



# USING OF QUEUING THEORY TO MINIMIZE WAITING TIME IN HOSPITAL SYSTEM

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## Abstract:

This study applies queuing theory to hospital management. The purpose of this study is to describe and categorize issues of urgency in medical cases with respect to patient allocation issues. It employs first-come-first-served, queue-discipline, Poisson arrival of patients and exponential service time, first-come-first-served, queue-discipline.

A queue model used to estimate patient wait times, service consumption, model system design, and reservation system ratings. A queuing system helps minimize patient wait times and reduce server load. H. Doctors, nurses, hospital beds, etc. Queues are not new, but hospitals have recently started using them effectively.

**KEYWORDS:** Queuing system, FCFS, Poisson arrival

## INTRODUCTION:

In hospital systems, increasing resource utilization to reduce costs, reducing patient wait times for timely care, and improving patient satisfaction are important but conflicting goals. . Queuing models are popular with researchers and system designers because they can provide fairly accurate assessments of system performance, are analytical in nature, and can provide quick solutions for "what-if" analysis. . A considerable amount of research has been published on how cues can be used to analyze and design hospital environments. We review and classify this literature to motivate further research on the application of queuing models in medicine. Planning is the most important aspect of starting a hospital. If the planning is good, everything can be fine. If you don't plan carefully, you may not finish the job. Planning for a new hospital begins with setting goals for the hospital. Without it, an organization cannot have clear direction and focus. This is followed by an examination of the organization's external environment and the internal and external resources used to achieve the set goals. This exercise facilitates the selection of means by which goals can be achieved at reasonable cost. Cue models are widely used to model and analyze various healthcare systems, such as hospitals (Cochran and Roche, 2009), the pharmaceutical industry (Viswanadham and Narahari, 2001), and organ transplantation (Zenios, 1999). I'm here. Prater (2001) compiled a bibliography of medical queuing applications almost ten years ago. This paper reviews and classifies the application of queuing models in hospital management. The correct allocation of beds in different wards of the hospital was debated. First,

estimate the volume required for your system. Estimated demand is divided into two components he called controllable and uncontrollable. A controllable demand component is defined as patients who require advance appointments for physical care. The latter category includes emergencies

## 2. REVIEW OF LITERATURE:

The history of queuing theory goes back almost 100 years. How to describe the concept of statistical equilibrium and the equilibrium equation of state (later called the Chapman-Kolmogorov equation [1]). "Rational determination of the number of circuits" (Brook Meyer et al. (1960) [2]), which deals with optimization problems in queuing theory at the time and was defined by basic research in the 1950s and his 1960s, there is Special note. Hiller (1963) [3]'s paper on the economic model of the industrial queuing problem states that Hiller considers his M/M/1 queuing and Heymans (1968) [4] considers the optimal server capacity, possibly queuing It suggests that you have derived the standard in question. This is the first article that introduces general optimization techniques. Theoretically, it turns off depending on the state of the system on the M/G/1 queue. Since then, operational researchers trained in mathematical optimization techniques have described many more complex uses of queuing systems. A queuing system with impatient customers is discussed for a simple steady-state constrained process: B. M/M/1 system. Most of the work has been done on systems using Lock or Renegade. In a special case, a combined repulsion and repulsion analysis was performed. A study on impatient customers resisting and opposing the M/M/1 system was first developed by Udagawa and Nakamura (1957) [5]. Probability of resistance and stubbornness depends on 'N'. Ancker and Gafarian (1962, 1963) [6, 7] considered the combined occurrence of occlusion and regeneration in M/M/1 and M/M/R systems. Ancker and Gafarian (1963) [8] considered the M/M/1 system with further assumptions about customer blocking and abandonment behavior. In this system, incoming customers are blocked with probability  $n/N$  and therefore have a chance of joining the system.

$$E_n = 1 - n/N \quad (n=0, 1, 2, 3\dots N)$$

Where  $n$  is the number in your system and  $N$  is the maximum number allowed in your system. After entering the queue, each customer waits a certain amount of time for service. This is the density function random variable.

$$d(t) = \alpha e^{-\alpha t}$$

If the service is not started by then, it will leave the system and be lost. Ancker and Gafarian (1963) [9] considered another model. The model makes the same assumptions about abandonment behaviors, with the difference being that abandonment behaviors are more likely to involve inbound customers in the system.

