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Application of Differential Equations in Medical Field

Y. D Teja

Researcher, Graduate from Department of Electrical and Electronics Engineering, JNTUK UCEV, Andhra Pradesh, India.

ABSTRACT

Medical field is more significant because it helps in the treatment and diagnosis of ailments, diseases and conditions and it break downs the molecular mechanism of every disease. The application of Different Equations (DE) in medical field has been executed in the prevailing work. DE refers to the mathematical statement that has involved one or more derivatives and it has also signified the rate of change of unceasing wavering quantities. DE has been general in science and engineering and in various fields of quantitative study, since it could be directly measured and perceived for systems which endured changes. Considering the medical field, DEs are used in Pathology, Cardiology, and dengue fever and in cancer therapy have been highlighted in the work. Moreover, challenges and the implementation of the DE has been explored in the work. Literature from 2018 -2022 has been preferred for the work. During the research, choosing the appropriate paper for the topics have been a broad task. The results shown that the data gained by the mathematical models could imbibe the Ordinary Differential Equation, Partial differential equations, Linear and Non-Linear equations, Homogeneous and Non-Homogeneous equations could be more precise than the data obtained from the classical model. The obtainable work will enact as a guide for future analysis based on the application of differential equation in Medical Field.

KEYWORD

Different Equations, Cardiology, Linear and Non-Linear equations, Pathology, Variable Order.

1. INTRODUCTION

Mathematically, a Differential Equation (DE) has represented the relationship between one or more functions with their derivatives. In which, physical quantities have been represented by functions and rate of change has been represented by derivatives. In complex systems, DE has been used for modelling behaviour [1]. Ordinary or partial, homogeneous or heterogeneous and linear or non-linear are some of the types DE. It has been observed that greater complex problems have been resolved and derived with a solution by using simple non-linear Partial Differential Equations (PDEs). But it requires spatiotemporal features such as large length and timescale, hence numerical solution for PDE becomes complex. In order to resolve the finest features presented in the solution, it has been interacted computationally [2]. Similarly, proper solution for PDE through deep learning has been attained by minimizing the PDE residual. PDE residue have been reduced, by constraining the neural network. Hence, appropriate solution for PDE has been obtained, by replacing the traditional method with neural network [3]. Dependent variables like time and/or space have been investigated by Variable Order (VO) Fractional Differential Equations (FDEs). In real world phenomenon, VO-FDE has been considered as more effective and precise approach. VO-FDE has been suitable for modelling many variety of phenomenon which includes fields like medicine, science, and engineering [4].Nowadays, Stochastic Differential Equation (SDEs) have been used to determine the system stability. Though SDE has been widely used, time delays may results in oscillations which cause harmful effects to SDE applications. It has been used in many fields such as biological, medical, engineering and so on [5].Fractional Differential Equation (FDE) confined with derivative Caputo-Fabrizio (CF) have wide applications in the field of medicinal science and engineering [6].

Among scientists, fractional calculus has attained more popularity since it works on different subjects and better results have been attained. It has been considered as the fastest growing branch in mathematics [7]. The suggested paper stated that large number of investigations have been carried out on Impulsive Fractional Differential Equations (IFDEs) stability under finite and infinite delay. IFDEs have effective applications in many fields and provided the appropriate solutions [8]. Hence, DE has been considered as the most interesting research topic in recent decades. PDEs could be implemented in medical imaging, electrical signalling of nerves, proper distribution of oxygen to the healing tissues, and more. For few intricate issues the fractional PDE could be more precise than Integer-Order PDE. Thus, engendering numerical solutions of fractional PDEs has become more significant [9]. In biological science, mathematical modelling over infectious diseases by using fraction order differential equations have obtained more attention in past few years. Numerous mathematical model of diseases have been represented through a scheme of non-linear ordinary differential equation. The recommended paper has ruminated such mathematical model [10].

2. UTILIZATION OF DIFFERENTIAL EQUATION IN MEDICAL FIELD

A series –based method which has named DTM (Differential Transform Method) has been used for resolving the DE in Bio fluid Problems in the existing paper [11]. The formulation of the problem in the paper has been modelled based on the circumstances in which the patients has been fighting from renal diseases because of the blockage of Semi-Permeable Membrane in Haemodialysis. In the study eigenvalues have been calculated with various Sherwood membrane, Peclet numbers, axial direction, angles and radius in the purview of DTM. The outcomes has revealed that the angular displacement has been the essential parameter that might be preserved as an indicator for the purity in the medical terms of the blood.

2.1 IMPLEMENTATION OF DIFFERENTIAL EQUATION IN CARDIOLOGY

In cardiology, DE have been used to obtain the exact analytical values. Therefore, the suggested paper reviewed that by investigating fractional, time and PDE at various stages, action potential dynamics in cardiac tissues can be understood better. The main objective of the existing paper is to solve the models analytically. Based on the mathematical prediction, stationary solution stability, optimal control, fractional calculus model has been recently considered to elucidate the health related and biological process especially in dynamics of cardiac phenomena. By using the current paper, it has been observed that more data and information have been required for assessing the performance of derived models which includes clinical and experimental comparison with formal cardiac model. Implementation of the model in rhythm control and in cardiac pacemakers also have possibilities to be investigated [12].

2.2 EMPLOYMENT OF DIFFERENTIAL EQUATION IN PATHOLOGY

The objective of the existing study is to provide a new modular implementation that has merged the better features of a continuum mechanics, agent based model and particle tracking methods. The existing study has been used to manage the multistage nature of the adaptation phenomena. A common rationale of the method has viewed that the MM has been defined the famous PDE. The paper has utilised a technique which has been dependent completely on fixed grid and PDE. A keystones of the method could be that explicitation of the fluid-elastic interface interaction where the model has been amalgamated into the pair of coupled PDEs. In order to build the method, the incompressible Navier stokes system has been written as: : $\rho\ \partial V\ \partial t\ +\ (V.\nabla)V\ =\ -\nabla\ P\ +\ \mu V\ +\ F\ (7)\ \nabla\ \cdot\ V\ =\ 0$ [13].

2.3 MATHEMATICAL ANALYSIS OF DENGUE FEVER

Mathematical model has acted as a powerful tool to understand the mechanism of controlling dengue and transmission dynamics. Mathematically, compartmental models which is governed by Ordinary Differential Equation (ODE) has been used to explain the spreading of dengue disease and to control the interaction between human and mosquitoes. Theoretical examination of the model has been directed to gain the associated dengue-free equilibrium. Mathematical analysis of single-type control interventions that has executed could demonstrate that the open space spray of insecticide could be the most prominent to encompass Dengue spread. Matrix Method has been used to calculate the productive reproduction number. The assumed outcomes have demonstrated that the adaptation of any of the control interventions which has been mentioned in the work could lead to the rejection of the predominant of Dengue among the population. [14].

2.4 USE OF DIFFERENTIAL EQUATION IN CANCER THERAPY

The striving function of Reactive Oxygen Species (ROS) has been acting as critical secondary messengers in cancer and also in the process of cancer chemotherapy. Mathematical model has been used for greater understanding of vigorous frameworks in cancer progression, to predict chemotherapy responses and to optimize the protocols for drug dosage. In the existing study, dynamics of N species has been described through the development of ODE such as, $[ROS]1(t), [ROS]2(t) \ldots [ROS]N(t)$, for each ROS species, production and decay terms have been used for governing the dynamics, Pi(t) and Di(t), where I =1,2,…N and t refers to time. The spatiotemporal distribution of N – species for$[ROS]1(t), [ROS]2(t) \ldots [ROS]N(t)$, has been predicted by the mathematical model, where t refers to time and x refers to spatial position. In the framework of PDE, for each ROS there local dynamics are governed by production terms and decay terms, i.e. $Pi(x,t)$ and $Di(x,t)$, where I refers to 1, 2, 3,...N. Globally most of the chemotherapeutics have been increased the ROS to cytotoxic level in order to target the cancer cells but the long term ROS exposure has been subjected to decrease the effectiveness of chemotherapy [15].

The recommended study suggested that FDE has more benefits when used for modelling of biological model than the modelling which used classical integer – order mathematical model. In recent days, developing an efficient mathematical model of biological science for infectious diseases and finding its solution in numerical value is more important for mathematical research. Because of the above reason, fractional model of tumour immune system has been developed which used numerical models named Toufik - Atangana and Adams – BashforthKoulton methods, which has been used in the treatment of fractional tumour immune system cancer model. The characteristics of tumour and effector cells for different values of fractional - order differential equation have been found. For clear and definite values of fractional order, the disordered behaviour of tumour and effector cells have been found. The suggested study stated that in medical science, mathematical modelling through infectious disease model will provide a new platform for biologists in future [16].

Recently, Ansarizadeh [17] has suggested the model of system which has used partial differential equation and showed the nature and behaviour of tumour, immune and normal cells present in the tumour when using chemotherapeutic drug has been reviewed in the paper. And also they found that number of tumour cells present in the hazardous and risky part of tumour has been decreased, with the help of biological parameters. It has been proved that the nature and characteristics of tumour – immune system and the interaction between host immune cells and cancer cells and their evolution have been understood by using a mathematical model which involves fractional differential equation. At time‘t’, the amount of drug given at tumour site is referred as $U(x,t)$, the external influx chemotherapeutic drug is referred as $y(t)$ which has been used for the understanding of the biological behaviour of immune cells and tumour cells with chemotherapeutic drugs. By varying the values of x and t, the responses of the tumour cells have been obtained by the proposed technique. Based on the time history and time instant, cancer chemotherapy effect model has been changed and in order to develop efficient model fractional calculus has been used. Further, it has been also used for other non – linear partial differential equations with dynamic fractional order [17].

3. RECENT TRENDS APPLIED IN MEDICAL FIELD WITH DIFFERENTIAL EQUATIONS

To identify correlations, machine learning used huge design spaces and to identify causality multiscale modelling predicts system dynamics. Recent trends suggested that integrating multiscale modelling and machine learning has provided a better way to understand about biomedical, behavioural and biological system. Multiscale modelling approaches have been divided into two categories based on the scale of interest, i.e. ordinary differential equation - based approaches and partial differential equation - based approaches. Ordinary differential equations have been used widely during disease, development, pharmaceutical intervention or environmental changes in order to stimulate the system integral response. While partial differential equations are basically used to study inherently heterogeneous dimensional patterns and varying field at particular regions i.e. to study blood flow in cardiovascular system, contraction of heart etc. [18].

Nowadays, Artificial Neural Network (ANN) and Deep Learning (DL) has included both DE and PDE, algorithms and specific techniques in order to find the difficult patterns in large data by using computers. In recent days, deep learning has been used in MRI for medical image focusing with the help of differential and partial differential equation [19].

Single or 1 – D backward stochastic differential equations (BSDE) has avoided the computation of conditional expectation. In future, many attempts have been made to combine BSDE method in multidimensional case to the form of Markovian coupled forward-backward SDEs [20].

4. COMPARATIVE ANALYSIS

Comparative analysis of the existing research based on the methods adopted, results obtained are listed in the tabular column below.

Table 1 Comparative analysis of conventional researches

S. No	Technique	Objective	Conclusion	Advantages / Limitations	Reference
1	fractional, time and partial differential equations	To investigate action potential dynamics in cardiac tissue.	It helps to elucidate the health related and biological process especially in dynamics of cardiac phenomena.	More data and information have been required for assessing the performance of derived models.	[12]
2	PDE	To provide a new modular implementation in order to manage with the multistage nature of the adaptation phenomena.	The method could be the explicitation of the fluid-elastic interface interaction where the model has been amalgamated into the pair of coupled PDEs.	Navier stokes system has been used to build the method.	[13]
3	ODE	The objective is to understand the vigorous frameworks in cancer progression and to optimize drug dosage.	Chemotherapeutics have used increased ROS to target cancer cells.	The main challenge in cancer biology is to establish clinical method which finds ROS in cancer in vivo.	[15]
4	FDE	To develop an efficient mathematical model of biological science for infectious diseases	Mathematical modelling through infectious disease model will provide a new platform for biologists in future	It has been used in the treatment of fractional tumour immune system cancer model.	[16]

5. CHALLENGES IN ADOPTING DIFFERENTIAL EQUATION IN MEDICAL FIELD

The main drawback of using DE in cancer biology is establishing clinical method which finds ROS in cancer in spatiotemporal manner, within living (vivo) human body. Multidisciplinary collaboration between modelling, experimental and clinical areas have been needed to integrate with the modern mathematical model which incorporates the expertise and experimental techniques that has been required for ROS analysis, detection and clinical translation. Future generation models would be developed to increase the understandings about the working of the cancer redox biology and a new chemotherapeutics design will be proposed to defeat cancer [15].

In asthma, the current mathematical model has described the growth of airway smooth muscles qualitatively in inflammatory and normal environment for a period of short and long term. Based on the model, inflammation resolution speed, inflammatory episodes frequency and magnitude of infection has been responsible for the growth of airway smooth muscle in long term. The challenge of the existing study stated that it does not consider the mechanical interaction between the cells and extra-cellular matrix which affects the apoptosis rates, growth and also the total capacity of airway wall. Another challenge in using mathematical models in biological system is that mathematical models are very hard and complex in analytical solving and it requires computational model applications in order to obtain the numerical value of the model solution [21].

6. CONCLUSION

From this review, it is evident that in medical stream, the application of DE is more important for diagnosis, analysis, interpretation and treatment of various diseases. Data obtained using mathematical models which incorporates ODE, PDE, linear and non-linear differential equations, homogeneous and non-homogeneous differential equations are more accurate than the data obtained from classical model. It is also observed that, in medical field, mathematical findings are more efficient when it incorporates with the application of computational models. This review paper will act as a guide for future analysis of application of differential equation in medical field.

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