' v_{i+1}) = 10, (1 \le i \le n-1)$

and finally $f(v_n' v_1) = 10$.

By applying the above method of coloring pattern, the splitting graph of double crown is properly total colored with ten colors. Hence the theorem.

EXAMPLE3.4



$$X_T(S(C_4^{++})) = 10.$$

THEOREM 3.5:

The splitting graph of n – pan graph $S(P_n^*)$ is eight total colorable.

PROOF:

From the definition of splitting graph, the splitting graph of n – pan graph $S(P_n^*)$ obtained from the n – pan graph P_n^* taking a new vertex v' for every vertex $v \in V(P_n^*)$ and joining v' to all vertices of n – pan graph P_n^* adjacent to v. The order and size of splitting graph of n – pan graph is 2(n+1) and 3(n+1) respectively.

The vertex set and the edge set of $S(P_n^*)$ are given below,

$$V(S(P_n^*)) = \{ v_1, v' \} \cup \{ v_i, v_i' \,/\, 1 \le i \le n\}$$

$$E(S(P_n^*)) = \{(v\, v_1), (vv_1'), (v_1v'), (v_1v_n'), (v_nv_1'), (v_1v_n)\} \cup \{(v_i\, v_{i+1}) \,/\, 1 \le i \le n-1\} \cup$$

$$\{ (v_i\, v_{i+1}') \,/\, 1 \le i \le n-1\} \cup \{(v_i'\, v_{i-1}) \,/\, 2 \le i \le n\}$$

Now we construct the total coloring, define

$$f: V(S(P_n^*)) \cup E(S(P_n^*)) \longrightarrow \{ 1,2,\ldots\ldots\ldots.8\}$$

We assign the colors to the vertices and the edges as given below. We consider two cases as follows.

CASE (I): when 'n' is odd

STEP 1: We shall assign colors to the vertices as follows.

$f(v) = 1$ and $f(v') = 2$

$$f(v_i) = \begin{cases} 1 & \text{if } i \text{ is even}, \quad 1 \le i \le n-1 \\ 2 & \text{if } i \text{ is odd}, \quad 1 \le i \le n-1 \end{cases}$$

$$f(v_i') = \begin{cases} 1 & \text{if } i \text{ is even}, \quad 1 \le i \le n-1 \\ 2 & \text{if } i \text{ is even}, \quad 1 \le i \le n-1 \end{cases}$$

$f(v_n) = 3$ and $f(v_n') = 3$

STEP 2:

We shall assign colors to the edges.

$f(v_n v_1') = 1, f(v v_1) = 4, f(v_n v_1) = 7, f(v_1 v') = 8, f(v_{n-1} v_n') = 7, f(v v_1') = 5$ and $f(v_n' v_1) = 6$

$$f(v_i v_{i+1}') = \begin{cases} 3 & \text{if } i \text{ is odd}, \quad 1 \le i \le n-2 \\ 4 & \text{if } i \text{ is even}, \quad 1 \le i \le n-2 \end{cases}$$

$$f(v_i v_{i-1}') = \begin{cases} 3 & \text{if } i \text{ is even}, \quad 2 \le i \le n \\ 4 & \text{if } i \text{ is odd}, \quad 2 \le i \le n \end{cases}$$

$$f(v_i v_{i+1}) = \begin{cases} 5 & \text{if } i \text{ is odd}, \quad 1 \le i \le n-1 \\ 6 & \text{if } i \text{ is even}, \quad 1 \le i \le n-1 \end{cases}$$

CASE (II): when 'n' is even

STEP 1: In the same way of case 1, first we shall assign colors to the vertices.

$f(v) = 1$ and $f(v') = 1$

$$f(v_i) = \begin{cases} 1 & \text{if } i \text{ is even}, \quad 1 \le i \le n \\ 2 & \text{if } i \text{ is odd}, \quad 1 \le i \le n \end{cases}$$

$$f(v_i') = \begin{cases} 1 & \text{if } i \text{ is even}, \quad 1 \le i \le n \\ 2 & \text{if } i \text{ is odd}, \quad 1 \le i \le n \end{cases}$$

STEP 2: We shall assign colors to the edges.

$f(v\,v_1) = 4$, $f(v_1'\,v_n) = 4$, $f(v\,v_1') = 5$, $f(v_1\,v') = 8$ and $f(v_1\,v_n') = 7$

$$f(v_i\,v_{i+1}') = \begin{cases} 3 & \text{if } i \text{ is odd}, \quad 1 \le i \le n-1 \\ 4 & \text{if } i \text{ is even}, \quad 1 \le i \le n-1 \end{cases}$$

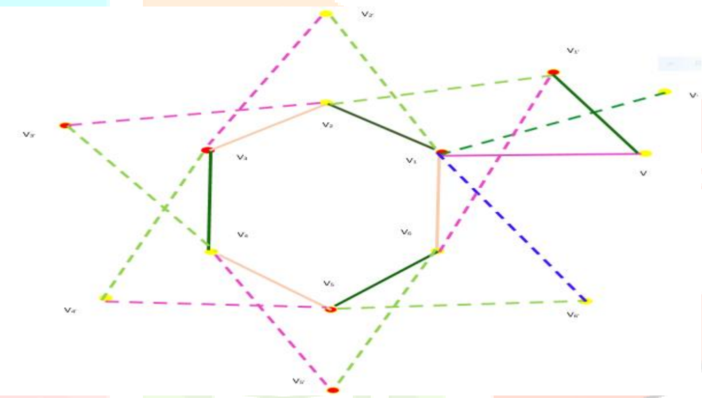
$$f(v_i\,v_{i-1}') = \begin{cases} 3 & \text{if } i \text{ is even}, \quad 2 \le i \le n \\ 4 & \text{if } i \text{ is odd}, \quad 2 \le i \le n \end{cases}$$

$$f(v_i\,v_{i+1}) = \begin{cases} 5 & \text{if } i \text{ is odd}, \quad 1 \le i \le n-1 \\ 6 & \text{if } i \text{ is even}, \quad 1 \le i \le n-1 \end{cases}$$

and finally $f(v_n\,v_1) = 6$

By applying the above method of coloring pattern, the splitting graph o n – pan is properly total colored with right colors. Hence the splitting graph of n – pan graph is eight colorable. Hence the theorem.

EXAMPLE3.6



$X_T(S(P_6^*)) = 10.$

IV.CONCLUSION:

In this paper, the total coloring of the splitting graph of bistar family, the splitting graph of double crown graph family and the splitting graph of n – pan graph family are discussed. Also, we obtained the total chromatic number of these families. All these facts highlight a wide scope for further studies in this area.

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