Two-Godown Inventory Model for Deteriorating Items using Multi-Objective Genetic Algorithm

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Abstract
In this chapter a deterministic inventory model has been developed for deteriorating items having a ramp type demand with the effects of inflation with two-storage facilities using Genetic Algorithm. The owned godown (OG) has a fixed capacity of W units; the rented godown (RG) has unlimited capacity using Genetic Algorithm. Here, we assumed that the inventory holding cost in RG is higher than those in OWG. Shortages in inventory are allowed and partially backlogged and it is assumed that the inventory deteriorates over time at a variable deterioration rate using Genetic Algorithm. The effect of inflation has also been considered for various costs associated with the inventory system. Numerical example is also used to study the behaviour of the model using Genetic Algorithm. Cost minimization technique is used to get the expressions for total cost and other parameters using Genetic Algorithm.

Keywords:- Inventory model, owned godown, rented godown, deteriorating items and Genetic Algorithm

1. Introduction
Many researchers extended the EOQ model to time-varying demand patterns. Some researchers discussed of inventory models with linear trend in demand. The main limitations in linear-time varying demand rate is that it implies a uniform change in the demand rate per unit time. This rarely happens in the case of any commodity in the market. In recent years, some models have been developed with a demand rate that changes exponentially with time. For seasonal products like clothes, Air conditions etc. at the end of their seasons the demand of these items is observed to be exponentially decreasing for some initial period. Afterwards, the demand for the products becomes steady rather than decreasing exponentially. It is believed that such type of demand is quite realistic. Such type situation can be represented by ramp type demand rate.

An important issue in the inventory theory is related to how to deal with the unfulfilled demands which occur during shortages or stock outs. In most of the developed models researchers assumed that the shortages are either completely backlogged or completely lost. The first case, known as backordered or backlogging case,
represent a situation where the unfulfilled demand is completely back ordered. In the second case, also known as lost sale case, we assume that the unfulfilled demand is completely lost.

Furthermore, when the shortages occur, some customers are willing to wait for backorder and others would turn to buy from other sellers. In many cases customers are conditioned to a shipping delay and may be willing to wait for a short time in order to get their first choice. For instance, for fashionable commodities and high-tech products with short product life cycle, the willingness of a customer to wait for backlogging is diminishing with the length of the waiting time. Thus the length of the waiting time for the next replenishment would determine whether the backlogging would be accepted or not. In many real life situations, during a shortage period, the longer the waiting time is, the smaller is the backlogging rate would be. Therefore, for realistic business situations the backlogging rate should be variable and dependent on the waiting time for the next replenishment. Many researchers have modified inventory policies by considering the “time proportional partial backlogging rate”.

