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Some New Form Of Nano Connectedness And Nano Compactness In Nano Topological Spaces

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Abstract

In this paper, we introduce the new concepts of Nλψg –compactness and Nλψg -connectedness in nano topological spaces and obtained some of their properties using Nλψg -closed sets.

AMS Subject Classification: 54A05, 54C10.

Key words : Nλψg -closed sets,Nλψg -compactness and Nλψg -connectedness.

1 Introduction

A.V.Archangelskii and R.Wiegandt[1] was introduce the concepts of Connectedness and disconnectedness in topological spaces. M.K.R.S.Veerakumar [14] was introduced the notion of ψ closed sets in topological spaces. Maki [5] introduced the notion of Λ-sets in topological spaces in 1986. Lellis Thivagar introduced [4] nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximation of X. The elements of nano topological space are called nano open sets. S.Krishnaprakash et.al [3] innovative some concept of nano compact space and nano connected in nano topology. P.Subbulakshmi and N.R.Santhi Maheswari [9]-[13]., introduced the new concepts of $N\Lambda_{\psi}(A)$ sets, $N\Lambda^{*}_{\psi}(A)$ sets, Nλψ generalized closed set , Nλψg -continuous functions in nano topological spaces we also introduced Nλψg -Open, Nλψg -Closed maps and Nλψg -homeomorphisms in nano topological spaces. The aim of this paper is to introduce the concepts of Nλψg - compactness and Nλψg -connectedness in nano topological spaces. We also investigate their properties.

2 Preliminaries

Definition 2.1. [7] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be in discernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

• The lower approximation of *X* with respect to *R* is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \cup_{x \in U} \{R(X) : R(X) \subseteq X\}$, where R(X) denotes the equivalence class determined by $X \in U$.

• The upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $U_R(X)$.That is $U_R(X) = \cup_{x \in U} \{R(X) : R(X) \cap X = \varphi\}$.

• The boundary of the region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [4] If (U,R) is an approximation space and $X, Y \subseteq U$, then
1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\varphi) = U_R(\varphi) = \varphi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $U_R(X \cup Y) \supseteq U_R(X) \cup U_R(Y)$
7. $U_R(X \cap Y) = U_R(X) \cap U_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$, whenever $X \subseteq Y$
9. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
10. $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
11. $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition 2.3. [4] Let U be the Universe and R be an equivalence relation on U and $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:
(1) U and $\varphi \in \tau_R(X)$.
(2) The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

We call $(U, \tau_R(X))$ is a nano topological space. The elements of $\tau_R(X)$ are called a nano open sets and the complement of a nano open set is called nano closed sets.

Throughout this paper $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U , U/R denotes the family of equivalence classes of U by R.

Definition 2.4. [4] If $(U, \tau_R(X))$ is a nano topological space with respect to X . Where $X \subseteq G$ and if $A \subseteq G$, then
(i) The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by Nint(A) , Nint(A) is the largest nano open subset of A.
(ii) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A) . Ncl(A) is the smallest nano closed set containing A.

Definition 2.5. [4] Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq G$. Then A is said to be
(i) Nano semi-open if $A \subseteq Ncl(Nint(A))$.
(ii) Nano semi-closed if $Nint(Ncl(A)) \subseteq A$.

Definition 2.6. [9] Let A be a subset of a nano topological space $(U, \tau_R(X))$. A subset $N\Lambda_\psi(A)$ is defined as $N\Lambda_\psi(A) = \cap\{H/A \subseteq H \text{ and } H \in N\psi O(U, \tau_R(X)\}$.

Definition 2.7. [9] A subset A of a nano topological space (U, $\tau_R(X)$) is called a $N\Lambda_\psi$ -set if A = $N\Lambda_\psi(A)$. The set of all $N\Lambda_\psi$ -sets is denoted by $N\Lambda_\psi(U, \tau_R(X))$.

