



# Measurement of Speed of Sound in Exhaust Gases using Mole Fraction

Atharva Prasanna Bagewadi

Bachelor of Technology Student

Department of Mechanical Engineering

Vishwakarma Institute of Information Technology, Pune, India

**Abstract:** When a vehicle is equipped with an IC Engine, it is bound to develop a lot of noise and vibrations. In order to have an efficient working of the engine it is necessary to attenuate the noise that is generated. In order to reduce the noise generated it is necessary to optimize the exhaust system of the vehicle. The sound waves travel through the exhaust header before they are attenuated inside the muffler. In this paper we will be calculating the speed of sound in the exhaust gases, which will help us in designing an efficient exhaust system. This paper will focus on the KTM Duke 390 engine. Usually, this parameter is calculated by performing simulation on Software like GT-Suite. This paper will provide a theoretical method to calculate the speed of sound using the mole fraction of the exhaust gases and we will be comparing these results with the results that we will simulate on GT-Suite software.

**Index Terms** – Exhaust system, GT-Suite, IC Engine, Muffler, Speed of Sound

## I. Introduction

IC Engines usually use fuels like gasoline and diesel. And in a 4-stroke engine (Suction-Compression-Power-Exhaust), exhaust gases are formed as the by-product of the generated power. Along with it noise and vibrations are also generated. Engine produces the vibratory forces due to unbalanced force from the engine parts during the operation. The vibrations caused by the engine at the support are torsional and longitudinal. The torsional vibrations are caused by the crankshaft while longitudinal vibrations are caused due to block. These vibrations are countered by robust mountings and rubber paddings.

Now to attenuate the noise generated by the IC Engine, mufflers and resonators are installed in the exhaust system. Noise is a type of sound and is described as unwanted, loud and unpleasant. As it is type of sound, the sound waves travel through the exhaust header finally to be attenuated at a muffler or a resonator.

While calculating the parameters for a muffler or a resonator we required to determine the speed of sound. Now as we know that velocity of sound changes as per the medium and exhaust gases are unlike Air, we cannot take its value as 343 m/s. as the sound travels in the exhaust gases their density, temperature are some of the factors that affect the speed of sound.

Usually the velocity of sound is calculated by the software like GT-Suite but this paper will show a theoretical approach to do the same. Finally, we will be comparing both the values to determine the deviation.

## II. Theory

If we focus of designing a Helmholtz resonator in the exhaust system in order to attenuate a particular frequency. A Helmholtz resonator consists of acoustical cavity contained within rigid wall and is connected to the environment through a small orifice. When the incident frequency matches with the natural frequency of the resonator, the sound energy is used to move the air or gas column inside the resonator. This creates motion and leads to work. As a result, the sound energy is converted to heat energy and the sound is attenuated. This phenomenon is called Acoustic Coupling. Hence, Helmholtz resonator behaves like an acoustical mass-spring system.

Now the main task is to find out the natural frequency of the resonator. That frequency is given by the formula below:

$$f_H = \frac{v}{2\pi} \sqrt{\frac{A}{V_0 L}}$$

In the above equation;  $f_H$  is the Helmholtz resonant frequency,  $v$  is the speed of sound in the gas,  $A$  is the cross-sectional area of the neck,  $V_0$  is the static volume of the cavity and  $L$  is the neck length.

In the above equation as we can see, velocity of sound is an important parameter which is required for the calculation of the natural frequency of the resonator.

Classically, the speed of sound in an ideal gas could be written as  $c_0 = (\gamma R_0 T / M)^{1/2}$ . Now, if we consider a mixture of ideal gases, the expression above could be rewritten. By assuming that the specific heat ratio  $\gamma$  is written as the mole weighted specific heat ratio for the mixture and that the molecular weight of the mix  $M$  is the mole fraction weighted sum of the mole weights of the constituents as:

$$\gamma = \frac{\sum_i^n x_i C_{pi}}{\sum_i^n x_i C_{vi}}$$

$$M = \sum_i^n x_i M_i$$

(Equation 1 & 2)

$C_{vi}$  is the specific heat at constant volume and  $C_{vi} = C_{pi} - R_0$ . And using these formulas above in the equation of speed of sound we arrive at:

$$c_0^2 = R_0 T \frac{\sum_{i=1}^n x_i C_{pi}}{\sum_{i=1}^n x_i M_i \sum_{i=1}^n x_i (C_{pi} - R_0)}$$

(Equation 3)

Where,  $R_0$  is the universal gas constant,  $T$  the absolute temperature,  $x_i$  the mole fraction of the  $i$ :th component,  $M_i$  the molecular weight and  $C_{pi}$  the heat capacity at constant pressure [1].

### III. Theoretical Calculations for Speed of Sound

Now considering the above formulas, the main parameter which is required is the mole fraction. We will be using the data sheet of KTM Duke 390 to find out the composition of the exhaust gases for gasoline at 1600 K.

Exhaust Gases	Weight Percentage at 1600 K	Weight
CO <sub>2</sub>	17	0.17
H <sub>2</sub> O	8.3	0.083
O <sub>2</sub>	1.1	0.011
N <sub>2</sub>	7.2	0.072
H <sub>2</sub>	0.035	0.00035
CO	1.4	0.014
HC	0.13	0.013
NO <sub>x</sub>	0.11	0.011
SO <sub>2</sub>	0.002	0.00002

Table 1. Composition of Exhaust Gases [2]

Exhaust Gases	Molar Mass of Exhaust Gases (Mi) (Kg/mol)	Cp at 1600 K (J/mol *K)	Cv at 1600 K (J/mol*K)
CO2	0.04401	58.9	50.586
H2O	0.01801528	48.1	39.786
O2	0.015999	36.8	28.486
N2	0.0280134	35.1	26.786
H2	0.0100784	32.7	24.386
CO	0.02801	35.5	27.186
HC	0.0130186	77.83	69.516
NOx	0.0460055	55.54	47.226
SO2	0.064066	57.34	49.026

Table 2. Properties of Exhaust Gases

Now that we have the weight for exhaust gases and also the molar-mass we can calculate the number of moles present.

Exhaust Gases	Number of Moles	Total Moles	Mole Percent	Mole Fraction (Xi)
CO2	0.00386276	0.01269888	30.4181073	0.30418107
H2O	0.00460720		36.2803703	0.3628037
O2	0.00068754		5.41420233	0.05414202
N2	0.00257020		20.2395713	0.20239571
H2	0.00034728		2.7347088	0.02734709
CO	0.00049982		3.9359499	0.0393595
HC	0.00009986		0.78634604	0.00786346
NOx	0.00002391		0.1882858	0.00188286
SO2	0.00000031		0.00245831	0.0000245831

Table 3. Molar Fraction Calculation Exhaust Gases

Exhaust Gases	A = Xi*Cpi	B= Xi*Mi	C= Xi*(Cvi)
CO2	17.9162652	0.01338701	15.38730376
H2O	17.4508581	0.00653601	14.43450811
O2	1.99242646	0.00086622	1.542289676
N2	7.10408951	0.00566979	5.421371559
H2	0.89424978	2.7561E-05	0.666886088
CO	1.39726221	0.00110246	1.070027339
HC	0.61201312	0.00010237	0.546636313
NOx	0.10457393	8.6622E-05	0.088919851
SO2	0.0014096	1.5749E-06	0.001205212
<b>Total:</b>	<b>47.473</b>	<b>0.028</b>	<b>39.159</b>

Table 5. Parameter calculation for Equation 1 &amp; 2

Finally, to calculate the speed of sound we will require the Equation 3.

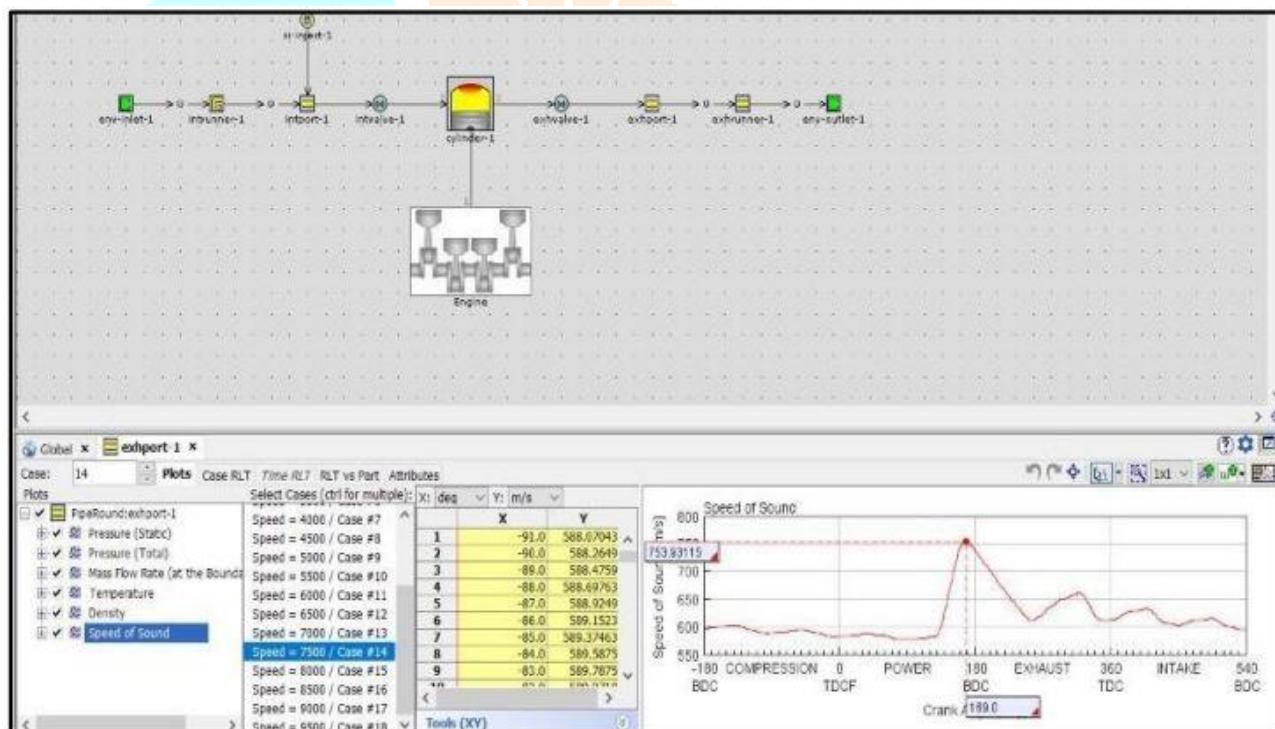
$$C_0^2 = R_0 T \frac{\sum_{i=1}^n x_i C_{pi}}{\sum_{i=1}^n x_i M_i \sum_{i=1}^n x_i (C_{pi} - R_0)}$$

For the above equation,

1.  $R_o * T = 13302.4$
2.  $A = X_i * C_{pi} = 47.473$
3.  $B = X_i * M_i = 0.028$
4.  $C = X_i * (C_{vi}) = 39.159$

Therefore, the theoretical value for  $C_0$  that is the speed of sound in the exhaust gas is 761.92 m/s.

#### IV. Simulation Results



The obtained value for speed of sound is 753.93115 m/s. The result is obtained when the engine speed is 7500 RPM and the crank angle is 169°.

Results	Speed of Sound (m/s)
Theoretical	761.92
Simulation	753.93115

## V. Conclusion

From the above results, we can see that this method to calculate the speed of sound in exhaust gases is accurate and precise. With the help of this method we can verify the speed of sound both theoretically and by simulations.

## VI. References

- [1] SPEED OF SOUND MEASUREMENTS IN GAS-MIXTURES AT VARYING COMPOSITION USING AN ULTRASONIC GAS FLOW METER WITH SILICON BASED TRANSDUCERS Torbjörn Löfqvist, Kęstutis Sokas, Jerker Delsing EISLAB, Department of Computer Science and Electrical Engineering Luleå University of Technology, Luleå, Sweden
- [2] <http://www.orientjchem.org/vol35no1/the-main-components-of-vehicle-exhaust-gases-and-their-effective-catalytic-neutralization/>

