



## Performance Analysis of Two-Hop Systems Over $\alpha - \eta - \kappa - \mu$ Channel

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**Abstract:** The increasing demand for new services, ubiquitous coverage and device/thing-oriented communications have been the main driving force to the development of a new, advanced generation relaying communication technology. Current technologies, such as 3G and even 4G, will not be able to meet the data requirements required for necessary connectivity. The wide diversity of scenarios for 5G with relay system opens the possibility for development of different solutions to meet the future demands. However recently developed  $\alpha-\eta-\kappa-\mu$  channel model can be used to model the behaviour of mm-Wave for aforesaid fading environment. This paper investigates performance matrices of fading model for two-hop communication system in closed-form formulas. More specifically, Amount of Fading, Average Bit Error Rate (ABER) and Average Channel Capacity.

**Index Terms:** Two-hop Communication, Signal to Noise Ratio (SNR), Amount of Fading, Bit Error Rate, Average Bit Error Rate, Source to Sink, Average Channel Capacity per Unit Bandwidth,  $t^{\text{th}}$ -Moment,  $\alpha-\eta-\kappa-\mu$  Fading Channel, Signal to Noise Ratio (SNR).

### 1. INTRODUCTION

Channel modelling plays an important role in the design of wireless communication systems where signals propagate in multipath fading environment. Regarding that various fading distributions have been proposed such as Rayleigh, Rician and Nakagami [1]. Besides these, some generalized fading distributions such as  $\kappa - \mu$ ,  $\eta - \mu$  and other similar distributions [2, 3] have also been proposed and studied. Since realistic environments contain non-linearities and are difficult to model, hence, some more fading distributions such as  $\alpha - \kappa - \mu$ ,  $\alpha-\eta - \mu$  [3] and their variants have been proposed [4,5,6]. Recent development of Machine-to-Machine communication, IoT Applications, Ad-hoc and Sensor Networks has created numerous applications of such wireless networks [7]. These networks need to be deployed in various challenging environments which requires the analysis of the state-of-art channels as well as performance of wireless communication systems over these channels.

In recent years, the relaying technique has played an important role in the wireless communication systems, as it can not only supply higher power density to coverage area but also construct flexible service networks. However, the device-to-device oriented communications, power and bandwidth resources are scarce and expensive in two-hop communication systems. As a result, it is crucial to make effort to enhance the utilization efficiency of power and bandwidth resource. It is obvious that, the new scenarios foreseen for the next, advanced generation, currently known as 5G, cannot be accomplished just by increasing the system throughput [8] but also reduction of latency, and better knowledge of the communication channel. Relaying communication system with two-hop and it's generalised version multi-hop are the most

promising one in fading environment for mm-Wave band channel (30 GHz to 300 GHz) [9], seems to be studied for all solutions.

The real benefit behind use of using mm-Waves is the high availability of bandwidth spectrum, but the signal propagation conditions at this band need to be investigated. It is very much certain that propagation mechanism in mm-Wave are the same as those found in any other band, but there must be some phenomena which are more or less strongly perceived. These propagation conditions are direct and indirect paths, a line-of-sight (LoS) and non-LoS (NLoS) situation. At higher frequencies, multipath clustering phenomenon also drive the environmental condition of wave propagation. In addition, nonlinearity of the medium is also noticeable and will have a strong impact in the propagation of the wave. Some authors have already assessed the use of some of the mentioned distributions to model real field data in emerging scenarios, e.g. mm-Wave band [10,11]. The  $\alpha - \eta - \kappa - \mu$  distribution [12] discuss all the possible scenario in the form of parameters such as  $\alpha$  represent the non-linear environment,  $\eta$  is the ratio of total power of the in-phase and quadrature scattered wave of multipath cluster,  $\kappa$  is the ratio between the total power of dominant component and scattered wave and  $\mu$  is the real existence of total number of multipath clusters. However, the paper contributes in the derivation of  $t^{th}$ -Moment and Moment Generating Function of instantaneous Signal to Noise Ratio (SNR) for  $\alpha - \eta - \kappa - \mu$  channel. The derived expressions have been used to obtain Amount of Fading (AoF), ABER and Channel Capacity. Finally, the analytical results have been verified with the simulation results.

The remainder of this paper is organised as followed: In Section 2 Channel model has been presented and expressions for  $t^{th}$ -Moment of the channel have been derived. In section 3, various performance measures for two-hop wireless communications systems have been obtained. Section 4 discusses numerical and simulation result.

## 2. CHANNEL MODEL:

The probability density function (pdf) for the envelope of  $\alpha - \eta - \kappa - \mu$  channel can be obtained as [12]

$$f_R(r) = \int_{-\frac{2\pi}{\alpha}}^{\frac{2\pi}{\alpha}} f_{R,\theta}(r, \theta) d\theta \quad (1)$$

where  $f_{R,\theta}$  is the joint envelope and phase distribution of  $\alpha - \eta - \kappa - \mu$  channel and given as [12, Eq. 29]

Again, substituting  $\theta' = \frac{2\pi}{\alpha} \theta$  in Eq. (1), we have

$$f_R(r) = \frac{2\pi}{\alpha} \int_{-1}^1 f_{R,\theta}\left(r, \frac{2\pi}{\alpha} \theta'\right) d\theta \quad (2)$$

Using Legendre-Gauss quadrature integral approximation [13], above equation can be rewritten as

$$f_R(r) = \frac{2\pi}{\alpha} \sum_{a=1}^{N_a} \omega_a f_{R,\theta}\left(r, \frac{2\pi}{\alpha} \theta_a\right) \quad (3)$$

where  $\theta_a$ 's and  $\omega_a$ 's are the nodes and weights for Legendre-Gauss quadrature integral approximation.  $N_a$  is number of terms equal to 30, which is sufficient to neglect approximation error. Considering  $\gamma$  and  $\bar{\gamma}$  as instantaneous and average SNR and using relation  $\gamma = \bar{\gamma} \left(\frac{r}{\bar{r}}\right)^2$  pdf of instantaneous SNR can easily be obtained through random variable transformation and given as

$$f_\gamma(\gamma) = \frac{2\pi}{\alpha} \sum_{a=1}^{N_a} \omega_a f_{\gamma,\theta}\left(\gamma, \frac{2\pi}{\alpha} \theta_a\right) \quad (4)$$

where  $f_{\gamma,\theta}(\gamma, \frac{2\pi}{\alpha} \theta_a)$  is given as,

$$\begin{aligned}
 f_{\gamma,\theta} \left( \gamma, \frac{2\pi}{\alpha} \theta_a \right) &= A \frac{\gamma^{\frac{\alpha}{4}(\mu+2)-1} \left| \sin \left( \frac{2\pi}{\alpha} \theta_a \right) \right|^{\frac{\mu}{p+1}} \left| \cos \left( \frac{2\pi}{\alpha} \theta_a \right) \right|^{\frac{\mu p}{p+1}}}{2\gamma^{\frac{\alpha}{4}(\mu+2)}} \\
 &\times \exp \left( -B \left( \eta \sin^2 \left( \frac{2\pi}{\alpha} \theta_a \right) + p \cos^2 \left( \frac{2\pi}{\alpha} \theta_a \right) \right) \left( \frac{\gamma}{\gamma} \right)^{\frac{\alpha}{2}} \right) \\
 &\times \exp \left( C \cos \left( \frac{2\pi}{\alpha} \theta_a - \varphi \right) \left( \frac{\gamma}{\gamma} \right)^{\frac{\alpha}{4}} \right) \\
 &\times \frac{I_{\frac{\mu}{p+1}-1} \left( D \left| \sin \left( \frac{2\pi}{\alpha} \theta_a \right) \right| \left( \frac{\gamma}{\gamma} \right)^{\alpha/4} \right)}{\cosh \left( D \sin \left( \frac{2\pi}{\alpha} \theta_a \right) \left( \frac{\gamma}{\gamma} \right)^{\alpha/4} \right)} \times \frac{I_{\frac{\mu p}{p+1}-1} \left( E \left| \cos \left( \frac{2\pi}{\alpha} \theta_a \right) \right| \left( \frac{\gamma}{\gamma} \right)^{\alpha/4} \right)}{\cosh \left( E \cos \left( \frac{2\pi}{\alpha} \theta_a \right) \left( \frac{\gamma}{\gamma} \right)^{\alpha/4} \right)} \tag{5}
 \end{aligned}$$

The Envelope PDF for  $\alpha$ - $\eta$ - $\kappa$ - $\mu$  fading environment represented by Eq. (4) is plotted for the different values of  $\alpha$  and shown in Fig. 1

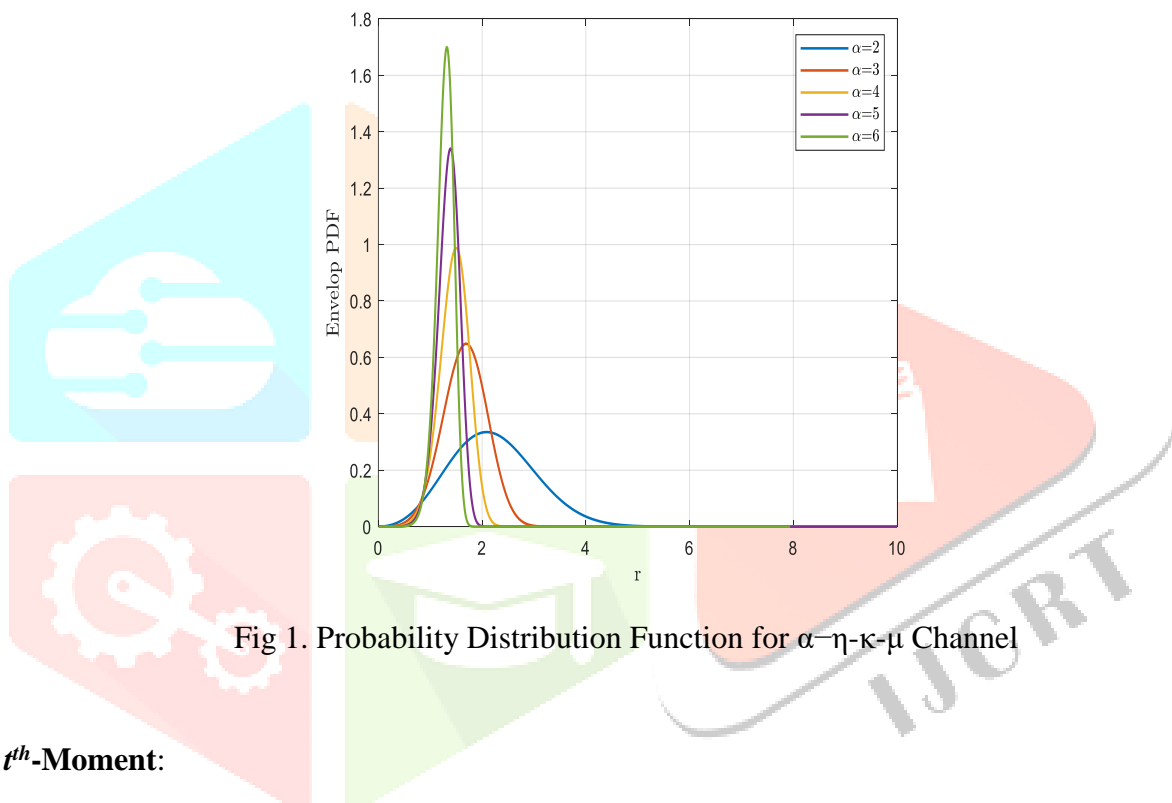


Fig 1. Probability Distribution Function for  $\alpha$ - $\eta$ - $\kappa$ - $\mu$  Channel

**2.1.  $t^{th}$ -Moment:**

The  $m_t$  of instantaneous SNR for  $\alpha$ - $\eta$ - $\kappa$ - $\mu$  channel can be obtained as,

$$m_t = E[\gamma^t] = \int_0^\infty \gamma^t f_\lambda(\gamma) d\gamma \tag{6}$$

Again,

$$m_t = F \sum_{a=1}^{N_a} G_a \int_0^\infty f_a(t, x) e^{-x} d(x) \tag{7}$$

Solving Eq. (7) through Gauss-Laguerre integral approximation,  $t^{th}$ -Moment of instantaneous SNR for  $\alpha$  -  $\eta$  -  $\kappa$  -  $\mu$  channel is given by

$$m_t = F \sum_{a=1}^{N_a} G_a \sum_{b=1}^{N_b} \omega_b f_a(t, x_b) \tag{8}$$

where  $x_b$  and  $\omega_b$  are the nodes and weights for Gauss-Laguerre integral approximation.  $N_b$  is the number of terms. Accuracy of the numerical results depends on the value of  $N_b$ .