Definition 2.8. [10] Let A be a subset of a nano topological space (U, $\tau_R(X)$). A subset N(Λ, ψ) closed if A = B ∩ C , where B is $N\Lambda_\psi$- set and C is a Nψ-closed set.

Definition 2.9. Let (U, $\tau_R(X)$) be a nano topological space and A ⊆ G. Then A is said to be

(i) Nano semi generalized closed [2] (briefly Nsg closed)if Nscl(A) ⊆ G, whenever A ⊆ G and G is NS -open in (U, $\tau_R(X)$).
(ii) Nano ψ -closed [14] (briefly Nψ -closed) if Nscl(A) ⊆ G, whenever A ⊆ G and G is Nsg -open in (U, $\tau_R(X)$) .
(iii) Nano λψ generalized closed [10] (briefly Nλψg -closed) if Nψcl(A) ⊆ H , whenever A ⊆ H and H is N(Λ, ψ) -open in (U, $\tau_R(X)$).

Definition 2.10. [8] A function f : (U, $\tau_R(X)$) → (V, τ'_R (Y)) is said to be

(i) Nano λψ generalized continuous [8] (briefly Nλψg -continuous) if the inverse image of every nano open set in (V, τ'_R (Y)) is Nλψg -open in (U, $\tau_R(X)$) .
(ii) Strongly nano λψ generalized irresolute [13] (briefly Nλψg -irresolute) if the inverse image of every Nλψg -open set in (V, τ'_R (Y)) is Nλψg –open in (U, $\tau_R(X)$).

Definition 2.11. [3] A nano topological space (U, $\tau_R(X)$) is said to be nano connected if U cannot be expressed as the union of two disjoint non-empty nano open set.

Definition 2.12. [3] A collection $\{A_i : i \in I\}$ of nano open sets in a nano topological spaces (U, $\tau_R(X)$) is said be nano open cover of a subset A in (U, $\tau_R(X)$) if $A \subseteq \cup_{i \in I} A_i$.

Definition 2.13. [3] A nano topological spaces (U, $\tau_R(X)$) is said be nano compact if every nano open cover of U has a finite subcover.

3 Nλψg - Connected

In this section, the concept of Nλψg -connected spaces is introduced. Also their basic properties and characterizations are discussed.

Definition 3.1. A nano topological spaces (U, $\tau_R(X)$) is said to be Nλψg -connected if U cannot be written as a union of two disjoint non empty Nλψg -open sets.

Definition 3.2. A subset G of a nano topological space (U, $\tau_R(X)$) is said to be Nλψg -connected set in U if G cannot be expressed as the union of two disjoint non empty Nλψg -open sets in (U, $\tau_R(X)$) .

Theorem 3.3. For a nano topological spaces (U, $\tau_R(X)$) the following statements are equivalent.

(i) U is Nλψg -connected.
(ii) The only subsets of U which are both Nλψg -open and Nλψg -closed are the empty set φ and U .
(iii) Each Nλψg -continuous function of U into a discrete space V with at least two points is a constant function.

Proof. (i) ⇒ (ii) Let U be a Nλψg -connected space. Let A be Nλψg –open and Nλψg -closed subset of U . Then U − A is both Nλψg -open and Nλψg - closed in U . That implies, U is the union of disjoint Nλψg -open sets A and U ⊆ A. Since U is Nλψg -connected either A = φ or U − A = φ . That is, A = φ or A = U .

(ii) ⇒ (i) Suppose that U = A ∪ B , where A and B are disjoint non empty Nλψg -open subsets of U . Then A and B are proper subsets of U . Since A = U − B , A is Nλψg -closed subset of U , then A is both Nλψg -open and Nλψg -closed subset of U . Therefore by assumption, A = φ or A = U , which is a contradiction. Thus U is Nλψg -connected.

(ii) ⇒ (iii) Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be Nλψg - continuous, where V is discrete space with atleast two points. Then U is covered by Nλψg –open and Nλψg -closed covering $\{f^{-1}(y) : y \in V\}$. By part (ii), $f^{-1}(y) = \varphi$ or U , for each $y \in V$. If $f^{-1}(y) = \varphi$, for all $y \in V$, then f fails to be a function. Therefore there exists atleast one point say $y_1 \in V$, such that $f^{-1}(y_1) \neq \varphi$ and hence $f^{-1}(y_1) = U$, which shows that f is a constant function.

(iii) ⇒ (ii) Let G be both Nλψg -open and Nλψg - closed in U . Suppose that $G \neq \varphi$. Let V be a discrete space with atleast two points, fix y_0 and y_1 in V and $y_0 \neq y_1$. Define $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ by $f(x) = \{y_0\}$, for $x \in G$ and $f(x) = \{y_1\}$, for $x \notin G$. Let F be a nano open set in V. If F contains y_0 alone, then $f^{-1}(F) = G$. If F contains both y_0 and y_1 , then $f^{-1}(F) = U$.

Otherwise f−1(F) = φ . In all case $f^{-1}(F)$ is Nλψg -open in U . Therefore f is Nλψg -continuous function. Then by assumption f is a constant function. Therefore $f(x) = y_0$ or $f(x) = y_1$, for all x in U . If $f(x) = y_0$, for all x in U , then G = U . If $f(x) = y_1$, for all x in U , then G = φ.

Theorem 3.4. If a space U is Nλψg - connected space, then it is nano connected.

Proof. Let U be a Nλψg - connected space. Suppose that U is not nano connected then U = A∪B, where A and B are disjoint non empty nano open sets in U . Since every nano open set is Nλψg -open, A and B are disjoint non empty Nλψg -open sets in U . This contradicts the fact that U is Nλψg - connected. Hence U is nano connected.

Remark 3.5. The following example shows that the converse of the above theorem need not be true in general.

Example 3.6. Let U = {a, b, c, d} and with U/R = {{a}, {b}, {c, d}} and X = {a, d} . Then $\tau_R(X)$ = {U, φ, {a}, {c, d}, {a, c, d}} . Then NλψGO(U, $\tau_R(X)$) = {U, φ, {a}, {c}, {d}, {a, b}, {a, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}} . Here U is nano connected but not Nλψg -connected because U can be written as union of two disjoint non empty Nλψg -open sets {d} ∪ {a, b, c} .

Theorem 3.7. $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is Nλψg -continuous surjection and U is Nλψg -connected, then V is nano connected.

Proof. Let U be Nλψg -connected. Suppose that V is not nano connected. Then V = A∪B , where A and B are disjoint non empty nano open sets of V. Since f is Nλψg -continuous and onto, $U = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty Nλψg -open sets in U . This contradicts the fact that U is Nλψg - connected. Therefore V is nano connected.

Theorem 3.8. If $f : (U, \tau_R(X)) \to \left(V, \tau'_R(Y)\right)$ is Nλψg -irresolute surjection and U is Nλψg -connected, then V is Nλψg -connected.

Proof. Assume that V is not Nλψg -connected. Then there exist disjoint non empty Nλψg -open sets A and B in V such that V = A ∪ B . Since f is Nλψg -irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are Nλψg -open sets in U. As f is a surjective function, $f^{-1}(A) \neq \varphi$ and $f^{-1}(B) \neq \varphi$, where $U = f^{-1}(V) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ which is a contradiction. This shows that V is Nλψg -connected.

4 Nλψg -separated

In this section, the concept of Nλψg -separated set is introduced. Also, their basic properties and characterizations are discussed. Further, the relationship of Nλψg -separated sets with Nλψg -connected sets are examined.

Definition 4.1. Two non-empty subsets G and H of a spaces U are called Nλψg -separated if Nλψgcl(G) ∩ H = G ∩ Nλψgcl(H) = φ.

Theorem 4.2. Any two disjoint non-empty Nλψg -closed sets are Nλψg -separated.

