Neighborhood Prime Labeling Of Some Graphs

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Abstract: Let G be the graph with n vertices, a bijective function \( f: V(G) \rightarrow \{1,2,\ldots,n\} \) is said to be a neighborhood prime labeling if for every vertex \( v \in V(G) \), \( \deg(v) > 1 \), \( \gcd(f(p) | p \in N(v)) = 1 \). A graph which admits neighborhood prime labeling is called neighborhood prime graph.

Here we discuss about the coconut tree graph, the Jewel graph, \( K_{1,3} \ast K_{1,n} \) graph, The Jelly fish graph which admits an NPL.

Keywords: Neighborhood prime labeling, Neighborhood prime graph, \( K_{1,3} \ast K_{1,n} \) graph, Adjacent vertices

I. Introduction: Roger Entringer introduced the concept of prime labeling and was introduced in 1980’s by Tout et al [6] which paved way for many researches in this zone. Motivated by the study of prime labeling, S.K.Patel and N.Shrimali in [3] introduced the concept of neighbourhood prime labeling in 2015, in which they have recognized the enough condition for a graph to admit neighbourhood prime labeling and proved that paths, cycles, helm, closed helm and flower have neighbourhood prime labeling.

II. BASIC TERMINOLOGY OF NEIGHBOURHOOD PRIME LABELING OF SOME GRAPHS

Definition 1.1: Let G be the graph with n vertices, a bijective function \( f: V(G) \rightarrow \{1,2,\ldots,n\} \) is said to be a neighborhood prime labeling if for every vertex \( v \in V(G) \), \( \deg(v) > 1 \), \( \gcd(f(p) | p \in N(v)) = 1 \). A graph which admits neighborhood prime labeling is called neighborhood prime graph.

Definition 1.2: The double star graph \( (K_{n,1,1}) \) is a tree getting from the star graph \( K_{1,n} \) by attaching new pendent edges of the exiting n pendent vertices which consisting total \( 2n + 1 \) vertices and \( 2n \) edges.

Definition 1.3[3]: Coconut tree graph is obtained by identifying the middle vertex of \( K_{1,m} \) with a pendent vertex of the path \( P_{n} \).

Definition 1.4[6]: The Jewel graph \( J_{n} \) is the graph with vertex set \( V(J_{n}) = \{u,v,x,y,u_{j}|1 \leq j \leq n\} \) and the edge set \( E(J_{n}) = \{ux,uy,vx,vy,xy,u_{j}u_{j}|1 \leq j \leq n\} \)

Definition 1.5[2]: Let \( G = K_{1,3} \ast K_{1,n} \) be the graph obtained from \( K_{1,3} \) by joining root of a star \( K_{1,n} \) at each pendent vertex of \( K_{1,3} \).

Definition 1.5[5]: The Jelly fish graph \( f(n,m) \) is obtained from a 4-cycle \( (v_{1},...,v_{4}) \) collected with an edge \( v_{1}v_{3} \) and affixing \( n \) pendent edges to \( v_{2} \) and \( m \) pendent edges to \( v_{4} \).

III. NEIGHBOURHOOD PRIME LABELING OF SOME GRAPHS

Theorem 3.1: Every Double star graph \( (K_{1,n,1}) \) is a neighbourhood prime labeling.

Proof: Let \( G = (V(G), E(G)) \) be a graph of double star graph \( (K_{1,n,1}) \) with

vertex set \( \{v,v_{i},u_{j}|i \in 1,2,\ldots,n\} \) obtained from the suppose \( \{v,v_{1},v_{2},...,v_{n}\} \) and \( \{u_{1},u_{2},...,u_{n}\} \) be the vertices and \( \{e_{1},e_{2},...,e_{n-1}\} \) be the edges which are denoted in figure.

Note that the path consist \( n \) vertices and \( n - 1 \) edges.

If \( G = (K_{1,n,1}) \) then total number of vertices \( 2n + 1 \) and total number of edges \( 2n \).

Now we define a vertex labeling \( f: V(G) \rightarrow \{1,2,\ldots,p\} \) as follows:
Then the double star graph admits NPL. Hence it is Neighborhood prime graph.

Theorem 3.2: The coconut tree graph $CT_{m,n}$ admits a neighbourhood prime labeling.

Proof: Let $V = \{v_i | 1 \leq i \leq m\} \cup \{u_j | 1 \leq j \leq n\}$ be the vertex set of coconut tree where $v_i$ are the vertices of the path $P_m$ and $u_j$ are the $n$ new pendant vertices at an end vertex of the path $P_m$.

Let $E = \{e_i = v_i v_{i+1} | 1 \leq i \leq m\} \cup \{e_j = v_j u_j | i = m, 1 \leq j \leq n\}$ be the edge set of coconut tree. Here the coconut tree has $|V(G)| = m + n$ vertices and $|E(G)| = m + n - 1$ edges.

An injective function $f: V(G) \to \{1, 3, ..., 2(m + n) - 1\}$ such that

Case(i): $m \equiv 1 \text{ (mod 2)}$

$$f(v_i) = \begin{cases} i, & \text{if } i \text{ is odd} \\ m + i, & \text{if } i \text{ is even} \end{cases}$$

$$f(u_j) = 2m + 2j - 1, 1 \leq j \leq n$$

The vertices with degree greater than one are $N(u_i) \ni \{u_{i-1}, u_{i+1}\}$ where $2 \leq i \leq m - 2$.

Where $f(u_{i-1}) = i - 1$ and $f(u_{i+1}) = i + 1$

Then the $\gcd\{f(p) | p \in N(u_i)\} = 1$

Now $N(u_m) \ni \{u_{m-1}, v_i\}$ here $1 \leq i \leq m$

Here $f(u_{m-1}) = 5$ and $f(u_j) = 2m + 2j - 1, 1 \leq j \leq n$

Then the $\gcd\{f(p) | p \in N(u_m)\} = 1$

Then the coconut tree graph $CT_{m,n}$ admits NPL. Hence it is Neighborhood prime graph.
Theorem 3.3: Let G be the Jewel graph admits an NPL.

Proof: Suppose $G^*$ be the graph defined by

$G^* = G - \{uw_j|1 \leq j \leq m - 2\}$. Here $V(G^*) = \{u, v, x, y, w_j|1 < j \leq m\}$ and

$E(G^*) = \{ux, vx, uy, vy, uw_{m-1}, uw_{m}, vw_j|1 \leq j \leq m\}$. Then $|V(G^*)| = m + 4$ and

$|E(G^*)| = m + 6$. Let $f: V(G^*) \to \{1, 2, ..., 2m + 9, 2(m + 6)\}$ is defined as follows:

$f(u) = 1$ \quad $f(v) = 3$

$f(w_j) = 2m + 9 - 2j; 1 \leq j \leq m - 2$

$f(w_{m-1}) = 9$ \quad $f(w_m) = 5$

$f(x) = 2(n + 6)$ \quad $f(y) = 2(m + 9)$

The vertices with degree grater than one are $N(x) \ni \{f(u), f(v)\}$

Here $f(u) = 1$ and $f(v) = 3$.

Then the the $\gcd\{f(p)|p \in N(x)\} = 1$

Now consider $N(y) = \{f(u), f(v)\}$

Here $f(u) = 1$ and $f(v) = 3$.

Then the $\gcd\{f(p)|p \in N(y)\} = 1$

Now, $N(u) \ni \{f(x), f(y), f(w_{m-1}), f(w_m)\}$

Here $f(x) = 2m + 12$ \quad $f(y) = 2m + 9$

$f(w_{m-1}) = 9$ \quad $f(w_m) = 5$

Then the $\gcd\{f(p)|p \in N(u)\} = 1$

Now, $N(v) \ni \{f(x), f(y), f(w_{m-1}), f(w_m), f(w_j)\}; 1 \leq j \leq m - 2$

Here $f(w_j) = 2m + 9 - 2j; 1 \leq j \leq m - 2$

$f(x) = 2(m + 6)$ \quad $f(y) = 2m + 9$

$f(w_{m-1}) = 9$ \quad $f(w_m) = 5$

Then the $\gcd\{f(p)|p \in N(w_j)\} = 1$

Now $N(w_{m-1}) \ni \{f(u), f(v)\}$

Here $f(u) = 1$ and $f(v) = 3$
Then the $\gcd(f(p)|p \in N(w_{m-1}) = 1$

Now $N(w_{m}) \ni \{f(u), f(v)\}$

Here $f(u) = 1$ and $f(v) = 3$

Then the $\gcd(f(p)|p \in N(w_{m})) = 1$

Here note that either $n$ is odd or $n$ is even.

The Jewel graph which admits NPL. Hence it is neighborhood prime graph.

**Theorem 3.4:** The graph $K_{1,3} \ast K_{1,n}$ admits an NPL for all $n \geq 2$.

**Proof:** Let $G = K_{1,3} \ast K_{1,n}$ with $V(G) = \{x, u, v, w, v_{j}, w_{j} | 1 \leq j \leq n\}$ and $E(G) = \{xu, xv, xw, uu_{j}, vv_{j}, ww_{j} | 1 \leq j \leq n\}$. Hence $|V(G)| = 3n + 4$ and $|E(G)| = 3(n + 1)$

Define $f: V(G) \rightarrow \{1, 2, \ldots, 6n + 7\}$ by

- $f(u) = 1$
- $f(v) = 3$
- $f(w) = 2n + 5$
- $f(x) = 4n + 7$
- $f(u_{j}) = 6n + 9 - 2j; 1 \leq j \leq n$
- $f(v_{j}) = 4n - 2j + 7; 1 \leq j \leq n$
- $f(w_{j}) = 2n - 2j + 5; 1 \leq j \leq n$

The vertices with degree greater than one are $N(u) \ni \{f(x), f(u_{j})\}; 1 < j \leq n$

Here $f(x) = 4n + 7$ and $f(u_{j}) = 6n - 2j + 9$

Then the $\gcd(f(p)|p \in N(u)) = 1$

$N(v) \ni \{f(x), f(v_{j})\}; 1 \leq j \leq n$

Here $f(x) = 4n + 7, f(v_{j}) = 4n - 2j + 7$

Then the $\gcd(f(p)|p \in N(v)) = 1$

Now $N(w) \ni \{f(x), f(w_{j})\}; 1 \leq j \leq n$

Here $f(x) = 4n + 7, f(w_{j}) = 2n - 2j + 5$

Then the $\gcd(f(p)|p \in N(w)) = 1$

Now $N(x) \ni \{f(u), f(v), f(w)\}$

Here $f(u) = 1$ which is relatively prime to remaining numbers.

So, The graph $K_{1,3} \ast K_{1,n}$ admits NPL. Hence it is Neighborhood prime graph.
**Theorem 3.5:** The Jelly fish graph $J(n, m)$ admits an NPL.

**Proof:** Let $G$ be the Jelly fish $J(n, m)$ graph. Let $V(G) = \{u, v, x, y, u_i, v_j | 1 \leq i \leq n, 1 \leq j \leq m\}$

And $E(G) = \{ux, vx, uy, vy, xu_i, yv_j | 1 \leq i \leq n\} \cup \{vx, yv_j | 1 \leq j \leq m\}$

Then $|V(G)| = n + m + 4$ and $|E(G)| = n + m + 5$

Define $f: V(G) \rightarrow \{1, 2, \ldots, 2(n + m + 4), 2n + 2m + 11\}$ as follows:

$f(u) = 2n + 2m + 11$  
$f(v) = 2n + 2m + 7$  
$f(x) = 2n + 1$  
$f(y) = 2n + 3$  
$f(u_i) = 2i - 1; 1 \leq i \leq n$  
$f(v_j) = 2n + 2j + 3; 1 \leq j \leq m$  

The vertices with degree greater than one as follows:

Now $N(u) \supset \{f(u_i), f(x), f(y)\}; 1 \leq i \leq n$

Here $f(u_i) = 2i - 1$  
$f(x) = 2n + 1$  
$f(y) = 2n + 3$

Then the $\gcd\{f(p) | p \in N(u)\} = 1$

Now $N(v) \supset \{f(v_j), f(x), f(y)\}; 1 \leq j \leq m$

$f(v_j) = 2n + 2j + 3$  
$f(x) = 2n + 1$  
$f(y) = 2n + 3$

Then the $\gcd\{f(p) | p \in N(v)\} = 1$

Now $N(x) \supset \{f(u), f(v), f(y)\}$

Here $f(u) = 2n + 2m + 11$  
$f(v) = 2n + 2m + 7$  
$f(y) = 2n + 3$

Then the $\gcd\{f(p) | p \in N(x)\} = 1$

Now $N(y) \supset \{f(u), f(v), f(x)\}$

Here $f(u) = 2n + 2m + 11$  
$f(v) = 2n + 2m + 7$  
$f(x) = 2n + 1$

Then the $\gcd\{f(p) | p \in N(y)\} = 1$

So, The Jelly fish graph $J(n, m)$ is NPL. Hence it is Neighborhood prime graph.
Fig 5: NPL of The Jelly fish graph $J(n, m)$.

Reference:


