MINIMUM NEIGHBORHOOD DOMINATION OF GRAPHS

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Abstract: In this paper, we define a new domination parameter called minimum neighbourhood domination. Also we define and study the minimum neighborhood domination number of some class of graphs

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I. INTRODUCTION

Let $G = (V, E)$ be a non-trivial simple graph with $|V| = p, |E| = q$. Let $D$ be the subset of $V$ is a dominating set if every vertex in $V - D$ is adjacent to at least one element in $D$. The minimum cardinality of dominating set is called domination number and it is denoted by $\gamma(G)$. [5] There was need for minimum neighbourhood dominating set in the case that, management has to pass information to each and every student of the institution. For that, management uses the concept of dominating set and chooses a set of students as dominating set. But it left some students unaware of information because some students were dominated by many students and those dominating students lavishly thought other will pass the information. To reduce this problem we introduce a concept called minimum neighbourhood domination by imposing a condition on dominating set.

II. Main Results

Definition 2.1 Minimum neighbourhood dominating set

Let $G = (V, E)$ be a non-trivial simple graph. A subset $D \subseteq V(G)$ is a minimum neighbourhood dominating set if $D$ is a dominating set and if for every $v_i \in D$, $|\cap_{i=1}^{n} N(v_i)| < \delta(G)$ holds.

Definition 2.2 Minimum neighbourhood dominating number

The minimum cardinality of minimum neighbourhood dominating set of a graph $G$ is called as minimum neighbourhood dominating number and it is denoted by $\gamma_{mn}(G)$.
Theorem 2.3 For a path graph $P_n$, $\gamma_{mn}(P_n) = \begin{cases} 
+2 & \text{if } n = 2, \\
+3 & \text{if } n = 3k, k = 2,3\ldots, \\
+\left\lfloor \frac{n}{3} \right\rfloor + 1 & \text{if } n \neq 3k, k = 2,3\ldots 
\end{cases}$

Proof. Let $P_n$ be a path graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$. Let $D \subseteq V(G)$ be a minimum neighbourhood dominating set of the path graph. To compute minimum neighbourhood dominating number, we consider the following cases:

Case 1: Let $n=2$ and the vertex set of $P_2$ be $\{v_1, v_2\}$. Here minimum neighbourhood dominating set is $D = \{v_1, v_2\}$. Therefore $\gamma_{mn}(P_2) = 2$.

Case 2: $n = 3k, k = 2,3\ldots$.

Let $k=2$, $n=6$ and the vertex set of $P_6$ be $\{v_1, v_2, \ldots, v_6\}$. Here every vertex not in the set $D = \{v_2, v_3\}$ is adjacent to at least one element of $D$ and $|N(v_2) \cap N(v_3)| < \delta(P_6)$. So that $D = \{v_2, v_3\}$ is a minimum neighbourhood dominating set of $P_6$. Therefore $\gamma_{mn}(P_6) = 2$. As proceeding for any $n = 3k, k = 2,3\ldots$, every vertex not in the set $D = \{v_1, v_{1+3(1)}, v_{1+3(2)} \ldots, v_{1+3(l)}\}, l = 0,1,2,3\ldots \left\lfloor \frac{n}{3} \right\rfloor - 1$ is adjacent to at least one element of $D$ and $|N(v_1) \cap N(v_{1+3}) \cap N(v_{1+3(2)}) \ldots \cap N(v_{1+3(l)})| < \delta(P_{3k})$, $i = 0,1,2,3\ldots \left\lfloor \frac{n}{3} \right\rfloor - 1$. So that $D = \{v_1, v_{1+3(1)}, v_{1+3(2)} \ldots, v_{1+3(l)}\}, l = 0,1,2,3\ldots \left\lfloor \frac{n}{3} \right\rfloor - 1$ is a minimum neighbourhood dominating set of $P_{3k}$. Therefore $\gamma_{mn}(P_{n=3k}) = \left\lfloor \frac{n}{3} \right\rfloor + 1$.

Case 3: $n \neq 3k, k = 2,3\ldots$.

Let $n=3$ and the vertex set of $P_3$ be $\{v_1, v_2, v_3\}$. Here every vertex not in the set $D = \{v_1, v_2\}$ is adjacent to at least one element of $D$ and $|N(v_1) \cap N(v_2)| < \delta(P_3)$. So that $D = \{v_1, v_2\}$ is a minimum neighbourhood dominating set of $P_3$. Therefore $\gamma_{mn}(P_3) = 2$. Similarly for any $n \neq 3k, k = 2,3\ldots$, every vertex not in the set $D = \{v_1, v_{1+3(1)}, v_{1+3(2)} \ldots, v_{1+3(l)}\}, l = 0,1,2,3\ldots \left\lfloor \frac{n}{3} \right\rfloor$ is adjacent to at least one element of $D$ and $|N(v_1) \cap N(v_{1+3}) \cap N(v_{1+3(2)}) \ldots \cap N(v_{1+3(l)})| < \delta(P_{n=3k})$, $i = 0,1,2,3\ldots \left\lfloor \frac{n}{3} \right\rfloor$. So that $D = \{v_1, v_{1+3(1)}, v_{1+3(2)} \ldots, v_{1+3(l)}\}, l = 0,1,2,3\ldots \left\lfloor \frac{n}{3} \right\rfloor$ is a minimum neighbourhood dominating set of $P_{n=3k}$. Therefore $\gamma_{mn}(P_{n=3k}) = \left\lfloor \frac{n}{3} \right\rfloor + 1$.

Theorem 2.4 For a cycle graph $C_n$, $\gamma_{mn}(C_n) = \begin{cases} n+2 & \text{if } n = 2, \\
+3 & \text{if } n = 3k, k = 2,3\ldots, \\
+\left\lfloor \frac{n}{3} \right\rfloor + 1 & \text{if } n \neq 3k, k = 2,3\ldots 
\end{cases}$

Proof. Let $C_n$ be a cycle graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$. Let $D \subseteq V(G)$ be a minimum neighbourhood dominating set of the cycle graph. To compute minimum neighbourhood dominating number, we consider the following cases:

Case 1: $n = 3k, k = 2,3\ldots$.

Let $k=2, n=6$ and the vertex set of $C_6$ be $\{v_1, v_2, \ldots, v_6\}$. Here every vertex not in the set $D = \{v_1, v_2\}$ is adjacent to at least one element of $D$ and $|N(v_1) \cap N(v_2)| < \delta(C_6)$. So that $D = \{v_1, v_2\}$ is a minimum neighbourhood dominating set of $C_6$. Therefore $\gamma_{mn}(C_6) = 2$.

As proceeding for any $n = 3k, k = 2,3\ldots$, every vertex not in the set $D = \{v_1, v_{1+3(1)}, v_{1+3(2)} \ldots, v_{1+3(l)}\}, l = 0,1,2,3\ldots \left\lfloor \frac{n}{3} \right\rfloor - 1$ is adjacent to at least one element of $D$ and $|N(v_1) \cap N(v_{1+3}) \cap N(v_{1+3(2)}) \ldots \cap N(v_{1+3(l)})| < \delta(C_{3k})$, $i = 0,1,2,3\ldots \left\lfloor \frac{n}{3} \right\rfloor - 1$. So that $D = \{v_1, v_{1+3(1)}, v_{1+3(2)} \ldots, v_{1+3(l)}\}, l = 0,1,2,3\ldots \left\lfloor \frac{n}{3} \right\rfloor$ is a minimum neighbourhood dominating set of $C_{3k}$. Therefore $\gamma_{mn}(C_{n=3k}) = \left\lfloor \frac{n}{3} \right\rfloor$.

Case 2: $n \neq 3k, k = 2,3\ldots$.

Let $n=3$ and the vertex set of $C_3$ be $\{v_1, v_2, v_3\}$. Here every vertex not in the set $D = \{v_1, v_2\}$ is adjacent to at least one element of $D$ and $|N(v_1) \cap N(v_2)| < \delta(C_3)$. So that $D = \{v_1, v_2\}$ is a minimum neighbourhood dominating set of $C_3$. Therefore $\gamma_{mn}(C_3) = 2$. Similarly for any $n \neq 3k, k = 2,3\ldots$, every vertex not in the set $D = \{v_1, v_{1+3(1)}, v_{1+3(2)} \ldots, v_{1+3(l)}\}, l = 0,1,2,3\ldots \left\lfloor \frac{n}{3} \right\rfloor$ is adjacent to at least one element of $D$ and $|N(v_1) \cap N(v_{1+3}) \cap N(v_{1+3(2)}) \ldots \cap N(v_{1+3(l)})| < \delta(C_{n=3k})$, $i = 0,1,2,3\ldots \left\lfloor \frac{n}{3} \right\rfloor$. So that $D = \{v_1, v_{1+3(1)}, v_{1+3(2)} \ldots, v_{1+3(l)}\}, l = 0,1,2,3\ldots \left\lfloor \frac{n}{3} \right\rfloor$ is a minimum neighbourhood dominating set of $C_{n=3k}$. Therefore $\gamma_{mn}(C_{n=3k}) = \left\lfloor \frac{n}{3} \right\rfloor + 1$. 
Theorem 2.5 Let G be a graph, then γ_{mn}(G) = 2. When G is (i) Complete graph K_n, (ii) Complete bipartite graph K_{m,n}, (iii) Crown graph H_{n,n}, (iv) Wheel graph W_{1,n}, (v) t-fold wheel graph W_{t,n}, t ≥ 1.

Proof. Let K_n be a complete graph with vertex set V = \{v_1, v_2, ..., v_n\}. Let D ⊆ V(G) be a minimum neighbourhood dominating set of the complete graph.

Let n = 2 and the vertex set of K_2 be \{v_1, v_2\}. Here D = \{v_1, v_2\}. Therefore γ_{mn}(K_2) = 2.

As proceeding for any n, every vertex not in the set D = \{v_1, v_2\} is adjacent to at least one element of D and |N(v_1) ∩ N(v_2)| < δ(K_n) = n − 1. So that D = \{v_1, v_2\} is a minimum neighbourhood dominating set of K_n. Therefore γ_{mn}(K_n) = 2.

In similar way we obtain minimum neighbourhood dominating number of (ii)complete bipartite graph K_{m,n}, (iii) crown graph H_{n,n}, (iv) wheel graph W_{1,n}, (v) t-fold wheel graph W_{t,n}, t ≥ 1

Theorem 2.6 Let G be a graph, then γ_{mn}(G) = 2. When G is (i) Flower graph F_{1,n,n}, (ii) Friendship graph F_m, (iii) Barbell graph B_n, (iv) Fan graph F_{1,n}, (v) Double Fan graph F_{2,n}, (vi) Generalized Fan graph F_{m,n}, (vii) Windmill graph W_{d,m,n}, (viii) Shell graph C(n, n − 3), (ix) Shell Flower graph \[C(n, n − 3) \cup K_2\]^k (x) Jewel graph J_n.

Proof. Let F_{1,n,n} be a Flower graph with vertex set V = \{u_1, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n\}. Let D ⊆ V(G) be a minimum neighbourhood dominating set of the Flower graph.

Let n = 3 and the vertex set of F_{1,3,3} be \{u_1, v_1, v_2, v_3, w_1, w_2, w_3\}. Here every vertex not in the set D = \{u_1, w_1\} is adjacent to at least one element of D and |N(u_1) ∩ N(w_1)| < δ(F_{1,3,3}). So that D = \{u_1, w_1\} is a minimum neighbourhood dominating set of F_{1,3,3}. Therefore γ_{mn}(F_{1,3,3}) = 2.

Let n = 4 and the vertex set of F_{1,4,4} be \{u_1, v_1, v_2, v_3, v_4, w_1, w_2, w_3, w_4\}. Here every vertex not in the set D = \{u_1, w_1\} is adjacent to at least one element of D and |N(u_1) ∩ N(w_1)| < δ(F_{1,4,4}). So that D = \{u_1, w_1\} is a minimum neighbourhood dominating set of F_{1,4,4}. Therefore γ_{mn}(F_{1,4,4}) = 2.

As proceeding for any n, every vertex not in the set D = \{u_1, w_1\} is adjacent to at least one element of D and |N(u_1) ∩ N(w_1)| < δ(F_{1,n,n}). So that D = \{u_1, w_1\} is a minimum neighbourhood dominating set of F_{1,n,n}. Therefore γ_{mn}(F_{1,n,n}) = 2.

In similar way we obtain minimum neighbourhood dominating number of (ii) Friendship graph F_m, (iii) Barbell graph B_n, (iv) Fan graph F_{1,n}, (v) Double Fan graph F_{2,n}, (vi) Generalized Fan graph F_{m,n}, (vii) Windmill graph W_{d,m,n}, (viii) Shell graph C(n, n − 3), (ix) Shell Flower graph \[C(n, n − 3) \cup K_2\]^k (x) Jewel graph J_n.

3 Conclusion

A new domination parameter called minimum neighbourhood domination was introduced. Minimum neighborhood dominating number was defined and minimum neighborhood dominating number of some class of graphs are found.

References
