Asymmetric Volatility Modelling Of Indian Stock Market During Different Regime Of Indian Financial Cycle

Avik Das, Dr. Devanjali Nandi Das
Research Scholar, Assistant Professor
Department of Management
Techno India University, West Bengal, India

Abstract: This study aims to evaluate the characteristics of conditional volatility of the Indian stock market and compare the nature of volatility between the boom-and-bust phase of the Indian Financial cycle. The asymmetric volatility modelling is conducted using Exponential GARCH (EGARCH), Asymmetric power GARCH (APARCH) and GJR-GARCH on S&P Nifty 50 during July 2001 to June 2017 to arrest the nature of Indian financial cycle. We perform Value at Risk back-testing for adequacy of model fitness and compare half-life estimation for understanding the nature of persistence. Our study identifies APARCH (1,1) to be the best model for explaining boom phase volatility and EGARCH (1,1) as the best model for bust phase. The study also reveals the existence of inverse leverage effect in the boom-and-bust phase of the financial cycle. Our study originally contributes by comparing and documenting the empirical performance of GARCH processes during the boom-and-bust phase of Indian Financial Cycle. These empirical results would be beneficial for better risk management, portfolio optimisation and asset pricing.

Index Terms - Asymmetric GARCH models, EGARCH, APARCH, GJR GARCH, Financial Cycle, Half-life estimation, Leverage effect.

I. INTRODUCTION

Volatility refers to the uncertainty in the stock market and investors’ perception of risk majorly begins with uncertainty of future outcomes. This makes volatility forecasting a vital factor in risk management, portfolio management and valuation of asset pricing. Volatility is only observable through a conditional variance process. Subsequently, a number of models have been established over the time, especially designed to arrest the conditional volatility of financial time series. The primary intention of constructing such models is to forecast future volatility that would enable improvement in portfolio allocation, having an improved risk management process. Further, policy makers would able to gauge stock market volatility as an indicator for the susceptibility of external and internal shock of the economy and financial market.

There are prominent evidences that the Global financial crisis (GFC) of 2007-08 has significant negative spillover on the real economy. The weaknesses inside the financial framework are found to be cyclical developments of financial factors like overvaluation of asset prices, prolonged credit booms and lower interest rates for a much longer time have preceded many financial crises throughout the world economy. These developments encouraged researchers to further investigate the relationship between financial cycles and factors of asset prices. Boom phase of Financial Cycles is generally characterized by increased capital flow, increased asset prices and high liquidity and massive credit growth in the real economy. These symptoms can induce formation of asset price bubbles as seen during GFC.

As Borio (2014) illustrated financial cycle thoughtfully, as an “self-reenforcing interaction” between risk and asset value which ultimately evolved to a boom-and-bust phenomenon. The cyclical changes of financial variables may exaggerate economic fluctuations, cause imbalances, lead to economic uncertainty and/or threaten financial stability of country. Many researchers provide evidences of cyclical features of financial markets as well. Financial cycles are longer than business cycles and can provides an early warning signal of uncertainty and risk aversion of financial market participants (Menden, 2017). In addition, financial crises mature at or near the peak of financial cycles (Borio, 2014) indicating a bust phase of financial cycle.

Our study is important to explore the nature of volatility of the Indian stock market during the financial cycle to understand the dynamic nature of volatility and leverage effect during boom-and-bust phases of the financial cycle. Secondly, this study will enhance the volatility forecasting of the Indian market, which will be helpful for risk management, portfolio allocation and asset pricing. The current study focuses on adequacy of different asymmetric GARCH models to capture the predictable features of volatility on the Indian stock market during the Indian Financial Cycle established by Behera and Sharma (2019). This paper is adding value to the current body of literature, for investors and portfolio managers, through connecting the characteristics of boom-and-bust phases with a testing procedure of asymmetric GARCH model extensions for different volatility regimes. Volatility estimation is performed on Nifty 50, major stock market indices in India. The fitted models are then evaluated to judge the model performance in terms of Value at Risk (VaR) at 5% significance level.
II. LITERATURE REVIEW

Modelling and forecasting of stock market volatility has immense importance for investors and academicians due to the application of asset pricing, risk mitigation and portfolio strategy. Plethora studies have been conducted across the globe using family of GARCH models for analysing the volatility of financial time series (Bekaert & Wu, 2000), (Jayasuriya, Shambora, & Rossiter, 2009), (Talpsepp & Rieger, 2010), (Horpestad, Lyócsa, Molnár, & Olsen, 2019), (Iqbal, Manzoor, & Bhatti, 2021). The effect that volatility in equity markets seems to be asymmetric: i.e., negative returns tend to increase volatility to a larger extent than positive returns, is well documented. The observation was first recognized by Black (1976) and Christie (1982) who elucidated the lopsidedness with the leverage effect, meaning that a decrease in the value of the stock increases financial leverage, which makes the stock riskier. Asymmetricity and leverage is a vital issue in the context of financial markets, and the GARCH type of models may provide a good illustration of volatility. Many researchers found asymmetric GARCH models like EGARCH, GJR-GARCH, APARCH etc. has superior predictability than symmetric GARCH and OLS methods (Pagan & Sossounov, 2003), (Awartani & Corradi, 2005), (Balaban & Bayar, 2005), (Hansen & Lunde, 2006)).

Heteroscedastic behaviour of Indian stock market was investigated by Karmakar (2007) using EGARCH model. It was observed that asymmetric volatility rises during the declining phase of market and returns are not statistically correlated to the risk. Mohanty (2009) compared 4 major indices of India using symmetric and asymmetric models and concluded EGARCH to be appropriate model to explain existing asymmetricity. Bordoloi and Ramanarayanan (2011) applied TGARCH and EGARCH on BSE 500 to capture asymmetric nature of India market and found negative news impact increases stock market volatility more than good news. Vijayalakshmi & Gaur (2013) used GARCH family models to evaluate the accuracy and forecasting capabilities, found TARCH and PARCH models showcased better volatility forecast for stock market indices. Tripathy and Gil-Alana (2015) proposed asymmetric volatility model performed better than symmetric model in Indian market. Also, they suggested Generalised Error distribution was the most suitable for asymmetric model. Samineni et al. (2021) unveiled EGARCH model can capture lopsided volatility in Indian market between 2011 to 2020.

As far as our knowledge, no previous study has been conducted on the volatility modelling during Indian Financial Cycle comparing the dynamic change in the asymmetricity and half-life estimation. Therefore, the objective of this paper will be an attempt to compare the asymmetric volatility between 2001 to 2017 including boom and bust phases of the Indian Financial cycle using EGARCH, APARCH and GJR-GARCH models. Moreover, we will compare four alternative density functions namely, normal, student t, skewed student t and GED to identify the best model for volatility modelling in the Indian market in two different phases and try to identify the leverage effect and half-life measurement.

III. METHODOLOGY

In the empirical studies, it is strongly advocated that to use conditional heteroskedastic variance instead of homoskedastic variance and models. Predominantly, in high frequency models like financial time series analysis should involve working with heteroskedastic models. A few salient features of stock returns are well documented like heavy-tail distribution, volatility clustering, leverage effect, co-movements of volatilities and mean reversion (Poon & Granger, 2003). Such idiosyncratic behaviour can be best explained by the Autoregressive Conditional Heteroscedasticity (GARCH) suggested by Engle (1982) and further generalised by Bollerslev (1986). The symmetric GARCH (p, q) is inappropriate as it fails to arrest the behaviour of the heavy tails. Alexander (2008) recommends the use of asymmetric GARCH models for equities and commodities. Thus, we nominate to model our index returns using the exponential-GARCH, APARCH and GJR-GARCH and compare how they arrest the idiosyncrasies.

**EGARCH (Exponential – GARCH)**

Nelson (1991) introduced Exponential GARCH and removed the non-negativity constraints imposed by the GARCH model. By including γ parameter E-GARCH can capture the leverage effect. The model specification is as follows:

\[
\log(h_t) = \omega + \beta \log(h_{t-1}) + \gamma \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} + \alpha \frac{|\epsilon_{t-1}|}{\sqrt{h_{t-1}}} - \frac{\sqrt{2/\pi}}{\sqrt{h_{t-1}}}
\]

(1)

Here, \(\omega\) is a constant, \(\epsilon_t\) is the innovation or the shock process, \(h_t\) is the conditional standard deviation. \(\alpha, \beta \) respectively are the ARCH and GARCH parameters. \(\gamma\) is the leverage parameter.

The conditional variance is constrained to be non-negative by the assumption that the logarithm of \(h_t\) is a function of past values of \(\epsilon^2_t\). Given the error process parameterised as:

\[
\epsilon_t = \epsilon^2_{t-1}(h_t)^{1/2}
\]

(2)

From (1) we realize \(\gamma\) being a function of both the measure and sign of \(\epsilon^2_t\) which empowers \(h_t\) to respond asymmetrically to positive and negative values of \(\epsilon_t\), believed to be important for example in modelling the behaviour of stock returns.

The EGARCH model in (1) seizes the lopsidedness in the returns because of the multiplicative term \(\frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}\). The coefficient \(\gamma\) is typically negative. That ensures positive return shocks induce less volatility than negative return shocks (Engle & Ng, 1993). If \(\gamma \neq 0\), the impact is asymmetric in nature and if \(\gamma > 0\), it is an indication of the existence of leverage effect and if statistically significant, a positive innovation (good news) in the past increases’ volatility more than a negative innovation (bad news). If \(\gamma < 0\), then it specifies that leverage effect exists and if statistically significant, a negative shock (bad news) in the past increases’ volatility more than a positive shock (good news).
GJR-GARCH

GJR-GARCH named after Glosten, Jagannathan and Runkle (1993), is another extension of GARCH with a third variable to arrest the asymmetricities in the returns. GJR model formulation differs from EGARCH model in the way it responds to previous negative volatility. Fundamentally, GJR model has an extra variable to capture leverage effect in the data stream. The variable $\gamma$ enhances the volatility response to only negative market shocks. The generalized specification for the conditional variance is given by:

$$h_t = \omega + \alpha \varepsilon^2_{t-1} + \gamma \varepsilon_{t-1} \varepsilon_t + \beta \sigma^2_{t-1} \tag{3}$$

where $h_t = 1$ if $\varepsilon_t < 0$ and otherwise 0. In this particular model, positive shock and negative shock have diverse effect on the $h_t$. Positive shock ($\varepsilon_{t-1} > 0$) has an impact of $\alpha$ and negative shock ($\varepsilon_{t-1} < 0$) has an impact of $(\alpha + \gamma)$. If $\gamma > 0$ and significant, then it implies negative shocks would increase return volatility or conditional volatility and there is a leverage effect.

APARCH

Ding, Granger and Engle (1993) introduced the APARCH (Generalized Asymmetric Power ARCH Model), which estimates the optimal power term. They reported that the absolute returns and their power transformations have a vastly substantial “long memory” property as the returns are highly correlated. This model can illustrate the Fat-tails, high kurtosis and Leverage Effects. The general structure is as follows:

$$\sigma^8_t = \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma^8_{t-j} \tag{4}$$

where $\varepsilon_t$ is the innovation process, $\sigma_t$ is the conditional standard deviation. Here $\gamma$ is the leverage variable and $\delta$ is the power term. Also, $\omega > 0$, and $\alpha, \beta, \delta \geq 0$, $|\gamma| \leq 1$. In this model, conditional variance depends not only on the magnitude but also on the sign of $\varepsilon_t$ results in positive shock and negative shock have different predictability of future volatility.

Brook and Burke (2003) indicate that the lag order $(1,1)$ of GARCH models is adequate to arrest all of the volatility clustering existing in the data. We have also chosen the model orders to $(1,1)$ in essence of the literature. The GARCH models are estimated using a maximum likelihood (ML) methodology. The reason of using ML is to interpret the density as a function of the parameters set, conditional on a set of sample outcomes.

In the seminal paper of Engle (1982), the density function was the standard normal distribution. The failure to seize the fat-tails property of high-frequency financial time series has led to the use of non-normal distributions to better model excessive third and fourth moments. For the current study we would use the most commonly used distributions like the normal distribution, Student-t distribution, Skewed student-t distribution and the Generalized Error Distribution (GED). We would compare all these density functions for individual models.

Half-life estimation of Volatility

Volatility demonstrates another feature of mean reversion; the short-term volatility reverts to its long term mean levels at a rate of sum of $\alpha$ & $\beta$ (Persistence). Half-life measurement is the average number of periods (days) for the volatility shock to revert to the long-term volatility levels. Engle and Patton (2001) defined half-life “A further measure of the persistence in a volatility model is the half-life of volatility”. This is defined as the time taken for the volatility to move halfway back towards its unconditional mean following a deviation from it.” Thus, we can write the subsequent formula of half-life for the volatility shocks:

$$L_{half} = \frac{\ln \left( \frac{1}{2} \right)}{\ln (\alpha + \beta)} \tag{5}$$

It is important to note that if the value of $(\alpha + \beta)$ is closer to 1 then the half-life will be the longer.

IV. DATA AND EMPIRICAL RESULTS

Behera and Sharma (2019) identified Indian Financial cycle exhibited two troughs in 2001: Q2 & 2017: Q1 and only peak in 2008: Q3. Depending on that, we segregate boom phase (upturn) from July 2001 to December 2008 and bust phase (downturn) January 2009 to June 2017. We have selected the Nifty 50 index and collected daily data from National Stock Exchange of India (NSE) of a total 4106 data point. The daily returns calculated using log returns formula:

$$\text{Daily returns} = \ln (P_t - P_{t-1}) \tag{6}$$
where $P_t$ is the closing price on day $t$. The descriptive statistics for the return series of Nifty 50 are summarized in Table 1.

<table>
<thead>
<tr>
<th>Data points</th>
<th>Boom</th>
<th>Bust</th>
</tr>
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<tbody>
<tr>
<td>1877</td>
<td>2229</td>
<td></td>
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</tbody>
</table>

| Min.        | -0.13054 | -0.063802 |
| 1st Qu     | -0.00695 | -0.0053735 |
| Mean        | 0.00054  | 0.0005584 |
| 3rd Qu      | 0.00942  | 0.0063303 |
| Max.        | 0.07969  | 0.1633431 |
| Std. Dev    | 0.01680864 | 0.01176722 |
| Skewness    | -0.7997669 | 1.000162 |
| Kurtosis    | 6.425719  | 18.04062 |
| Shapiro Wilk (p-value) | 0.92923 | 0.92606 |
| ARCH LM     | 363.62   | 67.177 |
| (p-value)   | (0.0000) | (0.0000) |

| ADF         | -8.9343*** | -10.697*** |
| PP          | -1660.1*** | -2004.8*** |
| KPSS        | 0.23555*   | 0.12199*   |

Table 1: Descriptive Statistics of Daily returns of Nifty 50

The Nifty 50 return series exhibit asymmetric and leptokurtic (high peak) (see Figure 1 & Figure 2) properties. Interestingly, the boom phase is negatively (-0.7997669) skewed but the bust phase is positively skewed (1.000162). A further test of normality of Shapiro-Wilk yield test statistics of 0.9293 & 0.92606 (p-value < 0.01) confirms normality of both return series.

The descriptive graph (Figure 3) display Nifty 50 returns exhibits volatility clustering that means periods of large changes in returns tend to group and followed by relatively low changes in returns.

Estimation Results

For volatility analysis the EGARCH, APARCH & GJR-GARCH models are executed on Nifty 50 returns series under Normal, Student-t, Student skewed-t and Generalized Error Distributions (Table 2). To preserve space the results of all the models with other distributions declined to present, but they are available upon request. The standard of model selection is based on in-sample diagnosis including minimum values of Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), and highest maximum-likelihood (ML) values.

For the boom phase, by ranking of Maximum Likelihood and all information criteria favour APARCH (1,1) – Skewed Student-t distribution and for bust phase, EGARCH (1,1) – Student-t distribution model.
It is clearly evident from Table 2, all coefficients of EGARCH (1, 1) are significant at 1% significance level and ($\alpha + \beta < 1$). There is high persistence in the downward financial cycle as $\beta$ (0.987787) is quite high. For $\gamma$, the leverage term greater than zero and valued 0.110512 is significant and positive. There exists leverage effects and the positive shock in the past observations increase volatility than the negative innovations. The estimated EGARCH model as follows:

$$\sigma_t^{1.541003} = 0.000094 + 0.141845 (|\varepsilon_{t-1} - 0.487299 \varepsilon_{t-1}|)^{1.541003} + 0.802793 \sigma_{t-1}^{1.541003}$$

Moreover, $\delta$, the power term is 1.541003 is positive and greater than 1 and the $\gamma$ term is positive (0.487299) and significant at 1% level, inferring negative shocks can impact higher on volatility than the positive shocks.

To check the performance of the model APARCH (1, 1), we executed a historical Value-at-Risk (VaR) back-test with exceedance of 0.05 and at confidence level of 0.95. We used Kupiec’s unconditional coverage test Kupiec (1995) and Christoffersen’s conditional coverage test (Christoffersen, 1998). With a moving window of 496, refitting of 1 and expected exceedance of 24 we actually found 40 VaR exceedance (8.1%). We had a $p$-value of 0.004 in Kupiec test and 0.011 in Christoffersen test, which is less than 5% significance level. We accept the alternative hypothesis.

Figure 4 and Figure 5 are plots of Nifty Actual Returns vs expected returns at VaR = 0.05 and out of sample performance of APARCH (1, 1) model from back-test.
\[
\log(h_t) = -0.115615 + 0.987787 \log(h_{t-1}) + 0.110512 \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} - 0.079445 \left[ \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} - \sqrt{\frac{2}{\pi}} \right]
\] (8)

We test performance of EGARCH model using Kupiec unconditional test and Christoffersen’s conditional coverage test. The back test length of 495 and refitting of 1 we found expected exceedance to be 24.8 with 5% alpha. The actual exceedance found to be 16 only. The test reported p-value of 0.054 in Kupiec and 0.092 in Christoffersen test. In both the cases p-value is greater than 5% significance level, therefore we accept the null hypothesis and confirm that our model is adequate.

**Half-life analysis**

Persistence and half-life analysis is reported in Table 3. Persistence measures continuation of an external shock (positive or negative) after the shock is removed from system. In other words, level of persistence indicates after the external shock how soon the data series will revert to the long-term mean or it will push further from the mean. Half-life particularly identifies the number of days after the shock it will take to revert to the long-term mean. Our analysis show in the boom the half-life is ranging between 10-13 days in different models. Whereas, in the bust phase half-life increase to many folds and ranging between 57-74 days in different models. It means in the bust phase the volatility rises due to any external shock take larger time to revert to unconditional volatility. Homoscedastic Volatility represents the unconditional volatility is quite high during boom phase ranging between 0.01317628 to 0.01758322 compared to bust phase ranging between 0.00879765 to 0.01446384. It signifies in boom phase the unconditional volatility is higher than the bust phase.

**Table 3: Volatility and Half-life estimations**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Boom Phase</th>
<th>Bust Phase</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>EGARCH (1,1) skewed t</td>
<td>APARCH (1,1) skewed t</td>
</tr>
<tr>
<td></td>
<td>0.942786</td>
<td>0.939117</td>
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<tr>
<td>Persistence</td>
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<td>12</td>
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<tr>
<td>Half-life (days)</td>
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<tr>
<td>Heteroskedastic Volatility (HeV)</td>
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</tr>
<tr>
<td>Homoscedastic Volatility (HoV)</td>
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</tr>
<tr>
<td>Volatility Comparison</td>
<td>HeV&gt;HoV</td>
<td>HoV&gt;HeV</td>
</tr>
</tbody>
</table>

V. CONCLUSION

The current paper explores the evolution of asymmetricity of the Indian stock market during the Indian Financial cycle (2001 to 2017) established by Behera and Sharma (2019). By employing three different asymmetric GARCH models in two different phase of Indian Financial Cycle we tried to estimate the best asymmetric model of the GARCH family. We use Maximum-Likelihood, AIC and BIC to select optimal GARCH model. Further, we estimate the persistence of volatility using half-life estimate. Our observations suggest, APARCH (1,1) – skewed t distribution will be the best model for boom phase of Indian Financial cycle whereas, EGARCH (1,1) – t distribution will be the accurate model for bust phase. Our study also confirms the presence of asymmetricity in both the phases.

On an important note, the leverage term in EGARCH and APARCH is positive that signifies an inverse leverage effect, in which positive shock increases future volatility more than negative shock. During the bust phase all the asymmetric models exhibit higher level of persistence in Indian market as a result the half-life estimations have gained many folds compared to boom phase though homoscedastic volatility is higher in boom phase. It signifies shock to the system remain for a longer time before die down. According to the EGARCH model, any shock will continue for 57 days in the bust phase. The long memory property is quite evident in the bust cycle that enable us to forecast volatility more accurately as we observe in Value at risk (VaR) back testing.

Our study advocates the proof of higher homoscedastic volatility in the boom phase of the financial cycle due to the dominance of speculators and noise traders in the upcycle than the down cycle. One of the purposes of the numerous GARCH models is to offer decent predictions of volatility which can be used for portfolio allocation, risk measurement, option valuation, etc. Risk averse Investors may choose to adjust their portfolios by reducing portfolio allocation to securities whose volatilities are forecasted to rise. This study can be proven beneficial to the portfolio managers, traders and policy makers as they can formulate separate risk minimizing strategies using more advanced dynamic hedging techniques according to the phases of the financial cycle.
REFERENCES