Proof. Suppose G and H are disjoint non-empty Nλψg -closed sets. Then Nλψgcl(G) ∩ H = G ∩ Nλψgcl(H) = G ∩ H = φ . Hence A and B are Nλψg -separated.

Remark 4.3. The following example shows that the converse of the above theorem need not be true in general.

Example 4.4. Let U = {a, b, c, d} and with U/R = {{a}, {b}, {c, d}} and X = {a, c} . Then τR(X) = {U, φ, {a}, {c, d}, {a, c, d}} . Let A = {a} and B = {b, d} . Then A and B are Nλψg -separated but not both Nλψg -closed.

Theorem 4.5. If G and H are Nλψg -separated and A ⊆ G, B ⊆ H , then A and B are also Nλψg -separated.
Proof. Let G and H be Nλψg -separated. Then Nλψgcl(G) ∩ H = G ∩ Nλψgcl(H) = φ . Since A ⊆ G and B ⊆ H , Nλψgcl(A) ⊆ Nλψgcl(G) and Nλψgcl(B) ⊆ Nλψgcl(H) which implies, Nλψgcl(A) ∩ B ⊆ Nλψgcl(G) ∩ H = φ and hence Nλψgcl(A) ∩ B = φ .
Similarly A ∩Nλψgcl(B) = φ . Therefore A and B are Nλψg -separated.

Theorem 4.6. If G and H are both Nλψg -open and if A = G ∩ (U − H) and B = H ∩ (U − G) , then A and B are Nλψg -separated.
Proof. Let G and H be Nλψg-open subsets in U . Then U − G and U – H are Nλψg -closed. Since A ⊆ U −H , Nλψgcl(A) ⊆ Nλψgcl(U −H) = U –H and so Nλψgcl(A) ∩ H = φ . Since B ⊆ H , Nλψgcl(A) ∩ B = φ . Similarly, Nλψgcl(B) ∩ A = φ . Hence A and B are Nλψg -separated.

Theorem 4.7. The subsets G and H of a space U are Nλψg -separated if and only if there exist a Nλψg -open sets P and Q such that G ⊆ P and H ⊆ Q , G ∩ Q = φ and H ∩ P = φ .
Proof. Suppose G and H are Nλψg -separated. Then Nλψgcl(G) ∩ H = G ∩ Nλψgcl(H) = φ . Take Q = U − Nλψgcl(G) and P = U − Nλψgcl(H) . Then P and Q are Nλψg -open sets such that G ⊆ P and H ⊆ Q, G∩Q = φ , H ∩ P = φ .
Conversely, assume that P and Q are Nλψg -open sets such that G ⊆ P and H ⊆ Q, G ∩ Q = φ, H ∩ P = φ . Then G ⊆ U − Q, H ⊆ U − P and U−Q, U−P are Nλψg -closed which implies Nλψgcl(G) ⊆ Nλψgcl(U−Q) = U − Q _ U − H and Nλψgcl(H) ⊆ Nλψgcl(U − P) = U − P ⊆ U − G. Then Nλψgcl(G) ⊆ U − H and Nλψgcl(H) ⊆ U − G. Hence H ∩ Nλψgcl(G) = φ and G ∩ Nλψgcl(H) = φ . Then G and H are Nλψg -separated.

Theorem 4.8. Each two Nλψg -separated sets are always disjoint.
Proof. Let G and H be Nλψg -separated. Then G ∩ Nλψgcl(H) = φ = Nλψgcl(G)∩H . Now G∩H ⊆ G∩Nλψgcl(H) = φ . Then G∩H = φ . Hence G and H are disjoint.

Theorem 4.9. A nano topological space (U, τ_R(X)) is Nλψg - connected if and only if U is not the union of any two Nλψg -separated sets.
Proof. Let U is Nλψg -connected. Suppose U = G ∪ H , where G and H are Nλψg -separated sets. By the Definition 4.1, Nλψgcl(G) ∩ H = G ∩ Nλψgcl(H) = φ . Since G ⊆ Nλψgcl(G) , G ∩ H ⊆ Nλψgcl(G) ∩ H = φ . Therefore G and H are disjoint, also G ⊆ U − H , Nλψgcl(G) ⊆ U − H = G and Nλψgcl(H) ⊆ U−G = H. Hence G = Nλψgcl(G) and H = Nλψgcl(H). Therefore G and H are Nλψg -closed sets and hence G = U − H and H = U −G are disjoint Nλψg -open sets. That is, U is not Nλψg -connected, which is contradiction to U is a Nλψg -connected space. Hence U is not the union of any two Nλψg -separated sets.
Conversely, assume that U is not the union of any two Nλψg -separated sets. Suppose U is not Nλψg -connected, then U = G ∪ H , where G and H are non empty disjoint Nλψg -open sets in U . Since G = U −H and H = U −G, Nλψgcl(G) ∩H = (U −H) ∩H = φ and G ∩Nλψgcl(H) = G ∩ (U − G) = φ . That is, G and H are Nλψg -separated sets, which is a contradiction to our assumption. Hence U is Nλψg -connected.

Theorem 4.10. If F ⊆ G ∪ H , where F is a Nλψg -connected set and G,H are Nλψg -separated sets, then either F ⊆ G or F ⊆ H .
Proof. Suppose F ⊄ G and F ⊄ H . Let F_1 = G ∩ F and F_2 = H ∩ F. Since F ⊆ G ∪ H , F_1 and F_2 are non empty sets and F_1 ∪ F_2 = (G ∩ F) ∪ (H ∩ F) = (G ∪ H) ∩ F = F . Since F_1 ⊆ G, F_2 ⊆ H and G,H are Nλψg - separated sets, Nλψgcl(F_1)∩F_2 ⊆ Nλψgcl(G)∩H = φ and F_1∩Nλψgcl(F_2) ⊆ G ∩ Nλψgcl(H) = φ . Therefore F_1 , F_2 are

Nλψg -separated sets such that F = F_1 ∪ F2 . Hence by Theorem 4.9, F is not Nλψg -connected, which is a contradiction to F is Nλψg -connected. Hence either F ⊆ G or F ⊆ H .

Theorem 4.11. If F is Nλψg -connected, then Nλψgcl(F) is also a Nλψg -connected set.

Proof. Let F be a Nλψg -connected set. Suppose Nλψgcl(F) is not Nλψg-connected. Then by Theorem 4.9, there exist Nλψg -separated sets G and H such that Nλψgcl(F) = G ∪ H. Since F is Nλψg -connected set and F ⊆ Nλψgcl(F) = G H , by Theorem 4.10, either F ⊆ G or F ⊆ H . If F ⊆ G, then Nλψgcl(F) ⊆ Nλψgcl(G) . Since G and H are Nλψg -separated sets, by Theorem 4.8, G ≠ φ , H ≠ φ and Nλψgcl(F) ∩ H ⊆ Nλψgcl(G) ∩ H = φ and hence H ⊆ U − Nλψgcl(F). Also H ⊆ G ∪ H = Nλψgcl(F). Therefore H ⊆ (U −Nλψgcl(F)) ∩Nλψgcl(F) = φ . Which is a contradiction to H ≠ φ. Similarly, if F ⊆ H , we get a contradiction to G ≠ φ . Hence Nλψgcl(F) is a Nλψg -connected set.

Theorem 4.12. Let F be a Nλψg -connected subset of a space U. If G is a subset of U such that F ⊆ G ⊆ Nλψgcl(F) , then G is Nλψg -connected.

