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Analysis Of Area Of Various Geometrical Figures And Physical Perspectives

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Abstract

Plane surface area is analyzed using *ab* initio method. Trapezium is put in the centre of consideration with proper mathematical logic. General formula is developed similar to Heron's formula. Pythagoras theorem is proved in much simpler way.

Index Terms Trapezium area formula , Pythagoras theorem by rotation., Heron like formula

I. INTRODCTION TO CONCEPT OF AREA

The concept of an area plays pivotal role in geometry, agriculture, mathematics and physics. The extended version of neighborhood may be regarded as an area[1]. We call this finite version of *nbd* as a cell. In Euclidean geometry a basic cell can be chosen as a square. Here we are deliberately choosing square shape as cell in place of circle as repetition of a square will occupy the entire plane while repetition of a circle will leave the holes. Thus number of basic squares within a figure will be its area. The direction of area is perpendicular to the plane and hence the principle of linear superposition leads to the simple scalar sum of the individual parts [2]. In the similar fashion area of a part can be extracted from a big figure using proportionality. Since square is the simplest figure, we take it as a particular case of a generalized figure [3].

II. INTRODCTION TO GENERALIZED FIGURE

The generalized figure needs not be a unique one, but we choose it in the context of simplicity of formula. Figure 1, shows that a square is a special case when two circles of equal radii have a separation of radial length between their centers. With different and arbitrary separation can be incorporated into a simple generalized figure which is trapezium in this figure.

fig.1

III CONSTRUCTION AND AREA OF TRAPEZIUM

When three informations as inputs are given we can construct a triangle. A quadrilateral can be regarded as combination of two triangles with one side as a common side which is either of the diagonals of it. Hence we need 3+3-1=5 inputs in all for the constructions of any quadrilateral. In case of trapezium the five informations are (1) b_1 (2)b_2 (3) l_1(4)l_2 (5) $b_1 \parallel b_2$.

The length 'b_1' will decide points '1' and '2'. The length 'b_2' will decide points '1' and '5'. The arcs of radii 'l_1'and 'l_2' with centers '5' and '2' respectively shall decide point '3'. The arcs of radii 'l_1'and 'b_2' with centers '1' and '3' respectively must decide point '4', as shown in figure 2.

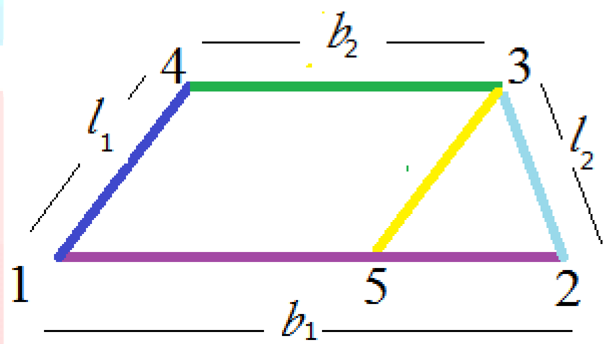

figure 2

Now we proceed to calculate the area of a trapezium. The basic concept in area is the square of which we need to know any side one side only. Its diagonal is to be decided by Pythagoras theorem and then nothing is unknown regarding the square. There are various methods to prove Pythagoras theorem, but our first author developed new, concise and simple method using rotation to accomplish the task.

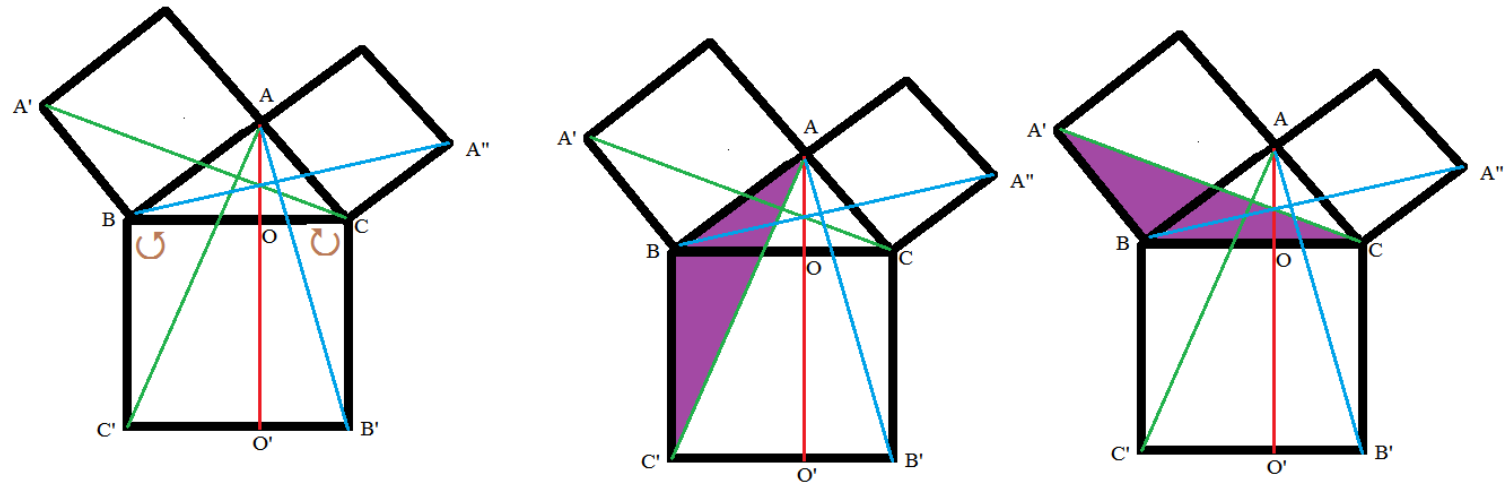

figure 3

Fig.3 shows, rectangle OBC’O’ and triangle ABC’ have the *common base* BC’ and the *common height* OB. If this triangle is rotated in counter clockwise direction with centre B about an angle of 90^0, we see new location is the triangle is A’BC. The square with base AB and triangle A’BC have the *common base* A’B and the *common height* AB. This shows that rectangle OBC’O’ has the area equal to square of side A’B=AB.

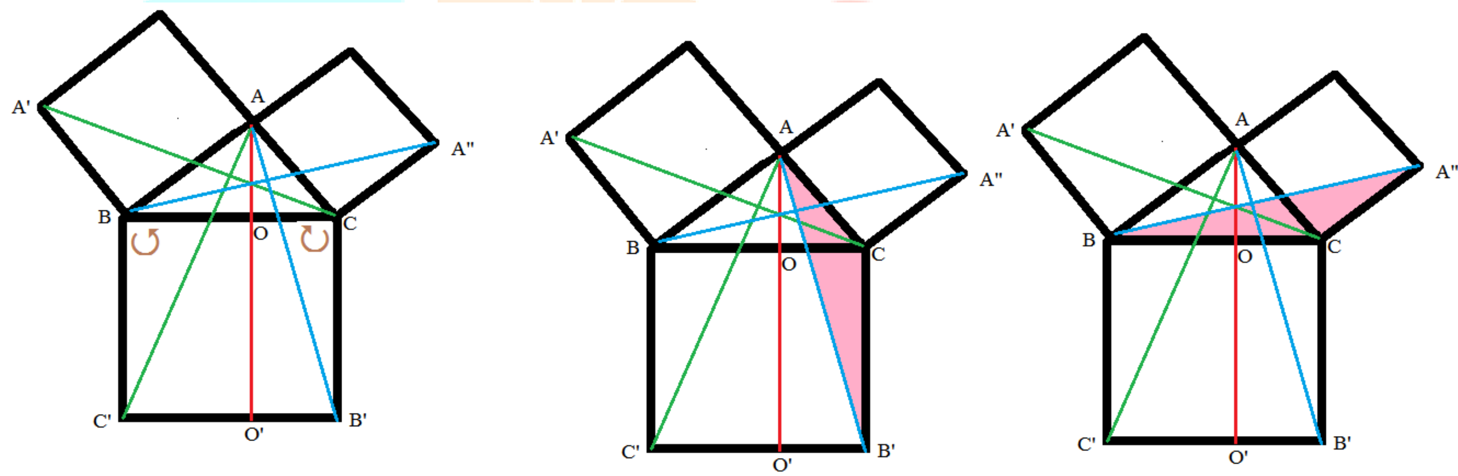

figure 4

In fig.4, we see similar rotation, in clockwise direction with centre C, yields that rectangle OCB’O’ has the area equal to square of side A’C=AC.

Combining these two we get square of BC= square of AB+ square of AC, i.e., $BC^2=AB^2+AC^2$.

One version of area of a trapezium lies in the vision that it is the figure between two parallel lines. The height of it is distance between these lines and any diagonal of it shows that two triangles have the bases on either of parallel lines. Thus area of trapezium is

$$A_{\Pi} = \tfrac{1}{2} height \times (sum\ of\ \ parallel\ \ sides)$$

$$A_{\Pi} = \tfrac{1}{2} h \times (b_1 + b_2) \qquad \ldots(1)$$

Its reduction yields area of various figures:

[1]. $b_2 = 0$, triangle,

[2]. $b_1 = b_2$, parallelogram, rectangle, rhombus,

[3]. $h = b_1 = b_2$, square.

Thus single formula can be used to calculate the area of various figures with the great simplicity provided that we know the height, i.e. the figure is constructed between two lines of known separation.

But when height is not known, it is to be calculated using Pythagoras theorem.

Close inspection of fig.2 shows that the height of the trapezium and triangle is the same.

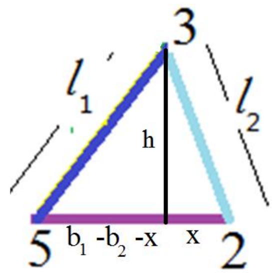

fig.5

$$l_1^2 = h^2 + (b_1 - b_2 - x)^2 ; \; l_2^2 = h^2 + x^2 \; ; \; 2x(b_1 - b_2) = (b_1 - b_2)^2 + l_2^2 - l_1^2 \qquad \ldots(2)$$

$$\begin{aligned}
& l_2^2 (b_1 - b_2)^2 = h^2 (b_1 - b_2)^2 + x^2 (b_1 - b_2)^2 \\
& \Rightarrow 4x^2 (b_1 - b_2)^2 = 4l_2^2 (b_1 - b_2)^2 - \tfrac{16}{4} h^2 (b_1 - b_2)^2 \\
& \Rightarrow \left\{(b_1 - b_2)^2 + l_2^2 - l_1^2\right\}^2 = 4l_2^2 (b_1 - b_2)^2 - 16A^2 \\
& \Rightarrow 16\Delta^2 = \left\{2l_2 (b_1 - b_2)\right\}^2 - \left\{(b_1 - b_2)^2 + l_2^2 - l_1^2\right\}^2 \\
& = \left[\left\{2l_2 (b_1 - b_2) + (b_1 - b_2)^2 + l_2^2\right\} - l_1^2\right]\left[\left\{2l_2 (b_1 - b_2) - (b_1 - b_2)^2 - l_2^2\right\} + l_1^2\right] \\
& = \left[\left\{(b_1 - b_2) + l_2\right\}^2 - l_1^2\right]\left[l_1^2 - \left\{(b_1 - b_2) - l_2\right\}^2\right] \\
& = \left[(b_1 - b_2) + l_2 + l_1\right]\left[(b_1 - b_2) + l_2 - l_1\right]\left[l_1 + (b_1 - b_2) - l_2\right]\left[l_1 - (b_1 - b_2) + l_2\right] \\
& = \left[(b_1 - b_2) + (l_1 + l_2)\right]\left[(b_1 - b_2) - (l_1 - l_2)\right]\left[(b_1 - b_2) + (l_1 - l_2)\right]\left[-(b_1 - b_2) + (l_1 + l_2)\right] \\
& = (\Delta b + \Sigma l)(\Delta b - \Delta l)(\Delta b + \Delta l)(-\Delta b + \Sigma l) = -(\Delta b + \Sigma l)(\Delta b - \Sigma l)(\Delta b + \Delta l)(\Delta b - \Delta l) \\
& A_\Lambda = \tfrac{i}{4}\sqrt{(\Delta b + \Sigma l)(\Delta b - \Sigma l)(\Delta b + \Delta l)(\Delta b - \Delta l)}
\end{aligned}$$

…(3)

$$A_\Lambda = \tfrac{1}{2} h (b_1 - b_2) = \tfrac{1}{2} h \Delta b \; ; \; A_\Pi = \tfrac{1}{2} h (b_1 + b_2) = \tfrac{1}{2} h \Sigma b$$

$$\therefore A_\Pi = \frac{\Sigma b}{\Delta b} A_\Lambda \qquad \ldots(4)$$

Using Eq.(4), we get, from Eq.(3),

$$A_\Pi = \tfrac{i}{4}\left(\frac{\Sigma b}{\Delta b}\right)\sqrt{(\Delta b + \Sigma l)(\Delta b - \Sigma l)(\Delta b + \Delta l)(\Delta b - \Delta l)} \qquad \ldots(5)$$

Where $\Delta x = x_1 - x_2$ and $\Sigma x = x_1 + x_2$, …(6)

IV. DISCUSSION OF THE RESULT

Our schematics runs in self- consistent way, and we see that the general formula, Eq. (5), looks like Heron's formula for triangle or Brahmgupta's Formula for cyclic quadrilateral except for the modulation factor.

This Heron like formula can be utilized to calculate area of different figures subjected to the conditions mentioned in conjunction with Eq.(1). The beauty of Heron's formula is its first term in product under square root covers non-negativity of semi-perimeter and others are concerned with triangular inequalities. Brahmgupta 's formula was obtained for cyclic quadrilateral and Heron's formula is for triangle and we that a triangle is always cyclic. Here we got Heron like formula for trapezium which needs not be cyclic.

V. CONCLUSION

In this paper we developed a simpler route based on *ab- initio* concepts of pure mathematical origin rather than computer methodologies. The authors think that scholars at university and college level must contribute in the pedagogy of basic natural sciences. In this way we claim that this paper will be extremely useful for teachers as well as students of fundamental mathematics.

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