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PHYSICAL MODEL AS A TOOL FOR STUDYING RIVER PROBLEMS- A REVIEW

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Abstract:

Before construction of hydraulic structures like barrages, dams, aqueducts, sluices, spars etc., prediction of the performance of such structures is required to be studied. Some different problems such as determination of co-efficient of discharge of an overflow structure like spillway or weir, determination of distribution of pressure on the surface of the overflow section, to design the energy dissipation device below spillways falls, weirs etc., are required to be solved before actual implementation of the structures. In most of the above cases analytical methods fail to solve the problem. Nowadays a study of hydraulic models has been recognised as a very useful tool by the engineers for solution of their difficult problems when analytical methods fail. In this paper, the theory of hydraulic study has been reviewed in detail. To get a comprehensive idea of the usefulness of the tool, a thorough discussion along with limitations of this technique of model study is done here. It is found that the technique of hydraulic experimentation is very useful in solving different flow problems.

Key Words. Hydraulic Model, Prototype, Undistorted models, Distorted models, Kinematic Similarity, Dynamic Similarity.

Introduction:

A number of problems encountered in the design and construction of hydraulic structures, for example Dams, Barrages, spillways, spars etc., are not amenable to mathematical, analytical or even empirical solution. In certain simple cases where conditions are idealised or where variables are not too many, problems of flow in hydraulics can be solved analytically or empirically, when analytical methods have been found inadequate. In nature a large number variables makes the hydraulic problem so complicated that our advanced knowledge of fluid mechanics today fails to tackle such problems analytically or empirically. In such cases we try to reproduce the conditions of nature as far as possible in the laboratory to arrive at the answer as nearly as possible. Laboratory experiments has been a very powerful tool in the advancement of our knowledge in all branches of science. Model of the structures is constructed in the laboratory to predict the performance of the hydraulic structures such as barrages, spars, spillways, aqueduct etc. Experiments are performed with the help of the models to obtain the desired information. The model is the small scale replica of the actual structure. The model is also called physical model or hydraulic scale model. The actual structure is known as prototype. The study of models of real structure is termed as Model analysis. Hydraulic model analysis is really an experimental technique of finding effects of hydraulic structure. It also helps to find solutions of complex flow problems. This technique has come to the help of investigators engaged in the field of hydraulics or hydraulic engineering.

The special feature of this experimental science is that we try to simulate the conditions obtaining in nature in a small scale model in the laboratory i.e we try to reproduced the river in small scale and predict the flow conditions in nature by closely observing the flow conditions in the laboratory. Now the question arises how and to what extent it is possible to predict the flow conditions in nature from just an experimental study of a small model in the laboratory. Here the author will try to analyse the basic principles of fluid mechanics on which this experimental science of hydraulic models has been built up.

Variables effecting the fluid flow:

There are three types of variables that influence fluid motion.

1) Geometry of the ground where motion takes place, e.g. shape of the channel, water body etc.
2) Physical properties of the fluid such as viscosity, density, surface tension.
3) External forces that cause the motion e.g. inertia force, gravity force, viscus force, pressure force, surface tension force etc.

The geometry of the system is characterised by linear variables length, breath, width, etc. and is denoted by a, b, c, d, etc.

The physical properties of fluid is characterised by surface tension σ, density ρ, viscosity μ, elasticity e. The external forces are characterised by gravity g, pressure p, etc.

The general equation of flow may be written as

$$v = F(a, b, c, d, g, \sigma, \rho, \mu, e, p,) , \quad (1)$$

where v is the velocity of fluid.

Classification of Models.

The hydraulic(physical) models are classified as flows:

1. Undistorted models,
2. Distorted models.

Undistorted Model.

The model which is geometrically similar to its prototypes is called undistorted model. In this case the scale ratios for linear dimensions of the prototype and its model are same.

Here, $$\frac{l_p}{l_m} = \frac{b_p}{b_m} = \frac{h_p}{h_m} = L$$

Where, L is the scale ratio of linear dimension. l_p , l_m are length of the prototype and model.

b_p , b_m are width of prototype and model. h_p , h_m are height of prototype and model.

The behaviour of the prototype can be easily predicted from the results of undistorted model.

Distorted Model:

The model which is not geometrically similar to its prototype is known as distorted model. For a distorted model different scale ratios for the linear dimensions are assumed.