Proof. Suppose G is not Nλψg -connected. By Theorem 4.9, there exist two non empty Nλψg -separated sets A and B such that G = A ∪ B . Since F ⊆ G = A ∪ B and by Theorem 4.10, F ⊆ A or F ⊆ B. If F ⊆ A implies that, Nλψgcl(F) ⊆ Nλψgcl(A). Now Nλψgcl(F) ∩ B ⊆ Nλψgcl(A) ∩ B = φ, which implies Nλψgcl(F) ∩ B = φ . Also A ∪ B = G Nλψgcl(F) , B ⊆ G ⊆ Nλψgcl(F). Hence Nλψgcl(F)∩B = B . Then B = φ , which is contradiction to B is non empty. Similarly, if F ⊆ B , we get a contraction to A ≠ φ. Hence G is Nλψg -connected.

Theorem 4.13. If G and H are Nλψg -connected subset of a space U such that G ∩ H ≠ φ , then G ∪ H is a Nλψg -connected subset of U.

Proof. Suppose that G ∪ H is not Nλψg -connected. Then by Theorem 4.9, there exist two Nλψg -separated sets F , K such that G ∪ H = F ∪ K . Since F and K are Nλψg -separated, F , K are non empty sets and F ∩ K ⊆ Nλψgcl(F) ∩ K = φ . Since G ⊆ G ∪ H = F ∪ K , H ⊆ G ∪ H = F ∪ K and G, H are Nλψg -connected, by Theorem 4.10, G ⊆ F or G ⊆ K and H ⊆ F or H ⊆ K .

Case (i): If G ⊆ F and H ⊆ F , then G ∪ H ⊆ F and so G ∪ H = F. Since F and K are disjoint, K = φ which is contradiction to K ≠ φ. Similarly, if G ⊆ K and H ⊆ K we get the contradiction.

Case (ii): If G ⊆ F and H ⊆ K, then G∩H ⊆ F ∩K = φ . Then G ∩H = φ , which is a contradiction to G∩H ≠ φ . Similarly, if G ⊆ K and H ⊆ F , we get the contradiction. Hence G ∪ H is Nλψg -connected subset of a (U, τ_R(X)).

Definition 4.14. A nano topological space (U, τ_R(X)) said to be Nλψg –disconnected if and only if it is not Nλψg -connected.

Remark 4.15. A nano topological space (U, τ_R(X)) is Nλψg –disconnected if and only if U can be expressed as the union of two disjoint non-empty Nλψg-open sets.

Theorem 4.16. A space U is Nλψg –disconnected if and only if there exists a non-empty proper subset of U which is both Nλψg -open and Nλψg -closed in U.

Proof. Let G be a non-empty proper subset of U which is both Nλψg –open and Nλψg -closed. We have to prove that U is Nλψg -disconnected. Let F = U − G. Then F is a non-empty set and G ∪ F = U and G ∩ F = φ. Since G is both Nλψg -open and Nλψg -closed, F is both Nλψg -open and Nλψg -closed. Thus U can be written as the union of two disjoint non-empty Nλψg -open sets. Hence U is Nλψg -disconnected.

Conversely, let U be Nλψg -disconnected. Then there exist non-empty Nλψg-open subsets G and F such that X = G ∪ F . Then F = U−G and G = U−F , which are Nλψg -closed in U . Hence G and F are both Nλψg -open and Nλψg -closed in U.

Example 4.17. Consider the nano topology on .Since is a non-empty proper subset of U which is both Nλψg -open and Nλψg -closed, the space U is Nλψg - disconnected.

Remark 4.18. Suppose a subset A is Nλψg -disconnected. Then there exists two disjoint Nλψg -open sets G and F such that G ∩ A ≠ φ , F ∩ A ≠ φ , (G ∩ A) ∩ (F ∩ A) = φ and (G ∩ A) ∪ (F ∩ A) = A.

Theorem 4.19. Let (U, τ_R(X)) be a nano topological space and let A be a subset of U. Then A is nano disconnected if and only if there exist non-empty sets G and F both Nλψg –open (Nλψg –closed) in U such that G ∩ A ≠ φ , F ∩ A ≠ φ , A ⊆ G ∪ F and G ∩ F ⊆ U − A.

