SINGLE AND MULTI TRANSMIT OF C SERVERS FUZZY MODELS

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Abstract

In this paper, we propose a single transmit and multi transmit fuzzy queuing models with single and multiple servers. Here we consider the arrival rate to follow pentagonal, heptagonal fuzzy numbers. Based on that we execute the characteristic of queuing model by applying ranking technique.

Keywords: Fuzzy number, arrival rate, service rate, servers

1. INTRODUCTION

Queuing models have a wide application in service organizations and one of such application area is real life situations having a policy of three class service channels. Different types of queuing models have been explored in ranging from queuing models having constant crisp values to queuing models with uncertain or fuzzy values. Therefore a possibility of having expressions such as arrival rate or service rate. This makes fuzzy queuing models more practical than the classical queuing models in many real situations. The conversion of fuzzy queues to crisp queues is used in number of methods. One such method is Ranking method. Different methodologies have been produced for positioning fuzzy numbers. Kao et al.[4] proposed a general method to develop the participation elements of the presentation estimate M/F/1/∞, F/M/1/∞, F/F/1/∞ and FM/FM/1/∞ lines, where F and FM signify the fuzzy time and exponential time. Likewise, Ke et al. [4] figured the enrolment capacity of the framework with the properties of arrival rate and service rate.

This paper deals with one of the queuing models with the technique for transformation from fuzzy to crisp qualities is known as the left and right positioning strategy applied in two types of fuzzy number. Pentagonal, heptagonal fuzzy queuing models also studied by Kaufmann, A., 1975[5], Julia Rose Mary and Christina.2018[7], K. Julia Rose Mary, P. Monica, and S. Mythili 2014[8].

The outline of this paper follows: Section 1 contains an introductory overview, Section 2 explain the basic definitions in fuzzy set theory, Section 3 describes the fuzzy queuing model, Section 4 depicts the plan of ranking, Section 5 explains numerical analysis, Section 6 gives the results and discussions and Section 7 gives conclusion.

2. Basic Definitions

2.1 Fuzzy sets:

A Fuzzy set can be characterized as: \( \tilde{A} = \{ x, \mu_{\tilde{A}}(x) : x \in X \} \) and X be a non-void set, \( \mu_{\tilde{A}}(x) : \epsilon [0, 1] \) is the membership of \( x \in X \) in \( \tilde{A} \).

2.2 Infinite framework limit:

If the arrival rate is not affected by the number of customers being served and waiting, i.e., systems with large population of potential customers (unlimited capacity).

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3. Fuzzy Queuing Models with three classes and three arrivals

Consider a fuzzy queueing model with three classes FM/F (H1, H2, H3)/1/FCFS and FM/F (H1, H2, H3)/1/FCFS without any priorities in entry rates, where FM signifies the fuzzy arrivals rate as a Poisson, while F(H1, H2, H3) indicates the fuzzified hyper exponential arrivals time rates with three classes in a First Come First Serve (FCFS) way, with infinite framework limit and population size.

In this model, customer arrive in gatherings by \( \tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3, \tilde{\mu}_1, \tilde{\mu}_2 \) and \( \tilde{\mu}_3 \) separately. Let \( \tilde{\lambda}_1 \) (x), \( \tilde{\lambda}_2 \) (y), \( \tilde{\lambda}_3 \) (z), \( \tilde{\mu}_1 \) (u), \( \tilde{\mu}_2 \) (v), \( \tilde{\mu}_3 \) (w) at that point be the fuzzy spoken to six sets as in Eq. (1), Eq. (2), Eq. (3) & Eq. (4):

\[
\begin{align*}
\tilde{\lambda}_1 &= \{(x, \tilde{\lambda}_1(x)) | x \in X \} \quad (1) \\
\tilde{\lambda}_2 &= \{(y, \tilde{\lambda}_2(y)) | y \in Y \} \quad (2) \\
\tilde{\lambda}_3 &= \{(z, \tilde{\lambda}_3(z)) | z \in Z \} \quad (3) \\
\tilde{\mu}_1 &= \{(u, \tilde{\mu}_1(u)) | u \in U \} \quad (4) \\
\tilde{\mu}_2 &= \{(v, \tilde{\mu}_2(v)) | v \in V \} \quad (5) \\
\tilde{\mu}_3 &= \{(w, \tilde{\mu}_3(w)) | w \in W \} \quad (6)
\end{align*}
\]

where X, Y, Z, U, V and W are crisp universal gathering of the arrival rate and service rate. Moreover, let \( f(x, y, z, u, v, w) \) mean the specific arrangement of interest. Consequently x, y, z, u, v, and w are fuzzy numbers and allegedly \( f(x, y, z, u, v, w) \) are fuzzy numbers.

Let \( Lq_1 \), \( Lq_3 \) and \( Lq_2 \) represent the single queuing model as in Eq. (7), Eq. (8) Eq. (9):

\[
\begin{align*}
Lq_1 &= \frac{\rho_1 + \rho_2 + \rho_3}{\mu_1 + \mu_2 + \mu_3} \quad (7) \\
Lq_2 &= \frac{\rho_1 + \rho_2 + \rho_3}{\mu_1 + \mu_2 + \mu_3} \quad (8) \\
Lq_3 &= \frac{\rho_1 + \rho_2 + \rho_3}{1 - \rho} \quad (9)
\end{align*}
\]

4. Ranking Technique

In this section, the way to change over the fuzzy numbers into crisp numbers is explained. Two sorts of fuzzy numbers; pentagonal and heptagonal fuzzy numbers are executed with the left and right ranking technique which is addressed by \( F(R) \rightarrow R \).

To begin assigning the innovation for this system, we consider the following cases

**Pentagonal fuzzy number**

Let a convex pentagonal fuzzy number \( \tilde{A} \) (z) = \( \tilde{A} \) (a1, a2, a3, a4, a5 ; w). Then the Left and right ranking index is portrayed Eq. (10)

\[
R(\tilde{A}) = \int_{z=0}^{w} \frac{L^{(-1)}(z) + R^{(-1)}(z)}{2} \quad (10)
\]

where \( L^{(-1)}(z) = [w(b-a) + a] + \frac{1}{2}[w(c-b) + b] \)
When \( w = 1 \)

According to the ranking index \( \mu \), \( \lambda \), \( \tilde{\lambda} \) computed the mean of queue length for each class. The two types of arrival customers;

Proceeding in the same way as in section 4.1,

In this section 5, numerical examples are considered a pipe company receiving three types of arrival customers; \( \tilde{A}_1, \tilde{A}_2, \tilde{A}_3 \) and the service time represented as a mixture of the exponential distribution \( \tilde{\mu}_1, \tilde{\mu}_2, \tilde{\mu}_3 \) respectively. Note that all parameters are in fuzzy environment and the management want to compute the mean of queue length for each class. The two types of fuzzy numbers considered are pentagonal and heptagonal fuzzy numbers as illustrated for the method in the following subsections.

**Example:1**

Assume that both arrival rate with three classes and rates are given as \( \tilde{A}_1[0.01, 0.03, 0.05, 0.07, 0.09; 1] \), \( \tilde{A}_2[0.002, 0.003, 0.004, 0.006, 0.008; 1] \), \( \tilde{A}_3[0.003, 0.004, 0.005, 0.006, 0.007; 1] \) and \( \tilde{\mu}_1[0.1, 0.4, 0.6, 0.7, 0.9; 1] \), \( \tilde{\mu}_2[0.2, 0.4, 0.6, 0.8, 0.14; 1] \), \( \tilde{\mu}_3[0.4, 0.8, 0.9, 0.10, 0.12; 1] \).

According to the ranking index \( \tilde{\lambda} \)

\[
R(\tilde{\lambda}) = \left( 2a_1 + 3a_2 + 2a_3 + 3a_4 + 2a_5 \right) / 4
\]

\[
R(\tilde{\lambda}_1) = R(0.01, 0.03, 0.05, 0.07, 0.09; 1)
\]

When \( w = 1 \)

\[
R(\tilde{\lambda}_1) = \frac{2(0.01) + 3(0.03) + 2(0.05) + 3(0.07) + 4(0.09)}{4} = \frac{0.78}{4} = 0.195
\]

\[
R(\tilde{\lambda}_2) = \frac{2(0.002) + 3(0.003) + 2(0.004) + 3(0.006) + 4(0.008)}{4} = \frac{0.071}{4} = 0.01775
\]

\[
R(\tilde{\lambda}_3) = \frac{2(0.003) + 3(0.004) + 2(0.005) + 3(0.006) + 4(0.007)}{4} = \frac{0.074}{4} = 0.0185
\]
Similarly, we get

\[
R(\lambda_1) = 0.195 \\
R(\lambda_2) = 0.01775 \\
R(\lambda_3) = 0.0185 \\
R(\mu_1) = 2.075 \\
R(\mu_2) = 1.440 \\
R(\mu_3) = 1.445
\]

\[
L_{q(1)} = \frac{\lambda_1(\rho_1 + \rho_2 + \rho_3)}{1 - \rho_1} \\
L_{q(2)} = \frac{\lambda_2(\rho_1 + \rho_2 + \rho_3)}{1 - \rho_2} \\
L_{q(3)} = \frac{\lambda_3(\rho_1 + \rho_2 + \rho_3)}{1 - \rho_3}
\]

where \( \rho_1 = \frac{\lambda_1}{\mu_1} \), \( \rho_2 = \frac{\lambda_2}{\mu_2} \), and \( \rho_3 = \frac{\lambda_3}{\mu_3} \)

the performance measures are,

\[
L_{q(1)} = (0.195) \left( \frac{0.0939}{2.075} + \frac{0.01232}{1.440} + \frac{0.01280}{1.445} \right) = 0.01387 \\
L_{q(2)} = \left( \frac{0.0177}{2.075} + \frac{0.01232}{1.440} + \frac{0.01280}{1.445} \right) = 0.01263 \\
L_{q(3)} = \left( \frac{0.0185}{2.075} + \frac{0.01232}{1.440} + \frac{0.01280}{1.445} \right) = 0.001306
\]

The performance measures for pentagonal fuzzy numbers are,

\[
L_{q(1)} = 0.01387 \quad Wq_{(1)} = 0.07112 \quad Ws_{(1)} = 0.5530 \quad Ls_{(1)} = 0.10783 \\
L_{q(2)} = 0.0012 \quad Wq_{(2)} = 0.07115 \quad Ws_{(2)} = 0.7655 \quad Ls_{(2)} = 0.01358 \\
L_{q(3)} = 0.00130 \quad Wq_{(3)} = 0.0705 \quad Ws_{(3)} = 0.76263 \quad Ls_{(3)} = 0.014108
\]

**Example 2**

Assume that both arrival rate with three classes and service rates are pentagonal fuzzy numbers in a FCFS (First Come First Service).

\[
\lambda_1 = [0.01, 0.03, 0.05, 0.07, 0.09; 4] \\
\lambda_2 = [0.002, 0.003, 0.004, 0.006, 0.008; 4] \\
\lambda_3 = [0.003, 0.004, 0.005, 0.006, 0.007; 4] \\
\mu_1 = [0.1, 0.4, 0.6, 0.7, 0.9; 4] \\
\mu_2 = [0.2, 0.4, 0.6, 0.8, 0.14; 4] \\
\mu_3 = [0.4, 0.8, 0.9, 0.10, 0.12; 4]
\]

When \( w = 4 \), similarly we get,

\[
R(\lambda_1) = \frac{4(0.78)}{4} = 0.78
\]
Similarly, we get,

\[ R(\tilde{\lambda}_2) = \frac{4[0.071]}{4} = 0.071 \]

\[ R(\tilde{\lambda}_3) = \frac{4[0.074]}{4} = 0.074 \]

\[ R(\tilde{\mu}_1) = 2.3 \]

\[ R(\tilde{\mu}_2) = 5.76 \]

\[ R(\tilde{\mu}_3) = 5.78 \]

The performance measures for pentagonal fuzzy number,

\[
\begin{align*}
Lq^{(1)} &= 0.01387 & Wq^{(1)} &= 0.01779 & Ws^{(1)} &= 0.10785 & Ls^{(1)} &= 0.13827 \\
Lq^{(2)} &= 0.00126 & Wq^{(2)} &= 0.01779 & Ws^{(2)} &= 0.01358 & Ls^{(2)} &= 0.19140 \\
Lq^{(3)} &= 0.0011 & Wq^{(3)} &= 0.0216 & Ws^{(3)} &= 0.01440 & Ls^{(3)} &= 0.19462
\end{align*}
\]

**Example 3**

Assume that both arrival rate with three classes and service rates are heptagonal fuzzy number in a First Come First Service (FCFS) Within framework limit and population size.

\[ \tilde{\lambda}_1 = [0.04, 0.05, 0.08, 0.10, 0.12, 0.14, 0.16; 1] \]

\[ \tilde{\lambda}_2 = [0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09; 1] \]

\[ \tilde{\lambda}_3 = [0.012, 0.013, 0.014, 0.018, 0.002, 0.008, 0.009; 1] \]

\[ \tilde{\mu}_1 = [0.65, 0.71, 0.81, 0.84, 0.85, 0.89, 0.99; 1] \]

\[ \tilde{\mu}_2 = [0.66, 0.73, 0.81, 0.89, 0.91, 0.95, 0.99; 1] \]

\[ \tilde{\mu}_3 = [0.85, 0.89, 0.90, 0.93, 0.94, 0.98, 0.99; 1] \]

According the ranking index of \( \tilde{\lambda} \) is,

\[
R(\tilde{\lambda}_1) = \frac{R \left( 0.04, 0.05, 0.08, 0.10, 0.12, 0.14, 0.16 \right)}{54} = \frac{w(2a_1+7a_2+7a_3+22a_4+7a_5+7a_6+2a_7)}{54}
\]

When \( w = 1 \)

\[
R(\tilde{\lambda}_1) = \frac{1(2(0.04)+7(0.05)+7(0.08)+22(0.10)+7(0.12)+7(0.14)+2(0.16))}{54} = 0.09818
\]

\[
R(\tilde{\lambda}_2) = 0.06518
\]

\[
R(\tilde{\lambda}_3) = 0.03366
\]

Similarly, we get,

\[
R(\tilde{\lambda}_1) = 0.09870
\]

\[
R(\tilde{\lambda}_2) = 0.06518
\]

\[
R(\tilde{\lambda}_3) = 0.03360
\]

\[
R(\tilde{\mu}_1) = 0.82555
\]

\[
R(\tilde{\mu}_2) = 0.86444
\]
The performance measures for heptagonal fuzzy number are as follows:

\[
\begin{align*}
L_q^{(1)} &= 0.3120 & W_q^{(1)} &= 1.405 & W_s^{(1)} &= 0.00030 & L_s^{(1)} &= 0.10902 \\
L_q^{(2)} &= 0.0206 & W_q^{(2)} &= 1.13792 & W_s^{(2)} &= 0.003161 & L_s^{(2)} &= 0.0060 \\
L_q^{(3)} &= 0.0013 & W_q^{(3)} &= 1.13834 & W_s^{(3)} &= 0.0340 & L_s^{(3)} &= 0.01150
\end{align*}
\]

Example 4

Assume that arrival rate with classes and service rates are heptagonal fuzzy number in a FCFS.

\[
\begin{align*}
\tilde{\lambda}_1 &= [0.04, 0.05, 0.08, 0.10, 0.12, 0.14, 0.16; 4] \\
\tilde{\lambda}_2 &= [0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09; 4] \\
\tilde{\lambda}_3 &= [0.012, 0.013, 0.014, 0.018, 0.002, 0.008, 0.009; 4] \\
\tilde{\mu}_1 &= [0.65, 0.71, 0.81, 0.84, 0.85, 0.89, 0.99; 4] \\
\tilde{\mu}_2 &= [0.66, 0.73, 0.81, 0.89, 0.91, 0.95, 0.99; 4] \\
\tilde{\mu}_3 &= [0.85, 0.89, 0.90, 0.93, 0.94, 0.98, 0.99; 4]
\end{align*}
\]

When \( w = 4 \)

\[
\begin{align*}
R (\tilde{\lambda}_1) &= \frac{4[5.33]}{54} = 0.39481 \\
R (\tilde{\lambda}_2) &= \frac{4[3.52]}{54} = 0.26074 \\
R (\tilde{\lambda}_3) &= \frac{4[1.818]}{54} = 0.134666
\end{align*}
\]

Similarly, we get,

\[
\begin{align*}
R (\tilde{\lambda}_1) &= 0.39481 \\
R (\tilde{\lambda}_2) &= 0.26074 \\
R (\tilde{\lambda}_3) &= 0.134666 \\
R (\tilde{\mu}_1) &= 3.3022 \\
R (\tilde{\mu}_2) &= 3.4577 \\
R (\tilde{\mu}_3) &= 3.7118
\end{align*}
\]

The performance measures for heptagonal fuzzy number are as follows:

\[
\begin{align*}
L_q^{(1)} &= 0.048 & W_q^{(1)} &= 0.0881 & W_s^{(1)} &= 0.1543 & L_s^{(1)} &= 0.3909 \\
L_q^{(2)} &= 0.02409 & W_q^{(2)} &= 0.0924 & W_s^{(2)} &= 0.099 & L_s^{(2)} &= 0.3816 \\
L_q^{(3)} &= 0.01244 & W_q^{(3)} &= 0.0923 & W_s^{(3)} &= 0.048 & L_s^{(3)} &= 0.36178
\end{align*}
\]

6. Results And Discussion

The obtains values are given in 1, 1-1 and 2, 2.1 which clarify various estimations of each class for a which range of membership function considered (heptagonal and pentagonal).
### Table 1

Performance measures: $W_s^{(1)}$, $W_s^{(2)}$, $W_s^{(3)}$ and $L_s^{(1)}$, $L_s^{(2)}$, $L_s^{(3)}$

<table>
<thead>
<tr>
<th>Membership Function</th>
<th>$W_s^{(1)}$</th>
<th>$W_s^{(2)}$</th>
<th>$W_s^{(3)}$</th>
<th>$L_s^{(1)}$</th>
<th>$L_s^{(2)}$</th>
<th>$L_s^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentagonal</td>
<td>0.55784</td>
<td>0.7655</td>
<td>0.76263</td>
<td>0.10784</td>
<td>0.0135</td>
<td>0.0141</td>
</tr>
<tr>
<td>Heptagonal</td>
<td>1.4058</td>
<td>1.13792</td>
<td>1.13834</td>
<td>0.10902</td>
<td>0.0060</td>
<td>0.01150</td>
</tr>
</tbody>
</table>

### Table 2

Performance measures: $W_s^{(4)}$, $W_s^{(5)}$, $W_s^{(6)}$ and $L_s^{(4)}$, $L_s^{(5)}$, $L_s^{(6)}$

<table>
<thead>
<tr>
<th>Membership Function</th>
<th>$W_s^{(4)}$</th>
<th>$W_s^{(5)}$</th>
<th>$W_s^{(6)}$</th>
<th>$L_s^{(4)}$</th>
<th>$L_s^{(5)}$</th>
<th>$L_s^{(6)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentagonal</td>
<td>0.13827</td>
<td>0.1914</td>
<td>0.1946</td>
<td>0.1078</td>
<td>0.01358</td>
<td>0.01440</td>
</tr>
<tr>
<td>Heptagonal</td>
<td>0.3909</td>
<td>0.3816</td>
<td>0.3617</td>
<td>0.15436</td>
<td>0.09949</td>
<td>0.04872</td>
</tr>
</tbody>
</table>

### Table 3

Performance measures: $W_q^{(1)}$, $W_q^{(2)}$, $W_q^{(3)}$ and $L_q^{(1)}$, $L_q^{(2)}$, $L_q^{(3)}$

<table>
<thead>
<tr>
<th>Membership Function</th>
<th>$W_q^{(1)}$</th>
<th>$W_q^{(2)}$</th>
<th>$W_q^{(3)}$</th>
<th>$L_q^{(1)}$</th>
<th>$L_q^{(2)}$</th>
<th>$L_q^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentagonal</td>
<td>0.07112</td>
<td>0.07115</td>
<td>0.0705</td>
<td>0.0138</td>
<td>0.0012</td>
<td>0.0012</td>
</tr>
<tr>
<td>Heptagonal</td>
<td>0.00030</td>
<td>0.00316</td>
<td>0.0340</td>
<td>0.3120</td>
<td>0.0206</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

### Table 4

Performance measures: $W_q^{(4)}$, $W_q^{(5)}$, $W_q^{(6)}$ and $L_q^{(4)}$, $L_q^{(5)}$, $L_q^{(6)}$

<table>
<thead>
<tr>
<th>Membership Function</th>
<th>$W_q^{(4)}$</th>
<th>$W_q^{(5)}$</th>
<th>$W_q^{(6)}$</th>
<th>$L_q^{(4)}$</th>
<th>$L_q^{(5)}$</th>
<th>$L_q^{(6)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentagonal</td>
<td>0.01779</td>
<td>0.01779</td>
<td>0.0219</td>
<td>0.01387</td>
<td>0.00126</td>
<td>0.0011</td>
</tr>
<tr>
<td>Heptagonal</td>
<td>0.08816</td>
<td>0.092421</td>
<td>0.092380</td>
<td>0.03484</td>
<td>0.024098</td>
<td>0.01244</td>
</tr>
</tbody>
</table>

Figure 1: Graphical representation table 1
From the outcomes shown in tables 1, 2, 3 and 4, its graphical representation is the positioning technique gives different arrangements of real values. For example, arrival and service rates for each class. In like converges between two classes in the entire framework. Here all execution proportions of class one is less than or equal to execution proportions of class two, class three in the two sorts of fuzzy numbers.

7. Conclusion:

In this section the results are discussed and the system evaluated, it is clear that ranking method gives various sets of real values, such as arrival rates and service rates for each class likewise different performance measurements are obtained which are given seem to converges between three classes in the whole system, it is also seen from table 1, 1.1 table 2, 2.1 that all performance measures of class one are less than performance measures of two in the system.

Also by using more types of fuzzy number lead us to obtain more real data and flexible choices in the system. So we conclude that arrival rate and service rate such as pipe company are collected the data to assume the fuzzy values were applied in the pentagonal and heptagonal, increasing the arrival rate and service rate are discussed heptagonal fuzzy number gives the maximum significant level. This paper extended using fuzzy values in a single (w=1) and a multi server (w=4) by analysing the graph heptagonal reaches the maximum point the waiting in a system and a queue.
Therefore the manager can take the best values and make optimal decisions. Another advantage of using the ranking method index is obtaining exact values inside the closed crisp interval, while also providing more than one solution of values in the queuing system with different types of membership functions.

8. Reference:


