On \((\varphi r)^\star\star\)-Closed Sets in Topological Spaces

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Abstract: The purpose of this paper is to introduce a new class of sets called generalized \(\alpha\) regular-star-star-closed sets (briefly \((\varphi r)^\star\star\)-closed set) in topological spaces. We compare \((\varphi r)^\star\star\)-closed sets with the other existing sets and also their characterizations are analyzed.

I. INTRODUCTION
The concept of generalization of closed set was introduced by Levine [3] in 1970. Further investigation on generalization closed set has lead to significant contribution to the theory of separation axiom, generalization of continuity and covering properties. The concepts of generalized b-closed sets in topological spaces was introduced by A.A.Omari and M.S.M.Nooran [1]. In this paper the non-empty topological space is denoted by \((X,\tau), (Y,\sigma), \) and \((Z,\eta)\) or \(X, Y\) and \(Z\) on which no separation axioms are assumed unless otherwise explicitly stated. In this paper we introduce a new class of sets called generalized \(\alpha\) regular-star-star-closed sets in topological spaces. Also their characterizations are analyzed and it is compared with the other existing sets.

II. PRELIMINARIES

Definition 2.1
Let \(A\) be an subset of topological space \((X, \tau)\). Then \(A\) is called
1. \(\alpha\)-open set \([6]\) if \(A \subseteq \text{int}(\text{cl}(\text{int}A))\).
2. generalized closed set (briefly g-closed set) \([3]\) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open.
3. generalized * closed set (briefly g*-closed set) \([7]\) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is g-open.
4. generalized \(\alpha\)-closed set (briefly g\(\alpha\)-closed set) \([5]\) if \(\alpha\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)-open.
5. \(\alpha\)-generalized closed set (briefly \(\alpha g\)-closed set) \([4]\) if \(\alpha\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open.
6. generalized pre regular-closed set (briefly gpr-closed set) \([2]\) if \(\alpha\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \(X\).
7. generalized \(\alpha\)-regular-closed set (briefly g\(\alpha\)-r-closed set) \([9]\) if \(\alpha\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \(X\).
8. regular closed set \([10]\) if \(A = \text{cl}(\text{int}(A))\).
9. gsp-closed set \([11]\) if \(\text{spcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).
10. generalized \(\alpha\)-regular-star-closed set (briefly \((g\alpha\varphi r)^\star\star\)-closed set) \([8]\) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \((g\varphi r)^\star\star\)-open in \(X\).

III. GENERALIZED \(\alpha\) REGULAR-STAR-STAR-CLOSED SETS IN TOPOLOGICAL SPACES

Definition 3.1
A subset \(A\) of a topological space \((X, \tau)\) is called generalized \(\alpha\) regular-star-star-closed set (briefly \((g\varphi r)^\star\star\)-closed set) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \((g\varphi r)^\star\star\)-open in \(X\).

Definition 3.2
A subset \(A\) of a topological space \((X, \tau)\) is called \((g\varphi r)^\star\star\)-open set if and only if \(A^C\) is \((g\varphi r)^\star\star\)-closed in \(X\).

Theorem 3.3:
Every \((g\varphi r)^\star\star\)-closed set is g-closed set.
From the above theorem and examples we have the following diagrammatic representation.
Theorem 3.15: If A and B are $(garr)^κ$-closed set in X then $AUB$ is $(garr)^κ$-closed set in X.

Proof:
Let A and B be $(garr)^κ$-closed set in X and U be any $(garr)^κ$-open set such that $AUB ⊆ U$. Therefore $cl(A) ⊆ U$, cl(B) ⊆ U. Hence $cl(AUB) = cl(A) ∪ cl(B) ⊆ U$. Therefore $AUB$ is $(garr)^κ$-closed set in X.

Theorem 3.16:
If a set A is $(garr)^κ$-closed set then $cl(A)$−A contains no non empty $(garr)^κ$-closed set.

Proof:
Let F be $(garr)^κ$-closed set in X. Such that $F ⊆ cl(A)$. $F ⊆ cl(A) ∩ A^c ⇒ F ⊆ cl(A)$ and $F ⊆ A^c$, $A ⊆ F^c$. Then $A ⊆ X−F$. Since A is $(garr)^κ$-closed set and $X−F$ is $(garr)^κ$-open then $cl(A) ⊆ X−F$. (i.e.) $F ⊆ X−cl(A)$. So $F ⊆ (X−cl(A))∩(cl(A)−A)$. Therefore, $F=∅$.

Theorem 3.17:
If B is $(garr)^κ$-closed set and B ⊆ A ⊆ cl(B) then A is $(garr)^κ$-closed.

Proof:
Let B be $(garr)^κ$-closed and O be any $(garr)^κ$-open set such that $A ⊆ O$. Then $B ⊆ O$ which implies $cl(A) ⊆ cl(B) ⊆ O$. Hence A is $(garr)^κ$-closed.

Theorem 3.18:
A is any $(garr)^κ$-open set if and only if $B ⊆ int(A)$ where B is $(garr)^κ$-closed and B ⊆ A.

Proof:
Let B be $(garr)^κ$-closed and B ⊆ A. Then $A^c ⊆ B^c$ which implies $cl(A^c) ⊆ B^c$. Since $A^c$ is $(garr)^κ$-closed set and B is $(garr)^κ$-open. Therefore we have $B ⊆ int(A)$. Conversely, assume that $B ⊆ int(A)$. Whenever B is $(garr)^κ$-closed and B ⊆ A. Let O be any $(garr)^κ$-open. The $O^c$ is $(garr)^κ$-closed. Therefore by assumption, $O^c ⊆ int(A)$ which implies $cl(A^c) ⊆ O$. Hence A is $(garr)^κ$-open.

Theorem 3.19:
If int(A)⊆B ⊆A and A is $(garr)^κ$-open then B is $(garr)^κ$-open.

Proof:
int(A)⊆B ⊆A implies $A^c ⊆ B^c ⊆ cl(A^c)$. Since A is $(garr)^κ$-open. $A^c$ $(garr)^κ$-closed. Therefore by theorem 3.17, $B^c$ is $(garr)^κ$-closed. Hence B is $(garr)^κ$-open.

REFERENCES
Closed set In topological spaces.