Proof. By Remark 4.18, A is Nλψg -disconnected if and only if there exist nonempty sets G and F both Nλψg -open (Nλψg -closed) in U such that G∩A ≠ φ , F ∩A ≠ φ , (G∩A)∩(F ∩A) = φ and (G∩[(.)-7A) ∪ (F ∩A) = A. Now (G∩A)∩ (F ∩A) = φ if and only if (G∩ F) ∩A) = φ if and only if G∩ F ⊆ U −A and (G∩A) ∪ (F ∩A) = A if and only if (G ∪ F)∩A = A if and only if A ⊆ G ∪ F .

5 Nλψg - compact spaces

In this section the concept of Nλψg - compact spaces using Nλψg -open sets are introduced and some of their properties are discussed.

Definition 5.1. A collection {A_i : i ∈ I} of Nλψg -open sets in a nano topological space (U, τ_R(X)) is called Nλψg -open cover of a subset A in (U, τ_R(X)) if A ⊆ $\sum_{i \in I}$(Ai) .

Definition 5.2. A nano topological space (U, τ_R(X)) is called Nλψg –compact if every Nλψg -open cover of (U, τ_R(X)) has a finite subcover.

Definition 5.3. A subset A of a nano topological space (U, τ_R(X)) is called Nλψg -compact relative to U if for every Nλψg -open cover of U has a finite subcover.

Definition 5.4. A subset A of a nano topological space (U, τ_R(X)) is called Nλψg -compact if A is Nλψg -compact of the subspace of (U, τ_R(X)) .

Theorem 5.5. A Nλψg -closed subset of Nλψg -compact space is Nλψg - compact relative to (U, τ_R(X)) .

Proof. Let A be a Nλψg -closed subset of a nano topological space U. Then U − A is Nλψg -open in U . Let H = {A_i : i ∈ I} be a Nλψg -open cover of A by Nλψg -open subsets in U . Then H ∪ {U − A} is a Nλψg – open cover of U . Since U is Nλψg -compact, then there exists a finite subcover say {A_1, A_2, ...A_n, (U − A)} . Then {A_1,A_2, ..A_n} is a finite Nλψg -open cover of A . Hence A is Nλψg -compact relative to U .

Theorem 5.6. Every Nλψg -compact space is nano compact.

Proof. Let U be Nλψg -compact. Let {A_i : i ∈ I} is a Nλψg -open cover of U . Since every nano open set is Nλψg -open. Since U is Nλψg -compact, then Nλψg -open cover {A_i : i ∈ I} of U has a finite subcover, say {A_i : i =1, 2, ...n} for U . Hence U is nano compact.

Theorem 5.7. Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be surjective, Nλψg –continuous function. If U is Nλψg -compact, then V is nano compact.

Proof. Let {A_i : i ∈ I} be a nano open cover of V . Since f is Nλψg -continuous function, then {$f^{-1}(A_i)$: i ∈ I} is Nλψg -open cover of U . Since U is Nλψg -compact, {$f^{-1}(A_i)$: i ∈ I} contains a finite subcover say {$f^{-1}(A_i)$: i

= 1..n} . Since f is surjective, then {A_1, A_2, ...A_n} is a finite subcover of {A_i : i ∈ I} , for V . Hence V is nano compact.

Theorem 5.8. If a function $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ is Nλψg –irresolute and a subset A of U is Nλψg compact relative to U, then the image f(A) is Nλψg -compact relative to V.

Proof. Let {A_i : i ∈ I} be any collection of Nλψg -open sets in V such that $f(A) = \cup_{i \in I} A_i$. Then $A \subseteq \cup_{i \in I} f^{-1}(A_i)$, where {$f^{-1}(A_i)$: i ∈ I} is Nλψg -open set in U . Since A is Nλψg compact relative to U , there exists finite subcollection {A_1, A_2, ..., A_n} such that $A \subseteq \cup_{i \in I} f^{-1}(A_i)$. Therefore $f(A) \subseteq \cup_{i \in I} A_i$. Hence f(A) is Nλψg -compact relative to V .

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