Supply chain management can be defined as: "Supply chain management is the coordination of production, stock, location and transportation between actors in supply chain to achieve the best combination of responsiveness and efficiency to a given market. Many researchers in the inventory system have focused on products that do not exceed deterioration. However, there are a number of things whose significance does not remain the same over time. The deterioration of these substances plays an important role and cannot be stored for long {Yadav, et. al. (1 to 10)}. Deterioration of an object can be described as deterioration, evaporation, obsolescence and loss of use or limit of an object, resulting in lower stock consumption compared to natural conditions. When commodities are placed in stock as inventory to meet future needs, there may be deterioration of items in the system of arithmetic that may occur for one or more reasons, etc. Storage conditions, weather or humidity. {Yadav, et. al. (11 to 20)}. It is generally claimed that management owns a warehouse to store purchased inventory. However, management can, for a variety of reasons, buy or give more than it can store in its warehouse and name it OW, with an additional number in a rented warehouse called RW located near OW or slightly away from it {Yadav, et. al. (21 to 53)}. Inventory costs (including holding costs and depreciation costs) in RW are usually higher than OW costs due to additional costs of handling, equipment maintenance, etc. To reduce the cost of inventory will economically use RW products as soon as possible. Actual customer service is provided only by OW, and in order to reduce costs, RW stocks are first cleaned. Such arithmetic examples are called two arithmetic examples in the warehouse {Yadav and swami. (54 to 61)}. Management of supply of electronic storage devices and integration of environment and nervous networks {Yadav and Kumar (62)}. Analysis of seven supply chain management measures in improving the inventory of electronic devices for storage by sending an economic load using GA and PSO and analysis of supply chain management in improving the inventory of storage and equipment using genetic calculation and model design and analysis of chain inventory from bi warehouse and economic difficulty of freight transport using genetic calculation {Yadav, AS (63, 64, 65)}. Inventory policies of inventory and inventory requirements and different storage costs under allowable payments and inventory delays An
example of depreciation of goods and services of various types and costs of holding down a Business-Loan and an inventory model with sensitive needs of prices, holding costs in contrast to loans of business expenses under inflation {Swami, et. al. (66, 67, 68)}. The objectives of the Multiple Objective Genetic Algorithm and PSO, which include the improvement of supply and deficit inventory, inflation, and a calculation model based on a genetic calculation of scarcity and low inflation by PSO {Gupta, et. al. (69, 70,)}. An example with two warehouses depreciation of items and storage costs under particle upgrade and an example with two warehouses of material damage and storage costs in inflation and soft computer techniques {Singh, et. al. (71, 72)}. Delayed alcohol supply management and refinement of particles and green cement supply system and inflation using particle enhancement and electronic inventory calculation system and distribution center using genetic calculations {Kumar, et. al. (73, 74.75)}. Example of depreciation inventory with two warehouses and stock-based stocks using a genetic inventory and vehicle inventory system for demand and inflation of stocks with two distribution centers using genetic inventory {Chauhan and Yadav (76 , 77)}. Marble Analysis Improvement of industrial reserves based on genetic engineering and multi-particle improvement {Pandey, et. al. (78)}. White wine industry in supply chain management using nervous networks {Ahlawat, et. al. (79)}. Best policy for importing damaged items immediately and payment of conditional delays under the supervision of two warehouses {Singh, et. al. (80)}.

2. Assumptions and Notations

In developing the mathematical model of the inventory system the following assumptions are being made:

1. The demand rate \( D(t) = (A_0 + 1) e^{-(\lambda_0 + 1)(t-[t-[\rho_0 + 1])]} \), \( (A_0 + 1) > 0, (\lambda_0 + 1) > 0 \).

2. The backlogging rate is \( \exp(-\delta_0)(t) \), when inventory is in shortage. The backlogging parameter \( \delta_0 \) is a positive constant.

The variable rate of deterioration in both storehouses is taken as \( \theta_0(t) = (\theta_0 + 1)t \) where \( 0 < (\theta_0 + 1) << 1 \) and only applied to on hand inventory.

In addition, the following notations are used throughout this paper:

- \( Y_{og}(t) \) - The inventory level in OG at any time t.
- \( Y_{rg}(t) \) - The inventory level in RG at any time t.
- \( W_0 \) - The capacity of the own godown.
- \( Q_o \) - The ordering quantity per cycle.
- \( T \) - Planning horizon.
- \( r_o \) - Inflation rate.
- \( G_1 \) - The holding cost per unit per unit time in OG.
3. Formulation and Solution of the Model

The inventory levels at OW are governed by the following differential equations:

\[ \frac{dY_{og}(t)}{dt} = -\left(\theta_0 + 1\right)Y(t) \]

\[ 0 \leq t < \mu_0 \quad \ldots \quad (1) \]

\[ \frac{dY_{og}(t)}{dt} + (\theta_0 + 1)Y(t) = -(A_0 + 1)e^{-\left(\theta_0 + 1\right)(\mu_0 + 1)} \]

\[ \mu_0 \leq t \leq t_1 \quad \ldots \quad (2) \]

And

\[ \frac{dY_{og}(t)}{dt} = -(A_0 + 1)e^{-\left(\theta_0 + 1\right)(\mu_0 + 1)}e^{-\left(\theta_0 + 1\right)t} \]

\[ t_1 \leq t \leq T \quad \ldots \quad (3) \]

with the boundary conditions,

\[ Y_{og}(0) = (W_0 + 1) \text{ and } Y_{og}(t_1) = 0 \quad \ldots \quad (4) \]