$$E_n = \{\beta/n\} \quad \text{if } (0 \leq \beta \leq 1, n=1, 2, 3\dots)$$

$$\text{If } E_n = 1 \quad (n = 0)$$

Where ' $n$ ' is the number of systems and  $\beta$  is a measure of customer willingness to queue. This issue removed the limit on the number of customers in the system. The above work on this topic is limited to special cases where blocking and denial depend on the number of customers in the system. Adam et al. (2001) [10] deal extensively with multiqueue queues. We have thus outlined the growth of queuing theory and identified important developments and directions. Seguin (2020) conducted research at his Adekunle Ajasin University in Akungba-Akoko, Nigeria, on performance modeling of health service delivery using queuing theory. The purpose of this study was to model an appropriate queuing system by determining patient waiting, arrival, and service times in an AUAA health environment and validating the model using simulation techniques bottom. The study was conducted at his AUAA Health Center Akungba Akoko. A suitable model was developed using analytical and simulation methods. A stopwatch was used to calculate the number of minutes each patient spent arriving and picking up a ticket or registering from the reception area to the final area (examination room area). Data on each patient's arrival time, waiting time, and service time were collected on weekdays

(Monday to Friday) for her 3 weeks. Data were calculated and analyzed using Microsoft Excel. Based on the analyzed data, a queuing system for real patient situations was modeled and simulated with PYTHON software. The results obtained from the simulation model showed that the average patient arrival rate on Friday of week 1 was lower than the average patient service rate (that is,  $5.33 > 5.625 (\lambda > \mu)$ ). In other words, there is a queue that goes on forever. Service facilities are manned at all times. After analyzing the AAUA health center system as a whole, we found that when the system was very busy, queue lengths increased. The study recommends that the AUAA health center should improve the quality of service to patients who visit the health center.

### **3. OBJECTIVE OF RESEARCH WORK:**

This research has two main objectives:

1. Investigate patient wait times at hospitals and investigate operational issues that may increase patient wait times at hospitals. A patient's experience of waiting has a fundamental impact on their perception of service quality.
2. The patients who need Emergency service, E.g.: Intensive care unit, maternity, accident etc.

### **4. QUEUING THEORY CONCEPT:**

Queues are a challenge for all healthcare systems. In today's world, considerable research projects are being carried out to improve queuing systems in various hospital environments. Unfortunately, this is not the case in developing countries. This paper seeks to contribute to this topic by analyzing public hospital queuing conditions and bringing practical value to how to improve hospital decision-making. Queuing theory is a powerful mathematical approach to analyzing queuing performance parameters in healthcare systems (Oscan, 2006). It is becoming a popular management tool for decision-making in developed countries.

### **5. Classification of queuing models**

Using Kendal & Lee notations Generally, any queuing models may be completely specified in the following symbolic form

A / B / C / D / E

A → Type of distribution of inter – arrival time

B → Type of distribution of inter – service time

C → Number of servers

D → Capacity of the system

E → Queue discipline

M → Arrival time follows Poisson distribution and service time follows an exponential distribution. [11]

#### **Model I: M / M / 1: ∞ / FCFS**

Where M -Arrival time follows a Poisson distribution

M → Service time follows a exponential distribution

1 → Single service model

$\infty \rightarrow$  Capacity of the system is infinite

FCFS  $\rightarrow$  Queue discipline is first come first served

### **Model II: M / M / 1: N / FCFS**

Where N  $\rightarrow$  Capacity of the system is finite

### **Model III: M / M / 1: $\infty$ / SIRO**

Where SIRO  $\rightarrow$  Service in random order

### **Model IV: M / O / 1: $\infty$ / FCFS**

Where D  $\rightarrow$  Service time follows a constant distribution or is deterministic

### **Model V: M / G / 1 : $\infty$ / FCFS**

Where G  $\rightarrow$  Service time follows a general distribution or arbitrary distribution

### **Model VI: M / Ek / 1: $\infty$ / FCFS**

Where Ek  $\rightarrow$  Service time follows Erlangen distribution with K phases.

### **Model VII: M / M / K: $\infty$ / FCFS**

Where K  $\rightarrow$  Multiple Server model

### **Model VIII: M / M / K: N / FCFS**

### **Model I: M / M / 1: $\infty$ / FCFS**

#### **6. Formulas:**

- Utilization factor traffic intensity / Utilization parameter / Busy period

$$\rho = \lambda / \mu$$

Where  $\lambda$  = Mean arrival rate,  $\mu$  = mean service rate

**Note:**  $\mu > \lambda$  in single server model only

- Probability that exactly zero units are in the system

$$P_0 = 1 - \lambda / \mu$$

- Probability that exactly 'n' units in the system

$$P_n = p_0 (\lambda / \mu)^n$$

- Probability that n or more units in the system

$$P_{n \text{ or more}} = (\lambda / \mu)^n$$

More than 'n' means n should be n+1

- Expected number of units in the queue / queue length

$$L_q = \lambda^2 / \mu (\mu - \lambda)$$

- Expected waiting time in the queue

$$W_q = L_q / \lambda$$

7. Expected number of units in the system

$$L = L_q + \lambda / \mu$$

8. Expected waiting time in the system

$$W = W_q + 1 / \mu$$

9. Expected number of units in queue that from time to time – (OR) non - empty queue size

$$D = \mu / \mu - \lambda$$

10. Probability that an arrival will have to wait in the queue for service

$$\text{Probability} = 1 - P_0$$

11. Probability that an arrival will have to wait in the queue more than w (where w > 0), the waiting time in the queue

$$\text{Probability} = (\lambda / \mu) e^{-(\lambda - \mu) w}$$

12. Probability that an arrival will have to wait more than v (v > 0) waiting time in the system is

$$= e^{-(\lambda - \mu) v}$$

13. Probability that an arrival will not have to wait in the queue for service = P<sub>0</sub>

### Model 1 - Problems

**Example:** In a municipality hospital patients arrival are considered to be Poisson with an arrival interval time of 10mins. The doctors (examination and dispensing) time many be assumed to be ED with an average of 6mins find:

- a) What is the chance that a new patient directly sees the doctor?
- b) For what proportion of the time the doctor is busy?  
What is the average number of patients in the system?
- c)
- d) What is the average waiting time of the system?
- e) Suppose the municipality wants to recruit another doctor, when an average waiting time of an arrival is 30mins in the queue. Find out how large should be to justify a 2nd doctor? [12, 13]

### Solution:

$$\lambda = \frac{1}{10} \times 60 = 6/\text{hr}$$

$$\mu = \frac{1}{6} \times 60 = 10/\text{hr}$$

Probability that a new patient straight away sees the doctor:

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{6}{10} = 0.4$$

(a) Proportion of time the doctor is busy:

$$\rho = \frac{\lambda}{\mu} = \frac{6}{10} = 0.6 \text{hr}$$

(b) Average number of patients in the system:

$$L = L_q + \frac{\lambda}{\mu} = \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} = \frac{6^2}{10(10-6)} + \frac{6}{10} = \frac{36}{40} + \frac{6}{10}$$

$$L = 1.5$$

(c) Average waiting in the system:

$$W = W_q + \frac{1}{\mu} = \frac{L_q}{\lambda} + \frac{1}{\mu} = \frac{\lambda^2}{\mu(\mu-\lambda)\lambda} + \frac{1}{\mu}$$

$$W = \frac{6}{10(10-6)} + \frac{1}{10} = \frac{6}{40} + \frac{1}{10} = 0.25$$

$$W_q = \frac{30}{60} = 0.5 \text{ hr}$$

$$W_q = \frac{L_q}{\lambda} = \frac{\lambda^2}{\mu(\mu-\lambda)\lambda} = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{6}{10(10-6)} = 0.5$$

(d)  $\frac{\lambda}{10(10-\lambda)} = 0.5$

$$\frac{\lambda}{100 - 10\lambda} = 0.5$$

$$\lambda = 100 - 10\lambda \times 0.5$$

$$\lambda = 50 - 5\lambda$$

$$\lambda = \frac{50}{6} = 8.33/ \text{hr.}$$

$\lambda$  The value of  $\lambda$  has to be increased from 6 to 8.33 to justify a second doctor.

## CONCLUSION:

In our study, the system not only serves people who request services in hospitals, but also uses their time for other activities. The benefits of using this open source can extend to society as a whole. He is not just one hospital that can benefit from the current system design and setup, he can also serve multiple hospitals at the same time. A single hospital can track its own queues with specific power users as queue managers. This design allows cost-sharing agreements between hospitals without spending budget on additional development. Multiserver queues can be used to estimate average wait times, queue lengths, number of servers, and service rates.

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