### 3. PERFORMANCE MEASURES:

Following performance matrices have been used to analyse the performance of wireless communication systems.

#### 3.1 Amount of Fading (AoF)

Fading in the wireless communication is defined as the fluctuation in strength of the signal received at the receiver and this can be analysed with the help of AoF.

For any arbitrary fading channel, it is given as,

$$AoF = \frac{m_2 - m_1^2}{m_1^2} = \left[ \left( \frac{m_2}{m_1^2} \right) - 1 \right] \quad (9)$$

Where  $m_1$  and  $m_2$  are the 1<sup>st</sup> and 2<sup>nd</sup> moments of instantaneous SNR. Calculating 1st and 2nd moments from Eq. (8) and substituting in above equation, AoF for  $\alpha - \eta - \kappa - \mu$  channel can be easily obtained.

#### 3.2. Average Bit Error Rate

Bit Error is used as an important parameter in characterising the performance of data channel. It is defined as the number of bit errors divided by total number of transferred bits during the studied time interval.

Average Bit Error Rate for coherent detection schemes for an arbitrary fading channel can be obtained as

$$\Lambda(\bar{\gamma}) = 2A_c \int_0^\infty Q(\sqrt{2B_c\gamma}) f_\gamma(\gamma) d\gamma \quad (10)$$

where  $A_c$  and  $B_c$  are constants for various modulation schemes.

From Eq. (4) and Eq. (10) ABER for QPSK modulation scheme over  $\alpha - \eta - \kappa - \mu$  channel is given as,

$$\Lambda(\bar{\gamma}) = F^0 \sum_{a=1}^{N_a} G_a^0 \sum_{b=1}^{N_b} \omega_b f_a(0, x_b) \int_0^{\pi/2} e^{-\frac{H_a x_b^{2/\alpha}}{\sin^2(\delta)}} d\delta \quad (11)$$

The above equation can be solved through numerical methods using any mathematical software like MATLAB.

#### 3.3. Average Channel Capacity

Average Channel Capacity per unit bandwidth (C) can be obtained as [13]

$$C = \int_0^\infty f_\gamma(\gamma) \log_2(1 + \gamma) d\gamma \quad (12)$$

Substituting Eq. (12) and using same substitution as in the derivation of  $t^{\text{th}}$ -moment followed by Gauss-Laguerre integral approximation, Average Channel Capacity per unit bandwidth for  $\alpha - \eta - \kappa - \mu$  channel can be obtained as

$$C = F^0 \sum_{a=1}^{N_a} G_a^0 \sum_{b=1}^{N_b} \omega_b f_a(0, x_b) \log_2(1 + H x_b^{2/\alpha}) \quad (13)$$

where  $x_b$  and  $\omega_b$  are the nodes and weights for Gauss-Laguerre integral approximation.

#### 4. RESULTS AND DISCUSSION:

To study the performance of wireless communication systems, variations in Amount of Fading, ABER and Average Channel Capacity per unit bandwidth have been analysed against various parameter sets of  $\alpha - \eta - \kappa - \mu$  channel. Moreover, all the analytical results have been verified with the simulation results.

For analysis, three different parameter sets of  $\alpha - \eta - \kappa - \mu$  channel have been considered as listed in Table. 1.

Table 1:  $\alpha - \eta - \kappa - \mu$  Distribution Parameters for Simulation

| Parameter set | A | H | K | M | P | Q |
|---------------|---|---|---|---|---|---|
| Set-1         | 2 | 3 | 4 | 3 | 2 | 1 |
| Set-2         | 3 | 3 | 5 | 2 | 1 | 1 |
| Set-3         | 4 | 2 | 3 | 3 | 1 | 1 |

Three different set of various parameters have been taken and the corresponding plot for ABER is shown in Fig 2 in case of two-hop communication system. It is evident from the results that the stated system outperforms in the  $\alpha - \eta - \kappa - \mu$  channel with parameter set 3 as compared to the channel having parameter sets 1 and 2 respectively. The performance over  $\alpha - \eta - \kappa - \mu$  channel with parameter set 3 is closer to the performance at AWGN.

