Bipolar Pentapartitioned Neutrosophic Set and its Topological Space

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Abstract: In this paper, we have introduced the new concept of Bipolar Pentapartitioned Neutrosophic set and discussed some of its properties. Also, we have investigated the properties of bipolar pentapartitioned neutrosophic topological spaces.

Keywords: Bipolar quadrpartitioned neutrosophic set, bipolar pentapartitioned neutrosophic set.

I. INTRODUCTION

To cope with uncertainty primarily based real and scientific issues, Prof. Zadeh [20] introduced the fuzzy set as a constructive tool. Later on Prof. Atanassov [11] extended the idea of fuzzy set theory to the intuitionistic fuzzy set (IFS), during which every element has both a membership degree and a non-membership degree. It’s quite clear that IFS are more useful than fuzzy set theory to deal the varied sorts of uncertainty model. In 2005, Smarandache [19] introduced the thought of a neutrosophic set (NS) as a further generalization of IFS from philosophical purposes of read. Gradually neutrosophic sets become more powerful technique to represent incomplete, inconsistent and indeterminate data that exists in our real universe. In neutrosophic set, truth membership functions (TA), indeterminacy membership functions (IA), and falsity membership functions (FA) are represented independently. However just in case of NS, all components lie in [0, 1]. Thus it is terribly powerful to use NS sets in real world issues. To resolve this problem Wang et al. [7] introduced single valued NS sets in 2010. Recently bipolar fuzzy set and set theoretical operations supported on fuzzy bipolar sets are introduced by Deli et al. in their paper [5]. They have shown that a bipolar fuzzy set consists two independent components, positive membership degree $T_+ \rightarrow [0, 1]$ and a negative membership degree $T_- \rightarrow [-1, 0]$. In a while, many researchers have studied bipolar fuzzy sets and applied it to completely different socio-economic model.

As continuation of neutrosophic set, Deli et al. [6] introduced the thought of bipolar neutrosophic sets, where each element has both positive and negative neutrosophic degrees. Here, the positive membership degree $T_+^A$, $I_+^A$, $F_+^A$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ corresponding to a bipolar neutrosophic set $A$ and the negative membership degree $T_-^A$, $I_-^A$, $F_-^A$ denotes the truth membership, indeterminate membership and false membership of a component $x \in X$ to some anti-property similar to a bipolar neutrosophic set $A$. Rama Malik and Surpatri Pranamik [18] has developed Pentapartitioned neutrosophic set and its properties. It is five valued logic set consisting truth membership, a contradiction membership, an ignorance-membership, an unknown membership and a falsity membership for each $x \in X$. Now pentapartitioned single valued neutrosophic set becomes an important tool in solving various types of decision making problems, medical diagnosis problems, clustering issues etc. The concept of Bipolar Quadr partitioned single valued neutrosophic sets was developed by Kalyan Sinha, et.al [8]. In this paper, we develop Bipolar pentapartitioned neutrosophic set and studied some of its properties.

II. PRELIMINARIES

2.1 Definition

Suppose $X$ be a non-empty set. A bipolar quadrpartitioned neutrosophic set (BQNS) $A$, over $X$ characterizes each element $x \in X$ by a positive truth-membership function $T_+^A$, a positive contradiction membership function $C_+^A$, a positive ignorance-membership function $U_+^A$, a positive falsity membership function $F_+^A$, a negative truth membership function $T_-^A$, a negative contradiction membership function $C_-^A$, a negative ignorance-membership function $U_-^A$, a negative falsity membership function $F_-^A$, such that for each $x \in X, T_+^A, C_+^A, U_+^A, F_+^A \in [0, 1], T_-^A, C_-^A, U_-^A, F_-^A \in [0, 1],$

and $T_+^A + C_+^A + U_+^A + F_+^A \leq 4, \quad -4 \leq T_-^A + C_-^A + U_-^A + F_-^A \leq 0$
2.2 Definition.
A BQNS \( B \) over \( X \) is said to be an absolute BQNS, denoted by \( B \), if and only if its membership values are respectively defined as \( T_\chi(x) = 1 \), \( C_\chi(x) = 1 \), \( U_\chi(x) = 0 \), \( F_\chi(x) = 0 \), \( T_\chi(x) = 0 \), \( C_\chi(x) = 0 \), \( U_\chi(x) = -1 \), \( F_\chi(x) = -1 \), for all \( x \in [0,1] \).

2.3 Definition
A BQNS \( B \) over \( X \) is said to be an empty BQNS, denoted by \( \emptyset \), if and only if its membership values are respectively defined as \( T_\chi(x) = 0 \), \( C_\chi(x) = 0 \), \( U_\chi(x) = 1 \), \( F_\chi(x) = 1 \), \( T_\chi(x) = -1 \), \( C_\chi(x) = -1 \), \( U_\chi(x) = 0 \), \( F_\chi(x) = 0 \), for all \( x \in [0,1] \).

2.4 Definition
A BQN set \( A = \{T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x), T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x)\} \) is contained in a BQN set \( B = \{T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x), T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x)\} \) if and only if \( T_\chi(x) \leq T_\chi(x) \), \( C_\chi(x) \leq C_\chi(x) \), \( U_\chi(x) \geq U_\chi(x) \), \( F_\chi(x) \geq F_\chi(x) \), \( T_\chi(x) \geq T_\chi(x) \), \( C_\chi(x) \geq C_\chi(x) \), \( U_\chi(x) \leq U_\chi(x) \) and \( F_\chi(x) \leq F_\chi(x) \).

2.5 Definition
The complement of BQNS \( A = \{T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x), T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x)\} \) is denoted by \( A^C \) and is defined as \( A^C = \{F_\chi(x), T_\chi(x), C_\chi(x), U_\chi(x), T_\chi(x), C_\chi(x), U_\chi(x), T_\chi(x)\} \).

2.6 Definition
The union of any two BQNS \( A = \{T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x), T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x)\} \) and \( B = \{T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x), T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x)\} \) is denoted by \( A \cup B \) and is defined as follows \( A \cup B = \{max \{T_\chi(x), T_\chi(x)\}, min \{C_\chi(x), C_\chi(x)\}, min \{U_\chi(x), U_\chi(x)\}, max \{F_\chi(x), F_\chi(x)\}\} \). 

2.7 Definition
The intersection of any two BQNS \( A = \{T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x), T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x)\} \) and \( B = \{T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x), T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x)\} \) is denoted by \( A \cap B \) and is defined as follows \( A \cap B = \{min \{T_\chi(x), T_\chi(x)\}, max \{C_\chi(x), C_\chi(x)\}, max \{U_\chi(x), U_\chi(x)\}, max \{F_\chi(x), F_\chi(x)\}\} \).

III BIPOLAR PENTAPARTITIONED NEUTROSOPHIC SETS

3.1 Definition
Suppose \( X \) be a non-empty set. A bipolar pentapartitioned neutrosophic set (BPNS) \( A \), over \( X \) characterizes each element \( x \) in \( X \) by a positive truth-membership function \( T_\chi(x) \), a positive contradiction membership function \( C_\chi(x) \), a positive ignorance-membership function \( U_\chi(x) \), a positive falsity membership function \( F_\chi(x) \), a positive unknown membership function \( T_\chi(x) \), a negative contradiction membership function \( C_\chi(x) \), a negative ignorance-membership function \( U_\chi(x) \), a negative falsity membership function \( F_\chi(x) \), a negative unknown membership function \( G_\chi(x) \) such that for each \( x \in X \), \( T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x) \in [0,1] \), \( T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x) \in [0,1] \), and \( T_\chi(x) + C_\chi(x) + U_\chi(x) + F_\chi(x) + G_\chi(x) \leq 5, -5 \leq T_\chi(x) + C_\chi(x) + U_\chi(x) + F_\chi(x) + G_\chi(x) \leq 0 \). 

When \( X \) is discrete, \( A \) is represented as \( A = \sum_{i=1}^{n} < T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x), T_\chi(x), C_\chi(x), U_\chi(x), F_\chi(x) \leq x_i, x_i \in X > \). 

3.2 Example
Consider the case where five different persons \( x_1, x_2, x_3, x_4, x_5 \) were asked to give their opinion on the statement “is there any climate change in India” in the year 2020. Each of the five persons will give their opinion in terms of degree of agreement, agreement or disagreement both, neither agreement nor disagreement, disagreement in terms of degree of positive and negative arguments and unknown agreement respectively. The aggregate of their opinion is very well expressed by a BPNS \( A \) as follows:
\[
A = \begin{cases} 
< 0.8, 0.7, 0.6, 0.4, 0.2, -0.5, -0.7, -0.3, -0.1 > / x_1 + \\
< 0.5, 0.7, 0.9, 0.2, 0.7, -0.2, -0.5, -0.6, -0.7, -0.3 > / x_2 + \\
< 0.7, 0.7, 0.4, 0.5, 0.2, -0.5, -0.8, -0.5, -0.2, -0.4 > / x_3 + \\
< 0.9, 0.3, 0.6, 0.2, 0.7, -0.2, -0.5, -0.6, -0.7, -0.3 > / x_4 + \\
< 0.2, 0.6, 0.6, 0.4, 0.7, -0.5, -0.4, -0.7, -0.9, -0.1 > / x_5 + 
\end{cases}
\]

Here according to \( x_i \), the degree of agreement with the statement is 0.8 and the degree of negative agreement with the statement is 0.5, the degree of both agreement and disagreement is 0.7 and the degree of negative argument of “both agreement and disagreement” is 0.8. The degree of neither agreement nor disagreement is 0.4, while the degree of negative argument of it is 0.3.

Similarly the degree of disagreement with the statement is 0.2 and degree of negative disagreement is 0.1 and the degree of unknown agreement is 0.6 and the degree of negative unknown agreement is 0.7. This is how BPNS set has been made.

### 3.3 Definition
A BPNS \( B \) over \( X \) is said to be an absolute BPNS, denoted by \( 1_X \), if and only if its membership values are respectively defined as \( T_A^+(\chi) = 1, C_A^+(\chi) = 1, G_A^+(\chi) = 0, U_A^+(\chi) = 0, F_A^+(\chi) = 0 \), \( T_A^- (\chi) = 0, C_A^- (\chi) = 0, G_A^- (\chi) = -1, U_A^- (\chi) = -1, F_A^- (\chi) = -1 \), for all \( \chi \in [0,1] \).

### 3.4 Definition
A BPNS \( B \) over \( X \) is said to be an empty BPNS, denoted by \( 0_X \), if and only if its membership values are respectively defined as \( T_A^+(\chi) = 0, C_A^+(\chi) = 0, G_A^+(\chi) = 1, U_A^+(\chi) = 1, F_A^+(\chi) = 1 \), \( T_A^- (\chi) = 1, C_A^- (\chi) = 1, G_A^- (\chi) = 0, U_A^- (\chi) = 0, F_A^- (\chi) = 0 \), for all \( \chi \in [0,1] \).

Remark
A BPNS is a generalization of a bipolar neutrosophic set. If we take average the components \( C, U^+, G^+ \) and \( C, U^-, G^- \) together respectively, we can easily get a bipolar SVN set.

### 3.5 Definition
A BPNS set \( A = \{ T_A^+, C_A^+, U_A^+, F_A^+, T_A^-, C_A^-, U_A^-, F_A^- \} \) is contained in a BPNS set \( B = \{ T_B^+, C_B^+, U_B^+, F_B^+, T_B^-, C_B^-, U_B^-, F_B^- \} \) if and only if \( T_A^+(\chi) \leq T_B^+(\chi), C_A^+(\chi) \leq C_B^+(\chi), U_A^+(\chi) \geq U_B^+(\chi), G_A^+(\chi) \geq G_B^+(\chi), F_A^+(\chi) \geq F_B^+(\chi) \), \( T_A^- (\chi) \geq T_B^- (\chi), C_A^- (\chi) \geq C_B^- (\chi), U_A^- (\chi) \leq U_B^- (\chi), G_A^- (\chi) \leq G_B^- (\chi), F_A^- (\chi) \leq F_B^- (\chi) \).

### 3.6 Definition
The complement of BPNS \( A = \{ T_A^+, C_A^+, U_A^+, F_A^+, T_A^-, C_A^-, U_A^-, F_A^- \} \) is denoted by \( A^C \) and is defined as \( A^C = \{ F_A^+, U_A^+, (1 - G_A^+), C_A^+, T_A^+, F_A^-, U_A^-, (1 - G_A^-), C_A^-, T_A^- \} \).

### 3.7 Example
Let \( X = [a,b] \). Then the BPNS R of X is given by
\[
R = \begin{cases} 
< a, (0.5, 0.7, 0.6, 0.2, 0.4, -0.1, -0.8, -0.5, -0.7, -0.7) > \\
< b, (0.4, 0.6, 0.7, 0.8, 0.3, -0.2, -0.5, -0.7, -0.8, -0.3) > 
\end{cases}
\]
\[
R^C = \begin{cases} 
< a, (0.4, 0.2, 0.2, 0.7, 0.5, -0.7, -0.7, -0.5, -0.8, -0.1) > \\
< b, (0.3, 0.8, 0.4, 0.6, 0.2, -0.3, -0.8, -0.3, -0.5, -0.2) > 
\end{cases}
\]

### 3.8 Definition
The union of any two BPNS \( A = \{ T_A^+, C_A^+, U_A^+, F_A^+, T_A^-, C_A^-, U_A^-, F_A^- \} \) and \( B = \{ T_B^+, C_B^+, U_B^+, F_B^+, T_B^-, C_B^-, U_B^-, F_B^- \} \) is denoted by \( A \cup B \) and is defined as follows
\[
A \cup B = \{ \max \{ T_A^+, T_B^+ \}, \max \{ C_A^+, C_B^+ \}, \min \{ G_A^+, G_B^+ \}, \min \{ U_A^+, U_B^+ \}, \min \{ F_A^+, F_B^+ \} \}
\]
\[
\min \{ T_A^-, T_B^- \}, \min \{ C_A^-, C_B^- \}, \max \{ G_A^-, G_B^- \}, \max \{ U_A^-, U_B^- \}, \max \{ F_A^-, F_B^- \} \}
\]
3.9 Example
Let \( X = \{p, q\} \). Then the bipolar pentapartitioned neutrosophic subsets \( A \) and \( B \) of \( X \) can be given as follows.

\[
A = \begin{cases} 
\langle p, (0.5,0.7,0.6,0.2,0.4, -0.1, -0.8, -0.5, -0.7, -0.7) \rangle \\
\langle q, (0.4,0.6,0.7,0.8,0.3, -0.2, -0.5, -0.7, -0.8,-0.3) \rangle 
\end{cases}
\]

\[
B = \begin{cases} 
\langle p, (0.9,0.2,0.4,0.7,0.5, -0.7, -0.5, -0.8,-0.1) \rangle \\
\langle q, (0.5,0.8,0.3,0.6,0.4, -0.3, -0.8,-0.3, -0.5,-0.2) \rangle 
\end{cases}
\]

Then the union of two BPNS \( A \) and \( B \) is

\[
A \cup B = \begin{cases} 
\langle p, (0.9,0.7,0.4,0.2,0.4, -0.7, -0.8, -0.5, -0.7,-0.1) \rangle \\
\langle q, (0.5,0.8,0.3,0.6,0.3, -0.8,-0.3, -0.5,-0.5) \rangle 
\end{cases}
\]

3.10 Definition
The intersection of any two BPNS \( A = \{T^+_A, C^+_A, G^+_A, U^+_A, F^+_A, T^-_A, C^-_A, G^-_A, U^-_A, F^-_A\} \) and \( B = \{T^+_B, C^+_B, G^+_B, U^+_B, F^+_B, T^-_B, C^-_B, G^-_B, U^-_B, F^-_B\} \) is denoted by \( A \cap B \) and is defined as follows,

\[
A \cap B = \{\min \{T^+_A, T^+_B\}, \min \{C^+_A, C^+_B\}, \max \{G^+_A, G^+_B\}, \max \{U^+_A, U^+_B\}, \max \{F^+_A, F^+_B\}, \\
\max \{T^-_A, T^-_B\}, \min \{C^-_A, C^-_B\}, \min \{G^-_A, G^-_B\}, \min \{U^-_A, U^-_B\}, \min \{F^-_A, F^-_B\}\}
\]

3.11 Example:
Let \( X = \{p, q\} \). Then the bipolar pentapartitioned neutrosophic subsets \( A \) and \( B \) of \( X \) can be given as follows.

\[
A = \begin{cases} 
\langle p, (0.5,0.7,0.6,0.2,0.4, -0.1, -0.8, -0.5, -0.7, -0.7) \rangle \\
\langle q, (0.4,0.6,0.7,0.8,0.3, -0.2, -0.5, -0.7, -0.8,-0.3) \rangle 
\end{cases}
\]

\[
B = \begin{cases} 
\langle p, (0.9,0.2,0.4,0.7,0.5, -0.7, -0.5, -0.8,-0.1) \rangle \\
\langle q, (0.5,0.8,0.3,0.6,0.4, -0.3, -0.8,-0.3, -0.5,-0.2) \rangle 
\end{cases}
\]

Then the intersection of two BPNS \( A \) and \( B \) is

\[
A \cap B = \begin{cases} 
\langle p, (0.5,0.2,0.6,0.7,0.5, -0.1, -0.7, -0.5, -0.8,-0.7) \rangle \\
\langle q, (0.4,0.6,0.7,0.8,0.4, -0.2, -0.5, -0.7, -0.8,-0.3) \rangle 
\end{cases}
\]

3.12 Proposition
The set-theoretic axioms are satisfied by any BPNS as it can be easily verified. Consider BPNS sets \( A, B, C \) over the same universe \( X \). Then the following properties holds all for BPNS over \( X \).

(i) \( A \cup B = B \cup A \)
(ii) \( A \cap B = B \cap A \)
(iii) \( A \cup (B \cup C) = (A \cup B) \cup C \)
(iv) \( A \cap (B \cap C) = (A \cap B) \cap C \)
(v) \( A \cap (A \cup B) = A \)
(vi) \( A \cup (A \cap B) = A \)
(vii) \( (A')' = A \)
(viii) \( (A \cup B)' = A' \cap B' \)
(ix) \( (A \cap B)' = A' \cup B' \)
(x) \( A \cup A = A \cup A \)
(xi) \( A \cap A = A \cap A \)
(xii) \( A \cup \emptyset = A \)
(xiii) \( A \cap \emptyset = \emptyset \)
(xiv) \( A \cup \emptyset = A \)
(xv) \( A \cap \emptyset = \emptyset \)

3.13 Definition
A bipolar pentapartitioned neutrosophic topology (BPNT) on a non empty \( X \) is a of BPN sets satisfying the following axioms.

1. \( 0 \in \tau \)
2. \( A \cap B \in \tau \) for any \( a, b \in \tau \)
3. \( A \cup \emptyset \in \tau \) for any arbitrary family \( \{ A_i \in \tau \} \)

The pair \( (X, \tau) \) is called Bipolar Pentapartitioned neutrosophic topological spaces (BPNTS).
Any BPN set in τ is called as BPN open set in X. The complement of BPN open set is BPN closed set.

### 3.14 Example
Let \( X = \{ p, q \} \). Then the bipolar pentapartitioned neutrosophic subsets \( A \) and \( B \) of \( X \) can be given as follows.

\[
A = \{< p, (0.5,0.7,0,6,0,2,0,4,\cdot,0.1,\cdot,0.8,\cdot,0.5,\cdot,0.7,\cdot,0.7)> \}
\[
B = \{< q, (0.4,0.6,0,7,0,8,0,3,\cdot,0.2,\cdot,0.5,\cdot,0.7,\cdot,0.8,\cdot,0.3)> \}
\]

Then \( \tau = \{ 0_X, 1_X, A, B, C \} \) is a bipolar pentapartitioned neutrosophic topology on \( X \).

### 3.15 Definition
Let \( (X, \tau) \) be a BPN topological space and \( A = \{ T_A^+, C_A^+, U_A^+, F_A^+, T_A^-, C_A^-, U_A^-, F_A^- \} \) be a BPN set in \( X \). Then the closure and interior of \( A \) is defined as

\[
\text{Int}(A) = \cup \{ F: F \text{ is a BPN open set in } X \text{ and } F \subseteq A \}
\]

\[
\text{Cl}(A) = \cap \{ F: F \text{ is a BPN closed in } X \text{ and } F \subseteq A \}
\]

Here \( \text{Cl}(A) \) is BPN closed set and \( \text{Int}(A) \) is a BPN open set in \( X \).

(a) \( A \) is BPN open set in \( X \) iff \( \text{Int}(A) = A \)

(b) \( A \) is BPN closed set in \( X \) iff \( \text{Cl}(A) = A \)

### 3.16 Example
Let \( X = \{ p, q \} \) and \( \tau = \{ 0_X, 1_X, A, B, C \} \) where

\[
A = \{< p, (0.5,0.7,0,6,0,2,0,4,\cdot,0.1,\cdot,0.8,\cdot,0.5,\cdot,0.7,\cdot,0.7)> \}
\]

Consider the BPN set \( D \) of \( X \) as

\[
D = \{< q, (0.7,0,8,0,3,0,1,0,2,\cdot,0.3,\cdot,0.9,\cdot,0.2,\cdot,0.7,\cdot,0.7)> \}
\]

Then \( \text{Int}(D) = A \) and \( \text{Cl}(D) = 1_X \).

### 3.17 Theorem
Let \( (X, \tau) \) be a BPN topological space and \( S, T \) be BPN set in \( X \). Then

1) \( \text{Int}(S) \subseteq S \) and \( S \subseteq \text{Cl}(S) \)
2) \( S \subseteq T \) \( \text{Int}(S) \subseteq \text{Int}(T) \)
3) \( S \subseteq T \) \( \text{Cl}(S) \subseteq \text{Cl}(T) \)
4) \( \text{Int}(\text{Int}(S)) = \text{Int}(S) \)
5) \( \text{Cl}(\text{Cl}(S)) = \text{Cl}(S) \)
6) \( \text{Int}(S \cap T) = \text{Int}(S) \cap \text{Int}(T) \)
7) \( \text{Cl}(S \cup T) = \text{Cl}(S) \cup \text{Cl}(T) \)
8) \( \text{Int}(1_X) = 1_X \)
9) \( \text{Cl}(0_X) = 0_X \)

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REFERENCES