The solutions of equations (1), (2) and (3) are given by:

\[ Y_{og}(t) = (W_0 + 1)e^{-\left(\theta_0 + 1\right)t^2/2}, \quad 0 \leq t < \mu_0 \quad \ldots \quad (5) \]

\[ Y_{og}(t) = (A_0 + 1)e^{-\left(\theta_0 + 1\right)(\mu_0 + 1)}\left(\frac{t_1 + t}{\theta_0 + 1}\right)\left(\frac{\theta_0 + 1}{\theta_0 + 1}\right)e^{-\left(\theta_0 + 1\right)t^2/2}, \quad \mu_0 \leq t \leq t_1 \quad \ldots \quad (6) \]

And

\[ Y_{og}(t) = (A_0 + 1)\frac{\theta_0 + 1}{\theta_0 + 1}e^{-\left(\theta_0 + 1\right)(\mu_0 + 1)}\left\{ e^{-\left(\theta_0 + 1\right)t} - e^{-\left(\theta_0 + 1\right)t_1/2} \right\}, \quad t_1 \leq t \leq T \quad \ldots \quad (7) \]

respectively.

The inventory level at RW is governed by the following differential equations:

\[ \frac{dY_{rg}(t)}{dt} + (\theta_0 + 1)Y(t) = -(A_0 + 1)e^{-\left(\theta_0 + 1\right)t} \]

\[ 0 \leq t < \mu_0 \quad \ldots \quad (8) \]

With the boundary condition \( Y_{rg}(0) = 0 \), the solution of the equation (3.8) is

\[ Y_{rg}(t) = (A_0 + 1)\left(\frac{\theta_0 + 1}{2}\left(\frac{\mu_0 + 1)^2 - t^2}{\theta_0 + 1}\right) + e^{-\left(\theta_0 + 1\right)t^2/2}, \quad \mu \leq t \leq t_1 \quad (9) \]
Due to continuity of \( Y_{\omega, t} \) at point \( t = \mu_0 \), it follows from equations (5) and (6), one has

\[
(W_0 + 1)e^{-(\theta_0 + 1)(\mu_0 + 1)^2 / 2} = (A_0 + 1)e^{-(\lambda_0 + 1)(\mu_0 + 1)} \left[ \frac{t_1 - (\mu_0 + 1)}{\theta_0 + 1} + \frac{t_1^3 - (\mu_0 + 1)^3}{6} \right] e^{-(\theta_0 + 1)(\mu_0 + 1)^2 / 2}
\]

\[
(W_0 + 1) = (A_0 + 1)e^{-(\lambda_0 + 1)(\mu_0 + 1)} \left[ \frac{t_1 - (\mu_0 + 1)}{\theta_0 + 1} + \frac{t_1^3 - (\mu_0 + 1)^3}{6} \right] \tag{10}
\]

The total average cost consists of following elements:

(i) Ordering cost per cycle = \( C_0 + 1 \) \( (11) \)

(ii) Holding cost per cycle in OG

\[
G_{HOG} = G_1 \int_0^{t_1} Y_{\omega, t}e^{-(\theta_0 + 1)(\mu_0 + 1)^2 / 2} dt + \int_0^{t_1} Y_{\omega, t}e^{-(\lambda_0 + 1)(\mu_0 + 1)^2 / 2} dt
\]

\[
\frac{t_1^2}{2} - \frac{(\theta_0 + 1)t_1^3}{6} + \frac{(\theta_0 + 1)t_1^4}{12}
\]

\[
- \frac{(\theta_0 + 1)\lambda_0 + 1)}{2 \theta_0 + 1}(\mu_0 + 1)^3 \tag{12}
\]

(iii) Holding cost per cycle in RG

\[
G_{HRG} = G_2 \int_0^{t_1} Y_{\omega, t}e^{-(\theta_0 + 1)(\mu_0 + 1)^2 / 2} dt
\]

\[
\frac{\lambda_0 + 1}{6}(3\lambda_1 - 2(\mu_0 + 1))
\]

\[
+ \frac{\lambda_0 + 1}{30}(4\lambda_1 - 3(\mu_0 + 1)) \tag{12}
\]
\[ G_{HGC} = G_2 \left[ \frac{(\mu_0 + 1)^2}{2} - \frac{3(\lambda_0 + 1) + (\eta_0 + 1)}{6} (\mu_0 + 1)^3 + \frac{(\theta_0 + 1) + (\lambda_0 + 1)(\eta_0 + 1)}{12} (\mu_0 + 1)^4 \right] \]

\[ - \left( \frac{(\eta_0 + 1)(\theta_0 + 1)}{20} - \frac{(\lambda_0 + 1)(\theta_0 + 1)}{30} \right)(\mu_0 + 1)^5 \]

(13)

(iv) Cost of deteriorated units per cycle
\[ G_{DC} = G_d \left[ \frac{1}{6} \left( \frac{\mu_0 + 1}{4} - \frac{\lambda_0 + 1}{12} \right) + \frac{\mu_0 + 1}{15} - \frac{\mu_0 + 1}{3} \right] \]

\[ \frac{1}{6} \left( \frac{\mu_0 + 1}{4} - \frac{\lambda_0 + 1}{12} \right) + \frac{\mu_0 + 1}{15} - \frac{\mu_0 + 1}{3} + \frac{36}{24} \left( \frac{\mu_0 + 1^4}{\mu_0 + 1^6} \right) \]

(v) Shortage cost per cycle

\[ G_{SC} = G_3 \left[ \int_{T_{1}}^{T} -Y_o(t) e^{-(\eta_0 + 1)(\eta_0 + t)} dt \right] \]
\[
\begin{bmatrix}
-\left(A_0 + 1\right)G_3 e^{-\left(A_0 + 1\right)\left(\frac{n_0}{2} + 1\right) - \left(\delta_0 + 1\right) t} \\
\left(\delta_0 + 1\right)^{T} e^{\left(-\left(n_0 + 1\right)\right) t}
\end{bmatrix}
\]

\[
\begin{bmatrix}
-\left(A_0 + 1\right)G_3 e^{-\left(A_0 + 1\right)\left(\frac{n_0}{2} + 1\right) - \left(\delta_0 + 1\right) t} \\
\left(\delta_0 + 1\right)^{T} e^{\left(-\left(n_0 + 1\right)\right) t}
\end{bmatrix}
\]

\[
(15)
\]

(vi) Opportunity cost due to lost sales per cycle

\[
G_{OPC} = G_4 e^{-\left(A_0 + 1\right)\left[1 - e^{-\left(\delta_0 + 1\right) t}\right]_t^T e^{-\left(\delta_0 + 1\right)\left(\mu_0 + 1\right) - \left(n_0 + 1\right)\left(\delta_0 + 1\right) t}}
\]

Therefore, the total average cost per unit time of our model is obtained as follows

\[
K(t_1, T) = \frac{1}{T} \left[ G_{DC} + G_{HOS} + G_{HRS} + G_{DC} + G_{SC} + G_{OPC} \right]
\]

4. Multi-Objective Genetic Algorithm

Discussions so far were limited to GA that handled the optimization of a single parameter. The optimization criteria are represented by fitness functions and are used to lead towards an acceptable solution. A typical single objective optimization problem is the TSP. There the sole optimization criterion is the cost of the tour undertaken by the salesperson and this cost is to be minimized. However, In real life we often face problem which require simultaneous optimization of several criteria. For example, in VLSI circuit design the critical parameters are chip area power consumption delay fault tolerance etc. While designing a VLSI circuit the designer may like to minimize area power consumption and delay while at the same time would like to maximize fault tolerance. The problem gets more complicated when the optimizing criteria are conflicting. For instance an attempt to design low-power VLSI circuit may affect its fault tolerance capacity adversely. Such problems are known as multi-objective optimization (MOO). Multi-objective optimization is the process of systematically and simultaneously optimizing a number of objective functions. Multiple objective problems usually have conflicting objectives which prevents simultaneous optimization of each objective. As GAs are population based optimization processes they are inherently suited to solve MOO problem. However traditional GAs are to be customized to accommodate such problem. This is achieved by using specialized
fitness functions as well as incorporating methods promoting solution diversity. Rest of this section presents the features of multi-objective GAs. Multi-objective GA is designed by incorporating pareto-ranked niche count based fitness sharing into the traditional GA process.

This is presented as **Procedure Multi-Objective-GA**

Step 1:- Generate the initial population randomly.

Step 2:- determine the pareto-optimal fronts $U_{p_{o1}}, U_{p_{o2}}, \ldots U_{p_{ok}}$.

Step 3:- If stopping criteria is satisfied then Return the pareto-optimal front $U_{p_{o1}}$

Stop

Step 4:- for each solution $x_{p}$, evaluate the fitness.

Step 5:- Generate the matting pool MP from population P applying appropriate selection operator.

Step 6:- Apply crossover and mutation operations on the chromosomes of the mating pool to produce the next generation $p'$ of population from MP.

Step 7:- Replace the old generation of population $p$ by the new generation of Population $p'$

Step 8:- Go to step 2.
5. Numerical Illustration

To illustrate the model numerically the following parameter values are considered.

\[ A_0 = 50 \text{ units}, \ G_0 = \text{Rs. 100 per order}, \ \eta_0 = 0.05 \text{ unit}, \ G_1 = \text{Rs. 3.0 per unit per year}, \ G_2 = \text{Rs. 12.0 per unit per year}, \ G_3 = \text{Rs. 4.0 per unit}, \ T = 1 \text{ year}, \ \lambda_0 = 0.2 \text{ unit}, \ G_4 = \text{Rs. 10.0 per unit}, \ \theta_0 = 0.002 \text{ unit}, \ \delta_0 = 0.1 \text{ unit}, \ \mu_0 = 0.2 \text{ year} \]

The parameter values are chosen for the minimization of total average cost and with the help of software, the optimal policy can be obtained such as:
The minimization of total average cost

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GA Results

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6. Conclusion

This study incorporates some realistic features that are likely to be associated with the inventory of any material. Decay (deterioration) overtime for any material product and occurrence of shortages in inventory are natural phenomenon in real situations using Genetic Algorithm. Inventory shortages are allowed in the model. In many cases customers are conditioned to a shipping delay, and may be willing to wait for a short time in order to get their first choice using Genetic Algorithm. Generally speaking, the length of the waiting time for the next replenishment is the main factor for deciding whether the backlogging will be accepted or not using Genetic Algorithm. The willingness of a customer to wait for backlogging during a shortage period declines with the length of the waiting time. Thus, inventory shortages are allowed and partially backordered in the present chapter and the backlogging rate is considered as a decreasing function of the waiting time for the next replenishment using Genetic Algorithm. Demand rate is taken as exponential ramp type function of time, in which demand decreases exponentially for the some initial period and becomes steady later on using Genetic Algorithm. Since most decision makers think that the inflation does not have significant influence on
the inventory policy, the effects of inflation are not considered in some inventory models. However, from a financial point of view, an inventory represents a capital investment and must compete with other assets for a firm’s limited capital funds using Genetic Algorithm. Thus, it is necessary to consider the effects of inflation on the inventory system using Genetic Algorithm. Therefore, this concept is also taken in this model using Genetic Algorithm. From the numerical illustration of the model, it is observed that the period in which inventory holds increases with the increment in backlogging and ramp parameters while inventory period decreases with the increment in deterioration and inflation parameters. Initial inventory level decreases with the increment in deterioration, inflation and ramp parameters while inventory level increases with the increment in backlogging parameter using Genetic Algorithm. The total average cost of the system goes on increasing with the increment in the backlogging and deterioration parameters while it decreases with the increment in inflation and ramp parameters. The proposed model can be further extended in several ways. For example, we could extend this deterministic model into a stochastic model using Genetic Algorithm. Also, we could extend the model to incorporate some more realistic features, such as quantity discount or the unit purchase cost, the inventory holding cost and others can also be taken fluctuating with time using Genetic Algorithm.

References:


