Playing Chess With Mathematics

Dr. Sunila Thomas                                             Sneha S Nair
Associate Professor M.Sc, B.Ed
Titus II Teachers College, Tiruvalla

ABSTRACT:
This article describes how mathematics relates to chess. At first glance, these two disciplines may seem unrelated. However, mathematics and chess have a large amount of unexpected connections. The relationship between chess and mathematics is seen in a number of ways. Every element of the chess game is well defined, which makes it theoretically possible to fully grasp it through logic and mathematics.

KEY TERMS USED:
• Chess: a two player board game.
• Algebraic notation: standard method for recording and describing the moves in a game of chess.
• Combinatorial game theory: a branch of mathematics and theoretical computer science that typically studies sequential games with perfect information.
• Game-tree complexity: the total number of possible games that can be played.

INTRODUCTION:
“A chess problem is an exercise in pure Mathematics.” – G. H. Hardy.

Chess is a recreational and competitive board game played between two players. It is played on a square chessboard with 64 squares arranged in an eight-by-eight grid. At the start, each player (one controlling the white pieces, the other controlling the black pieces) controls sixteen pieces: one king, one queen, two rooks, two knights, two bishops, and eight pawns. The objective of the game is to checkmate the opponent’s king, whereby the king is under immediate attack (in "check") and there is no way to remove it from attack on the next move. There are also several ways a game can end in a draw.

The game structure and nature of chess are related to several branches of mathematics. Many combinatorial and topological problems connected to chess, such as the knight’s tour and the eight queens puzzle, have been known for hundreds of years. The number of legal positions in chess is estimated to be about $10^{43}$, and has been proved to be fewer than $10^{47}$, with a game-tree complexity of approximately $10^{123}$. Zermelo’s theorem, in game theory, states that it is possible to solve chess.

GEOMETRY
Chess is a game that relies on geometry, namely lines. The chessboard itself is composed of sixty-four alternating light and dark squares. The board can be further divided into lines, more specifically, ranks, files and diagonals. For two-dimensional boards, each horizontal row of squares is called a rank, each vertical column of squares is called a file, and each oblique line of squares of the same colour is called a diagonal.
Good chess players employ simple geometric rules to figure out, with a simple glance at the chess board, what would be the result of the encounter.

**ALGEBRAIC NOTATION**

Algebraic notation is based on a system of coordinates to uniquely identify each square on the chessboard. Each square of the chessboard is identified by a unique coordinate pair, a letter and a number, from White's point of view. The vertical columns of squares, called files, are labelled a through h from White's left (the queenside) to right (the kingside). The horizontal rows of squares, called ranks, are numbered 1 to 8 starting from White's side of the board. Each piece type (other than pawns) is identified by an uppercase letter. English-speaking players use the letters K for king, Q for queen, R for rook, B for bishop, and N for knight. Each move of a piece is indicated by the piece's uppercase letter, plus the coordinate of the destination square.

**MATHEMATICAL PROBLEMS ON BOARD**

**The Knight’s Tour Problem**

The knight's tour problem is an instance of the more general Hamiltonian path problem in graph theory. The knight's tour problem is the mathematical problem of finding a knight’s tour. A knight’s tour is a sequence of moves of a knight on a chessboard such that the knight visits every square exactly once.

**The Eight Queens Puzzle**

Chess composer Max Bezzel proposed the eight queens puzzle in 1848. The eight queens puzzle is the problem of placing eight chess queens on an 8×8 chessboard so that no two queens threaten each other; thus, a solution requires that no two queens share the same row, column, or diagonal.

**GAME THEORY**

In game theory, Zermelo's theorem is a theorem about finite two-person games of perfect information in which the players move alternately and in which chance does not affect the decision making process. The game must satisfy the following criteria: there are two players in the game; the game is of perfect information; the board game is finite; the two players can take alternate turns; and there is no chance element present. Zermelo has stated that there are many games of this type however his theorem has been applied mostly to the game chess. When applied to chess, Zermelo’s Theorem states "either White can force a win, or Black can force a win, or both sides can force at least a draw".

**GAME TREE COMPLEXITY**

Combinatorial game theory has several ways of measuring game complexity. Game-tree complexity represents the number of distinct plays, or paths, for a given game. Each path consists of multiple different game states. For example, imagine every possible checkmate scenario in chess. Now count each possible game path that results in each different scenario. That is game-tree complexity. The Shannon number, named after the American mathematician Claude Shannon, is a conservative lower bound of the game-tree complexity of chess of $10^{120}$, based on an average of about $10^3$ possibilities for a pair of moves and a typical game lasting about 40 such pairs of moves.

**PLAYING CHESS**

Playing chess has strong resonances with doing mathematics. Chess promotes thinking skills of higher order, spatial reasoning skills, visual memory and capacity to predict and anticipate consequences. It requires constant calculation and correlation; to decide what piece is best to sacrifice at a certain point. It introduces a coordinates system and geometric concepts (files, rows, diagonals). Analysis of positions has a lot in common with mathematical problems.

**THE RICE AND CHESSBOARD STORY**

Legend has it that the game was invented by a mathematician in India. When chess was presented to a great king, the king offered the inventor any reward that he wanted. The inventor asked that a single grain of rice be placed on the first square of the chessboard. Then two grains on the second square, four grains on the third, and
so on. Doubling each time. The King thought that he’d got away lightly, but little did he realise the power of doubling to make things big very quickly. From the one grain of rice on the first square of the chessboard, the amount increases to the point that by the time to square 64, there are over 18 quintillion grains of rice on the board. In mathematics, it’s a demonstration of exponential growth.

CONCLUSION:
Chess is unarguably mathematical; the game structure and nature of chess are related to several branches of mathematics. Every element of the chess is well defined, which makes it theoretically possible to fully grasp it through logic and mathematics.

A chess problem is a mathematical problem which is formulated using a chessboard and chess pieces. Playing chess has strong resonances with doing mathematics. There are simple rules for the way each chess piece moves but beyond these basic constraints, the pieces can roam freely across the board. Mathematics also proceeds by taking self-evident truths (called axioms) about properties of numbers and geometry and then by applying basic rules of logic you proceed to move mathematics from its starting point to deduce new statements about numbers and geometry.

REFERENCES:
- Mathematics in the Game of Chess—www.ukessays.com