Here, $\frac{b_p}{b_m} = L_H = \frac{\text{Linear horizontal dimension of prototype}}{\text{Linear horizontal dimension of model}}$

$\frac{h_p}{h_m} = L_V = \frac{\text{Linear vertical dimension of prototype}}{\text{Linear vertical dimension of model}}$

For example, in case of problems connected with rivers (problems of erosion, flood control, river training etc.) it is generally necessary to construct models comprising of long length of rivers. In order to construct the model within practicable size, the horizontal scale chosen is comparatively small. Depth of water in a river is generally small compared to its width. So if the vertical and horizontal scale ratios are same, then the depth of water in the model of the river becomes very small which may not be measured accurately. Also capillary and viscous forces effect the flow of the model, while in the prototype their effect is negligible. Therefore, to reduce the effect of capillary and viscous forces in the model, the vertical scale of the model is required to be increased and hence both the depth of water in the model as well as velocity will increase. The vertical scale of such a model is different from its horizontal scale which increases the slope of the river bed in the model. As a result proper bed movement as in the prototype can be achieved in the model. Though there is no geometric similarity in such a model, the results obtained from it are found to be quite reliable for predicting the behaviour of the prototype. If the distortion of the model is not chosen properly serious distortion of the velocity and pressure distribution may occur in the model. To reproduce the conditions of prototype a judicious choice of distortion of scale of the model is required.

Besides this type of geometric similarity two other similarities i.e. Kinematic similarity and Dynamic similarity should be taken into account during construction of model.

Kinematic Similarity. The model is kinematically similar to its prototype if the ratio of the velocity /acceleration at any point in the model and at the corresponding point in the prototype is same.

Here, $\frac{V_{p_1}}{V_{m_1}} = \frac{V_{p_2}}{V_{m_2}} = \text{v}_r$ and $\frac{a_{p_1}}{a_{m_1}} = \frac{a_{p_2}}{a_{m_2}} = \text{a}_r$

Where, V_{p_1} and V_{p_2} velocity of fluid in prototype at points 1 and 2 respectively.

V_{m_1} and V_{m_2} velocity of fluid in model at points 1 and 2 respectively.

a_{p_1} and a_{p_2} acceleration of fluid in prototype at points 1 and 2 respectively.

a_{m_1} and a_{m_2} acceleration of fluid in model at points 1 and 2 respectively.

v_r and a_r are velocity ratio and acceleration ratio respectively.

Further the direction of velocity in the model should be same as that in the prototype.

Dynamic Similarity. The model is dynamically similar to its prototype if the ratio of the forces at any point in the model and at the corresponding point in the prototype is same.

Here, $\frac{(F_i)_p}{(F_i)_m} = \frac{(F_g)_p}{(F_g)_m} = \ldots = F_r$ (2)

Where, $(F_i)_p$, $(F_g)_p$ are Inertia force and Gravity force at a point in prototype.

$(F_i)_m$, $(F_g)_p$ are Inertia force and Gravity force at the corresponding point in model.

F_r is the force ratio.

Dimensionless Numbers:

We will now discuss in details the parameters of equation (2) and the conditions under which the value of each parameter may be same in the model and in the prototype.

1) If the model is geometrically similar to the prototype, then the dimensions, a/b, a/c, and a /d will be of same numerical value in the model and in the prototype.
2) The other four parameters are as follows.

i) Froude Number. $\sqrt{\frac{v^2}{ag}} = F$, this represents the effect of gravity force.

v = Velocity of fluid,
a = Linear dimension,
g = Acceleration due to gravity.

To achieve dynamic similarity between the model and prototype , the Froude number for both of them should be equal.

Hence the condition is $F_p = F_m$, where F_m and F_p are Froude numbers for model and prototype respectively.

i.e. $$\frac{v_p^2}{a_p\, g_p} = \frac{v_m^2}{a_m\, g_m}$$

Since the value of g is same in the prototype and model , $g_p = g_m$

Hence, $$\frac{v_p}{v_m} = \sqrt{\frac{a_p}{a_m}} = \sqrt{L} \qquad (3)$$

ii) Weber number, $\frac{v^2 a}{\sigma/\rho} = W$, this represents the effect of surface tension.

To achieve dynamic similarity between the model and prototype, the Weber number for both of them should be equal.

Hence the condition is $W_p = W_m$

Where W_m , W_p are Weber numbers for model and prototype respectively.

Hence, $$\frac{v_p^2\, a_p}{\sigma_p / \rho_p} = \frac{v_m^2\, a_m}{\sigma_m / \rho_m}$$

i.e. $$\frac{v_p}{v_m} = \sqrt{\frac{\sigma_p}{\sigma_m}} \cdot \sqrt{\frac{\rho_m}{\rho_p}} \cdot \sqrt{\frac{a_m}{a_p}} \qquad (4)$$

If same fluid is used in the model and the prototype

$$\frac{v_p}{v_m} = \sqrt{\frac{a_m}{a_p}} = \sqrt{\frac{1}{L}} \qquad (5)$$

iii) Reynolds Number, $\frac{va}{\mu/\rho} = R$, this represents the effect of viscous force.

To achieve dynamic similarity between the prototype and model , the Reynolds number for both of them should be equal.

Hence the condition is $R_p = R_m$, where R_P , R_m are Reynolds numbers for prototype and model respectively.

Hence, $\frac{v_p a_p}{\mu_p/\rho_p} = \frac{v_m a_m}{\mu_m/\rho_m}$

Hence, $\frac{v_p}{v_m} = \frac{\mu_p}{\mu_m} \cdot \frac{\rho_m}{\rho_p} \cdot \frac{a_m}{a_p}$ (6)

If the same fluid is used in the model as in the prototype, we have

$$\frac{v_p}{v_m} = \frac{a_m}{a_p} = \frac{1}{L} \quad (7)$$

iv) Cauchy Number , $\frac{v^2}{e/\rho} = C$, this represents the effect of elastic forces.

If Mach number is represented by M, then $C = M^2$

To achieve dynamic similarity between the prototype and model, the Cauchy number for both of them should be equal.

Hence the condition is $C_p = C_m$

where C_m , C_p are Cauchy numbers for model and prototype respectively.

Hence, $\frac{v_p^2}{e_p/\rho_p} = \frac{v_m^2}{e_m/\rho_m}$

Hence, $\frac{v_p}{v_m} = \sqrt{\left(\frac{e_p}{e_m} \cdot \frac{\rho_m}{\rho_p}\right)}$ (8)

If the same fluid is used in the model as in the prototype, we have

, $\frac{v_p}{v_m} = \sqrt{\left(\frac{e_p}{e_m}\right)}$ (9)

v) Euler's Number (E) , it is the square root of the ratio of the inertia force of a flowing fluid to the pressure force.

$E = \sqrt{\frac{F_i}{F_p}}$, where F_p = pressure intensity × Area = pA and F_i = inertia force = ρAV^2

$$E = \frac{V}{\sqrt{p/\rho}} \quad (10)$$

To achieve dynamic similarity between the prototype and model, the Euler's number for both of them should be equal.

Hence the condition is $E_p = E_m$

where E_m , E_p are Euler's numbers for model and prototype respectively.

Hence, $\frac{V_p}{\sqrt{p_p/\rho_p}} = \frac{V_m}{\sqrt{p_m/\rho_m}}$

Hence, $\frac{V_p}{V_m} = \sqrt{\left(\frac{p_p}{p_m} . \frac{\rho_m}{\rho_p}\right)}$ (11)

If the same fluid is used in the model as in the prototype, we have

, $\frac{V_p}{V_m} = \sqrt{\left(\frac{p_p}{p_m}\right)}$ (12)

Now the question arises whether it possible to design a hydraulic model so as to satisfy all the required conditions of perfect similitude with the prototype.

The relationship in equations (3) and (5) can not be satisfied unless L = 1, i.e. the model is of the same size as that of the prototype.

Hence it is clear that if same fluid is used in the model as in the prototype both the necessary conditions of equality of F and W in the model and the prototype can not be satisfied simultaneously.

However, if we consider the model fluid different from that of prototype fluid such that (from equation (3) and (4))

$\sqrt{\frac{\sigma_p}{\sigma_m}} . \sqrt{\frac{\rho_m}{\rho_p}} = L$ (13)

both the necessary conditions of equality of F and W can be satisfied.

From equations (3), (5) and (7) it is again found that the necessary conditions for F , W and R are different from each other.

Now if the model fluid is so chosen such that (from equation (3) and (6))

$\frac{\mu_p}{\mu_m} . \frac{\rho_m}{\rho_p} = L^{3/2}$ (14)

both the necessary conditions of equality of F and R can be satisfied but the conditions of equality of W can not be satisfied at the same time unless the chosen fluid satisfy the following condition. (From equations (13) and (14))

$\left(\frac{\mu_p}{\mu_m} . \frac{\rho_m}{\rho_p}\right)^{2/3} = L = \left(\frac{\sigma_p}{\sigma_m} . \frac{\rho_m}{\rho_p}\right)^{1/2}$ (15)

We shall now consider the equality of F and C. If the fluid is so chosen that the following condition hold good then both the necessary conditions of equality of F and C in the model and the prototype can be satisfied. (From equation (3) and (8))

$\left(\frac{e_p}{e_m} . \frac{\rho_m}{\rho_p}\right) = L$ (16)

We shall again consider the equality of F and E. If the fluid is so chosen that the following condition hold good then both the necessary conditions of equality of F and E in the model and the prototype can be satisfied. (From equation (3) and (11))

$$\left(\frac{p_p}{p_m}.\frac{\rho_m}{\rho_p}\right) = L \qquad (17)$$

To satisfy all the five necessary conditions of equality of F, W, R , C and E in the model and prototype the fluid is to be so chosen that the physical properties satisfies the following condition.(From equation (15), (16) and (17)).

$$\left(\frac{\mu_p}{\mu_m}.\frac{\rho_m}{\rho_p}\right)^{2/3} = \left(\frac{\sigma_p}{\sigma_m}.\frac{\rho_m}{\rho_p}\right)^{1/2} = \frac{e_p}{e_m}.\frac{\rho_m}{\rho_p} = \frac{p_p}{p_m}.\frac{\rho_m}{\rho_p} = L \qquad (18)$$

It is not possible to find a fluid for the model which satisfy the above condition. Hence it is clear that it is not possible to construct a model which satisfy all the conditions of equality of F, W, R and C for the prototype and model.

In practical application some simplification is required to design a model which is similitude with the prototype. In practical purpose to design a model we have generally two degrees of freedom, one the selection of scale and other the selection of model fluid. With these two degrees of freedom we can satisfy only two independent criteria. In some problem where bed movement is required we have another degrees of freedom i.e. selection of bed materials. Hydraulic problems may be classified in different groups according as the predominant force acting on it [1].

1) Froude Model law. Froude model law is applicable when the gravity force is predominant force and hence controls the flow in addition to the force of inertia, the effect of viscous force being small while the effect of surface tension and elastic force being negligible, Here models are designed based on equality of Froude number and geometrical similarity. Froude model law is applicable in the following fluid problem.
   i) Open channel flow (problem regarding scour, sedimentation etc.),
   ii) Free surface flow over structures such as , spillways, sluice, etc.
   iii) Flow of jet from a nozzle or orifice,
   iv) Where waves may formed on surface.

2) Weber Model Law. When surface tension is predominant force and hence controls the flow in addition to the force of inertia, Weber model law is applicable. Here models are designed based on equality of Weber number and geometrical similarity. Weber model law is applicable in the following fluid problems.
   i) Capillary rise in narrow passages,
   ii) Capillary movement of water in soil

3) Reynold's Model Law. When viscous force is predominant force and hence controls the flow in addition to the force of inertia, Reynold's model law is applicable. Here models are designed based on equality of Renold's number and geometrical similarity. Reynold's model law is applicable in the following fluid problems.
   i) Pipe flow
   ii) Resistance experienced by sub – marines, airplanes etc.

4) Cauchy Model Law. When elastic force is predominant force and hence controls the flow in addition to the force of inertia, Cauchy model law is applicable. Here models are designed based on

equality of Cauchy number and geometrical similarity. Cauchy model law is applicable in the following fluid problem.

i) Flow of aeroplane and projectile through air at supersonic speed, i,e. at a velocity more than the velocity of sound,

ii) Aerodynamic testing

iii) Under water testing of torpedoes.

5) Euler's Model Law. When pressure force is predominant force and hence controls the flow in addition to the force of inertia, Euler's model law is applicable. Here models are designed based on equality of Euler's number and geometrical similarity. Euler's model law is applicable in the following fluid problems.

i) Flow through a closed pipe

ii) Where phenomenon of cavitation takes place

Depending on which force is predominant, the similitude criteria of each problem may be reduced to the equality of only of the number F,W,R and C between the prototype and model in addition to the geometrical similarity. This simplification of the hydraulic model studies has been found justified by actual experience.

Transference Ratio: A value of a parameter of the prototype can be found from corresponding measured value of the parameter in the model with the help of some established transference ratios. The purpose of the model is thus achieved.

Let us consider a problem in open channel where the gravity force is predominant. Equation (3) represents the similitude condition of equality of Froude number F between the model and prototype.

Here, $\frac{v_p}{v_m} = \sqrt{\frac{a_p}{a_m}} = \sqrt{L}$

a) Scale ratio for time: time = $\frac{\text{Length}}{\text{Velocity}}$

i.e. $T=\frac{l}{v}$, where l = linear dimension of the model, v = velocity, T=time

Hence, $\frac{T_p}{T_m} = \frac{l_p}{l_m} \cdot \frac{v_m}{v_p} = L \times \frac{1}{\sqrt{L}} = \sqrt{L}$ (19)

b) Scale ratio for discharge:

Discharge = Q = Area × Velocity = $l^2 \times v$

Hence, $\frac{Q_p}{Q_m} = \frac{l_p^2 \times v_p}{l_m^2 \times v_m} = L^2 \times \sqrt{L} = L^{\frac{5}{2}}$ (20)

c) Scale ratio for acceleration:

f = acceleration = $\frac{v}{T}$

Hence, $\frac{f_p}{f_m} = \frac{v_p}{v_m} \times \frac{T_m}{T_p} = \sqrt{L} \times \frac{1}{\sqrt{L}} = 1$ (21)

Let us now consider the scale ratio of model to the prototype is 1 in 100. i.e $\frac{l_p}{l_m} = 100$

Hence, $\frac{v_p}{v_m} = \sqrt{100} = 10$,

$\frac{T_p}{T_m} = \sqrt{100} = 10,$

$\frac{Q_p}{Q_m} = (10^2)^{5/2} = 100{,}000$

It is clear from the above that a velocity of 1 ft/sec in the model corresponds to a velocity of 10 ft/sec in the prototype, a discharge of 1 cusec in the model corresponds to a discharge of 100,000 cusec in the prototype, a time of 1 sec in the model corresponds to 10 sec in the prototype and the acceleration is same for both in the prototype and the model.

Similarly for each similitude criteria (W, R and C) different set of transference ratios can be obtained to infer the parameter of prototype from the model.

Transference ratios for Distorted Models :

As discussed above distortion in the linear scale is required in order to reproduce the flow conditions in the model similar to those of the prototype.

Here, $\frac{l_p}{l_m} = \frac{b_p}{b_m} = L_H = \frac{\text{Linear horizontal dimension of prototype}}{\text{Linear horizontal dimension of model}}$

$\frac{h_p}{h_m} = L_V = \frac{\text{Linear vertical dimension of prototype}}{\text{Linear vertical dimension of model}}$

a) Scale ratio for velocity:

, $\frac{v_p}{v_m} = \sqrt{\frac{h_p}{h_m}} = \sqrt{L_v}$ (22)

b) Scale ratio for area of flow:

Let A_p =Area of flow in prototype= $b_p \times h_p$

A_m = Area of flow in model = $b_m \times h_m$

So, $\frac{A_p}{A_m} = \frac{bp \times hp}{bm \times hm} = \frac{b_p}{b_m} \times \frac{h_p}{h_m} = L_H \times L_V$ (23)

c) Scale ratio for discharge:

Q_p =Discharge through prototype = $A_p \times V_p$

Q_m = Discharge through model = $A_m \times V_m$

$\frac{Q_p}{Q_m} = \frac{Ap \times Vp}{Am \times Vm} = L_H \times L_V \times \sqrt{L_v} = L_H \times L_v{}^{3/2}$ (24)

Let us now consider the horizontal scale ratio of model to the prototype is 1 in 100.

i.e $\frac{l_p}{l_m} = 100 = L_H$

and the vertical scale ratio of model to the prototype is 1 in 50. i.e. $\frac{h_p}{h_m} = 50 = L_V$

This means that 0.5 ft. water depth in the model corresponds to a depth of 25 ft.(0.5×50) in the prototype and a discharge of 1 cusec in the model corresponds to a discharge of $100\times(50)^{3/2}$ cusec i.e. 35355 cusec in the prototype.

The linear scale ratio, L of the model should not be too large i.e. scale of the model should not be too small. It is observed by a number of researchers in the field of model studies, that there is a limit beyond which the scale of the model can not be further reduced, because when the scale of the model is too small, viscous force and surface tension effect the flow of the model while in the prototype their effect is negligible. They have also found that if the value of the Renolds number R in the model is kept above a certain limiting value by suitable selecting the scale, there is a reasonably amount of turbulence in the model and the flow conditions in the model do not differ appreciably from those of the prototype.

In open channel flow the first test of reliability is to examine whether the water levels and velocities, as actually observed in the prototype, are reproduced in the model when equivalent discharges are run in the model. If this condition is not satisfied adjustments are to be made to satisfy the condition. Adjustment are made on the factors like scale, distortion, friction i.e. bed materials. Calibration of the model is done using stage- discharge relationship and bed movements i.e. the model is said to be reliable and we may accept the model as reliable tool to study the river problem if stage –discharge relationship in the model represents the stage- discharge relationship of the river (prototype). If bed of the model is constructed with movable bed materials like fine sand, coal dust or any other suitable material, the movement of the bed material in the model is also to be synchronised with the movement of bed material in the actual river.

For most open surface flows gravity force is predominant, hence most often physical modelling is performed using Froude similitude. So far we have discussed physical modelling without considering sediment motion. Flow characteristic of open channel are directly dependent on the boundary shear force. The boundary shear stress τ_0 is defined as

$$\tau_0 = \frac{f}{8}\rho V^2 \qquad (25)$$

where f is coefficient of friction and V is the velocity of flow.

Bed – load motion:

Shields expressed his results in a similarity law. The critical Shields parameter i.e. the dimensionless critical shear stress is denoted as

$$\tau_c^* = \frac{\tau_c}{(\rho_s - \rho)g d_s} = F(\rho\frac{d_s V^*}{\mu}) \qquad (26)$$

Where, τ_c is critical shear stress, ρ_s is density of sediment, ρ is density of fluid, g is acceleration due to gravity, d_s is diameter of sediment, μ is dynamic viscosity, $V^* = \sqrt{\frac{\tau_0}{\rho}}$ is the shear velocity. From equation 21 it clear that Critical Shield parameter is a function of Reynolds Number, $R = \frac{va}{\mu/\rho}$

To achieve similarity of occurrence of bed load motion between the prototype and model , the following equations must be satisfied[2]. Hence for perfect model

$$(\tau_c^*)_p = (\tau_c^*)_m \quad (27)$$

$$(\rho \frac{d_s V^*}{\mu})_p = (\rho \frac{d_s V^*}{\mu})_m \quad (28)$$

From equation (22) we have,

$$(\frac{\tau_c}{(\rho_s - \rho) g d_s})_p = (\frac{\tau_c}{(\rho_s - \rho) g d_s})_m \quad (29)$$

If same liquid is used in the model as in prototype then from equation (25) and (29) we have

$$\frac{f_p V_p^2}{(SG-1)_p (d_s)_p} = \frac{f_m V_m^2}{(SG-1)_m (d_s)_m}$$, where $SG = \frac{\rho_s}{\rho}$ relative density of sediment particle.

Hence, $$\frac{(SG-1)_p (d_s)_p}{(SG-1)_m (d_s)_m} = \frac{V_p^2}{V_m^2} \frac{f_p}{f_m} \quad (30)$$

For Froudian undistorted model we have {Eq. (3)} , $\frac{v_p}{v_m} = \sqrt{\frac{a_p}{a_m}} = \sqrt{L}$

It is to be noted that flow resistance in the mode will be similar to the flow resistance of the prototype for Froude similitude.

$f_r = \frac{f_p}{f_m} = 1$ for undistorted model

and $f_r = \frac{L_V}{L_H}$ for distorted model with flat slope and wide channel

We have for undistorted model, $\frac{(SG-1)_p}{(SG-1)_m} \frac{(d_s)_p}{(d_s)_m} = L$

i.e. $$L_{SG} \, L_s = L \quad (31)$$

where, $L_{SG} = \frac{(SG-1)_p}{(SG-1)_m}$ is scale ratio of relative density and $L_s = \frac{(d_s)_p}{(d_s)_m}$ is scale ratio of sediment size(sediment diameter).

and for distorted model, $\frac{(SG-1)_p}{(SG-1)_m} \frac{(d_s)_p}{(d_s)_m} = L_V \frac{L_V}{L_H} = \frac{L_V^2}{L_H}$

Hence, $$L_{SG} \, L_s = \frac{L_V^2}{L_H} \quad (32)$$

Again from equation (23) for similarity of Reynolds number we have,

$$\frac{(d_s)_p V_p^*}{\mu_p} = \frac{(d_s)_m V_m^*}{\mu_m}$$

So, $\frac{(d_s)_p}{(d_s)_m} = \frac{\mu_p}{\mu_m} \frac{V_m^*}{V_p^*} = \frac{V_m^*}{V_p^*}$ (same fluid (water) is used in model as in the prototype)

$$\frac{(d_s)_p}{(d_s)_m} = \frac{1}{\sqrt{L}} = \sqrt{\frac{a_m}{a_p}} \quad (33)$$

From equation (33) it is clear that for $\sqrt{\frac{a_m}{a_p}} < 1$, $\frac{(d_s)_p}{(d_s)_m}$ <1 which implies that model grain size must be larger than prototype grain size. This is unacceptable and hence equations (28) and (33) should not be used in practice.

The equations of fluid motion are integrated experimentally in the hydraulic scale model. A model is set up by suitably scaling down the horizontal and vertical length from field data received on the basis of survey. The bed movement in the model is ensured by choosing appropriate scale distortion. Once the model is proved, the model becomes a powerful instrument and different river problems can be studied within that particular reach of the river. The results obtained from the model should not be mechanically scaled up to use in the prototype. To properly interpret the results obtained from a model, not only is it imperative to have substantial experience, but it is also essential to have an in depth knowledge of the behaviour of the prototype. Majumder [4] cited an example to explain this point. According to Majumder, two leading research institutes of India were simultaneously carrying out a study on a very important project. The project was of great significance and therefore it necessitated cross checking of the results obtained by one research institute with that of the other. The crucial point in question was whether Cellular Coffer Dam could be used in connection with the construction of the main structure of the project. Experiment was carried out, on the basis of the design of the Cellular Coffer Dam received from the project authorities, by both the above mentioned institutes. Both institutes obtained almost identical results from the model study, and on the basis of the model study, one institute gave clearance to the use of the Cellular Coffer Dam. However, the institute which had a more intimate knowledge of the behaviour of the prototype, thought otherwise. The point was debated at the High Power Technical Committee set up in connection with the project and the decision was taken that the same pilot experiment be carried out in the prototype itself. In the summer of 1966, a test cell was erected in the river bed for the purpose. However, it was washed away during the first flood and therefore, the idea of a Cellular Coffer Dam had to be dropped. Majumder has pointed out that this does not by any means undermine the level of expertise of the other institute, but that the institute which had not recommended the use of the Cellular Coffer Dam, had a greater acquaintance with the river and thus had the opportunity to study its behaviour more closely to predict the result beforehand.

Uncertainty and Errors in Model.

During construction and experimentation of physical model, certain uncertainties and errors are inevitable. If we are able to understand the uncertainties and errors very clearly, the model can be used in a very efficient way and the results obtained from the model can be interpreted most accurately. Some sources of uncertainties and errors are given below:

i) Huge number of data such as topography, cross section, water level, velocity, velocity direction etc. is required to construct the model. Uncertainty may occur due to measurement and observational error.
ii) The phenomenon of the problem may not have been thoroughly comprehended.
iii) As discussed above, all the similitude criteria cannot be satisfied and hence models are simplified according to predominant factors thereby creating uncertainty.
iv) Due to rounding errors, uncertainty arises.
v) Accurate experimental confirmation of the model may not be properly achieved .

Hence, sufficient experience and in depth knowledge of the prototype along with thorough understanding of the problem to be solved is mandatory in order to achieve appropriate results from the model study.

Conclusion:

It is not possible to construct a model which is perfectly similar to the prototype, but the similarity that can actually be achieved is justified for all practical purposes. It is found by researchers that use of this tool for solution of different hydraulic problems which are not amenable to analytical solution, is justified. Performance of a number of hydraulic structures (barrages, dams, spillway, etc.) are studied world wide and constructed accordingly.

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