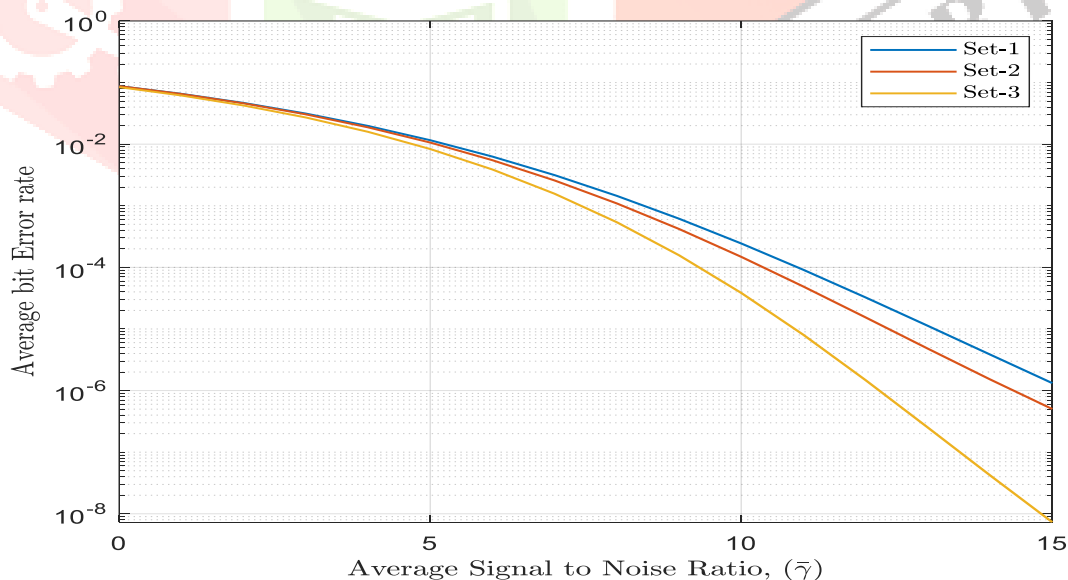


Fig 2. Average Bit Error Rate w.r.t. Average SNR for  $\alpha - \eta - \kappa - \mu$  Channel

Fig. 3 shows Amount of Fading for different parameters of  $\alpha - \eta - \kappa - \mu$  channel. It is evident from the results that AoF increases with increase in  $\alpha$ , whereas it decreases with increase in the value of either  $\eta$ ,  $\kappa$  or  $\mu$ .

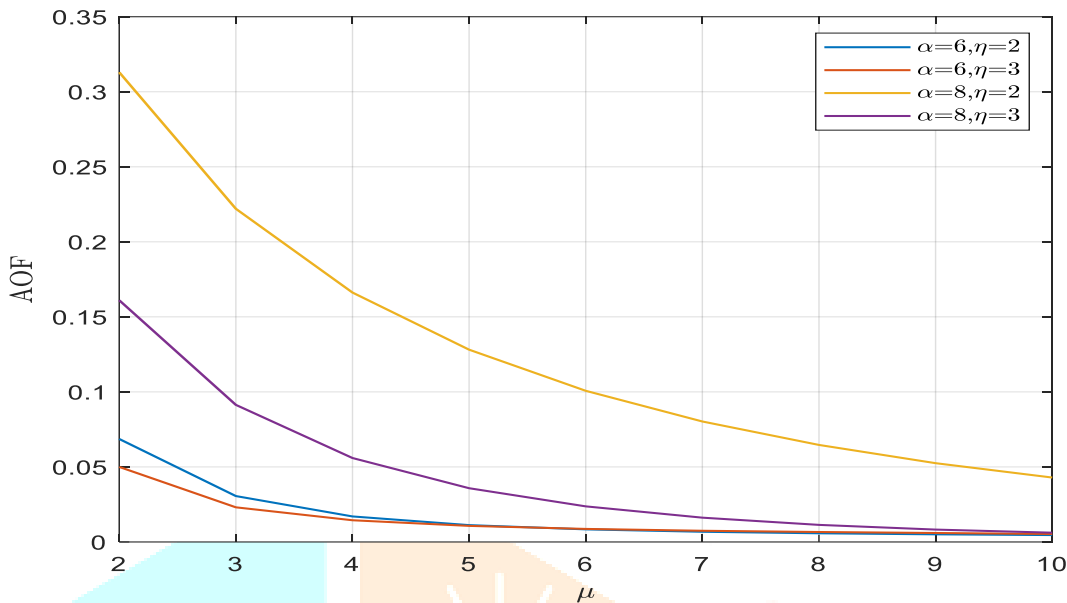


Fig 3. Amount of Fading over  $\alpha - \eta - \kappa - \mu$  Channel for various values of  $\alpha$  &  $\eta$

Fig. 4 represents Average Channel Capacity per unit Bandwidth for the above-mentioned parameter sets of the channel. Channel with parameter set-3 has maximum channel capacity per unit bandwidth followed by channel capacities of channels with parameter set-1 and 2 respectively. This is due to the fact that capacity increases with increase in  $\alpha$ ,  $\kappa$  and  $\mu$ , whereas, it decreases with increase in  $\eta$  [23].

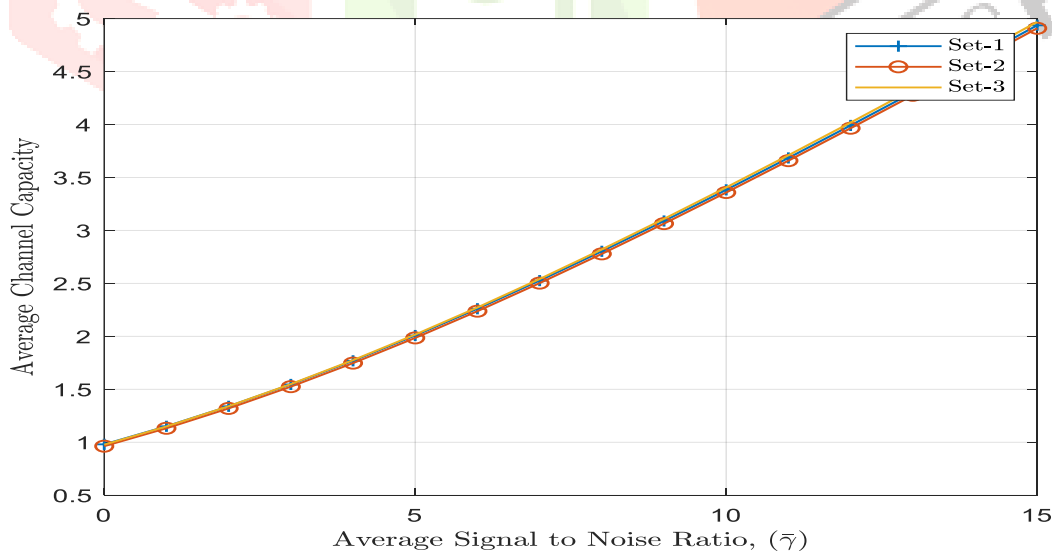


Fig 4. Average Channel Capacity w.r.t. Average SNR for various parameter set of  $\alpha - \eta - \kappa - \mu$  Channel

In the mathematical calculation for all above performance metrics, as  $N_b$  increases, the results become more and more accurate. However, higher value of  $N_b$  increases the computation burden. Therefore, choice of  $N_b$  depends on the requirement of accuracy as per the application and available computational resources. Number of terms for other parameter sets can also be obtained in similar manner.



## 5. CONCLUSION

This paper derives the expressions of  $t^{\text{th}}$ -Moment for  $\alpha$ - $\eta$ - $\kappa$ - $\mu$  fading channel through numerical integration technique. The derived expressions have further been used to obtain AoF, ABER and Channel Capacity. All these metrics have been obtained for two-hop communication system considering  $\alpha$ - $\eta$ - $\kappa$ - $\mu$  fading scenario. Derived analytical expressions have been validated through simulations. This analysis can also be carried out through closed form expressions. This analysis can be extended for the Multi-hop relaying system with H number of intermediate nodes. However closed form expressions are quite complex to compute due to presence of complex mathematical operations/functions and is an open future scope.

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