



# NETWORK-BASED TECHNIQUES FOR SMALL R&D PROJECTS

*K. MALARVIZHI<sup>1</sup>, A. NANDHITHA<sup>2</sup>, T. MYTHIL<sup>3</sup>*

*<sup>1</sup>Assistant professor, <sup>2</sup>PG Scholar, <sup>3</sup>PG Scholar  
Department of Mathematics  
Sri Krishna Arts and Science College, Coimbatore, India.*

## Abstract

Contemporary Project Management has designed mathematical-based instruments models for project preparation, scheduling and monitoring, expenditures and costs Capabilities. The two important project management techniques that assist the project managers to schedule the projects are Critical Path Method (CPM) And Program Evaluation and Review Technique (PERT). In this paper we are heading to analyze the total cost of the project which includes the direct cost, indirect cost and finding the cost slope by the given values. Some in several project network models activities are closely linked to each other, such as procurement activities of a basic nature study tools and events.

**Key words:** project management, Scheduling, Optimization, Crashing, cost slope.

Corresponding Author: K. Malarvizhi

## 1. Introduction

Planning is the most critical step of project management. The planning is where the project schedules are registered, the project deliverables and specifications are identified and the project schedule is drawn up.[1] A project consists of “n” number of activities. Relating the activities in logical order is called Scheduling. The project plan shows what is to be completed, what services are to be included and when the project is due. The most important techniques usually used for planning and scheduling are the network-based methods. A network is a graphical representation of a project operation and which is composed of activity and event that must be completed to reach in objective of the project. An activity is represented by “an arrow” (→). Activity consumes money, time, man power, material etc. It is simply called as resources. An activity must contain a starting node and an ending node. An event is denoted by circle. Event does not consume any resources. It only denotes when the activity starts and the activity ends. There evolved many planning tools such as CPM, PERT, MPM, PDM etc. In the following sections we are here to perform only two tools CPM and PERT. In 1950’s CPM was developed by DuPont and the emphasis was on the trade-off between the cost of the project and its overall completion time (e.g. for certain activities it may be possible to decrease their completion times by spending more money- how does it affect the overall completion time of the project?). PERT was developed by the US Navy for the planning and control of the Polaris missile program and focused on completing the program as soon as possible. In the addition PERT had the ability to cope with uncertain activity completion times (e.g. for a particular activity the most likely completion is 4 weeks but it could be anywhere between 3 weeks and 8 weeks).

## 2. Preliminaries

### Phases of Project Management:

#### 1.Planning

- Diving the project into distinct activities.
- Estimating time requirement for each activity.
- Establishing precedence relationships among the activities.
- Construction of the arrow diagram.

#### 2.Scheduling

Scheduling determines the start and end time of each and every activity.

#### 3.Controlling

This process is also carried out at the same time as implementation, and this is where consistency is tracked, as well as distance and cost/time allocations are monitored in order to keep everything within the budget.[3]

### Types of Network Schedule:

- Critical Path Method (CPM)
- Program Evaluation and Review Technique (PERT)

### Critical Path Method (CPM):

The Critical Path Method (CPM) is a step-by-step process, strategy or algorithm for organizing projects with several tasks requiring dynamic, interdependent interactions. CPM is an important project management tool because it identifies critical and non-critical tasks to prevent conflicts and bottlenecks. CPM is also used to evaluate the logic diagram of the project network in order to achieve optimum functional efficiency.

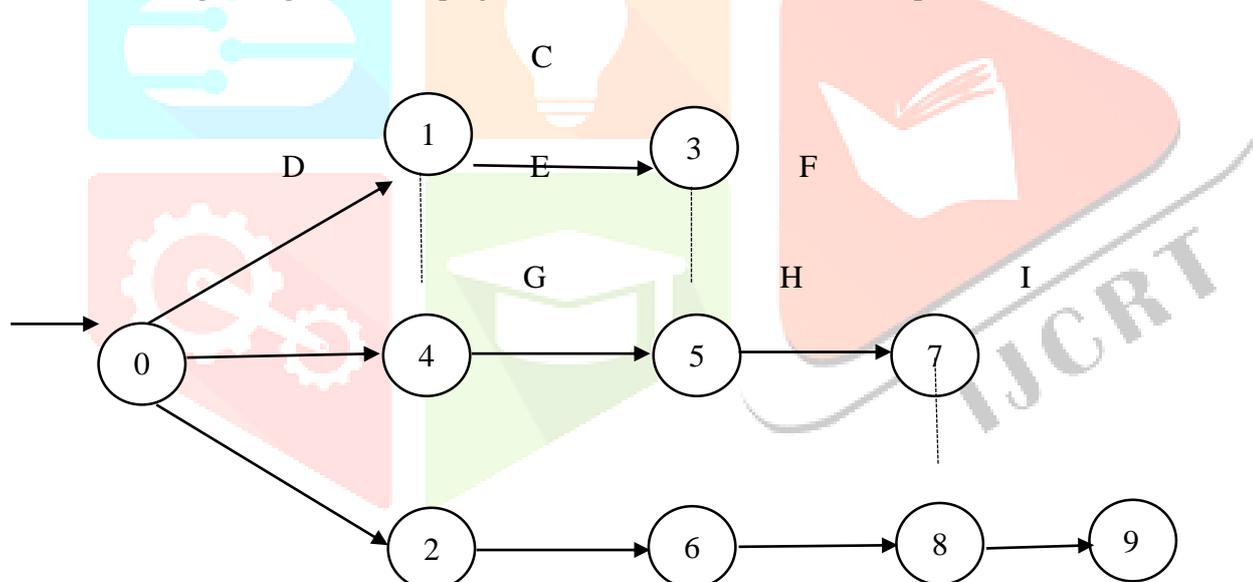


Figure 1. Network Diagram

### General Rules to Be Followed While Constructing CPM Network:

- A project must contain one starting node and one ending node. In Fig 1. 0 is the starting node and 9 is the ending node.
- Crossing is not allowed in the CPM network. One activity must not cross on another activity.
- Looping is not allowed. A network flow must be in only one direction.[6]
- Dummy activity is used when two activities start at same node and end at common node. In this situation add some other number and that number does not consume any man power, time etc. For ex. In Fig 1. The dummy activity is denoted by dotted lines.

**Important Terminologies of CPM and PERT:**

The following terminology is used for a PERT/CPM network.[5]

1. Activity – This is a separate activity that needs to be carried out as part of the initiative.
2. Arrow – It shows the direction of the activity.
3. Node – It is represented by a circle and indicates an event, a point in time where one or more activities begin and/or end. The start node is the node that represents the start of the project, while the finish node signifies the end of the project.
4. Immediate predecessors – These are the activities that must be completed no later than the start time of the activity.
5. Immediate successor – Provided the immediate predecessor to an activity, this activity becomes the immediate successor of one of these immediate predecessors. If an immediate successor has several immediate predecessors, then all of them must be completed before the operation can begin.
6. Path – A route through a project network is a path that follows a series of arcs from the start node to the finish node. The length of the path is defined as the sum of the duration of the path operation.
7. Slack time – These are the variations between the most recent period and the earliest time of the action. That is the period of time that an operation may be postponed without delaying the completion of the project.
8. Critical path – This is the direction that has the longest duration of the project. It's the fastest time a project will actually be done. If the slack is zero for an operation, it is on the critical path. Similarly, if the slack is positive, the operation is not on the vital path.
9. Dummy – It is put into the network to display a prior partnership, but it does not reflect the real passing of time.
11. Earliest Starting Time (EST) - It is the calendar period that an occurrence will occur when all previous events have been completed as quickly as possible. The early start time for an operation is equivalent to the highest of the early finishing times of its immediate ancestors.
12. Earliest Finishing Time (EFT) – It is time for the operation to be finished provide there are no delays in the process.
13. Latest Starting Time (LST) – It is the most recent period that the operation will begin without delaying the ensuing activities and the completion of the project.
14. Latest Finishing Time (LFT) – It is the most recent period when the operation can be done without delaying the ensuing activities and the execution of the project. The last completion period of an operation is equal to the smallest of the most recent start times of its immediate predecessors.[7]
15. Forward pass – The method of going through the project from start to finish is to decide the early start and end times for the tasks of the project.
16. Backward pass - It is the method of bringing the project from the end to the beginning to assess the latest start and completion dates for the project's activities.
17. Crashing – Crashing an operation refers to putting on additional spending in order to minimise the length of an activity below the intended duration. Crash point indicates the time and cost when the operation is absolutely crashes.

**Float and Its Types:**

Float or free time should only be correlated with non-critical tasks. By example, essential path operations cannot be float. There are four different types of floats. Total float, Free float, Interfering float, Independent float.

**Total Float:**

The total float of the operation represents the amount of time that the task may be delayed without extending the completion date of the project.

**Free Float:**

Free float is the component of the overall float within an operation that can be modified without impacting the float of future operations.

### Interfering Float:

The use of an operation float can have an effect on the float periods of other operations in the network.

### Independent Float:

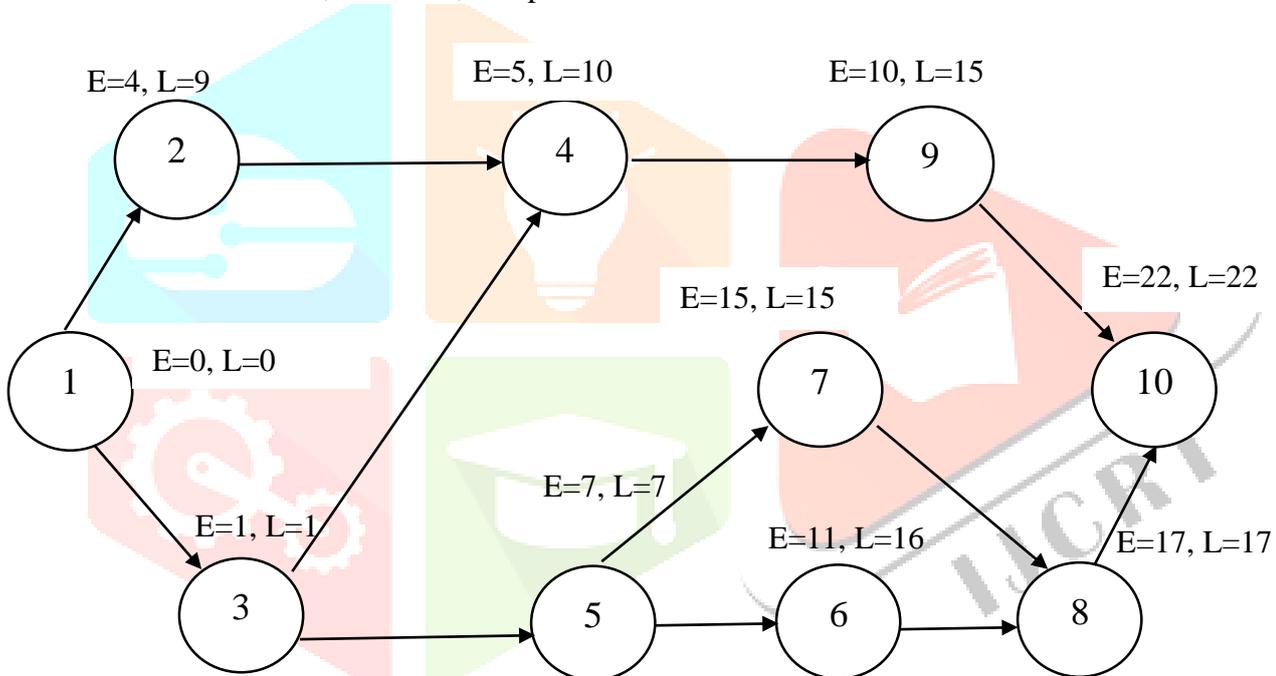
This is the period of time that can be postponed when all prior tasks are completed as late as possible and all active activities are begun as soon as possible.

### 3.Problem:

The project schedule has following characteristics

Activities	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
No. of. Hours	4	1	1	1	6	5	4	8	1	2	5	7

- Find critical path and project duration.
- Draw the network.
- Determine the total float, free float, independent float.



ACT	Duration	EST	EFT	LST	LFT	TF	FF	IF
1-2	4	0	4	5	9	5	0	0
1-3	1	0	1	0	1	0	0	0
2-4	1	4	5	9	10	5	0	-5
3-4	1	1	2	9	10	8	3	3
3-5	6	1	7	1	7	0	0	3
4-9	5	5	10	10	15	5	0	0
5-6	4	7	11	12	16	5	0	-5
5-7	8	7	15	7	15	0	0	0
6-8	1	11	12	16	17	5	5	0
7-8	2	15	17	15	17	0	0	0
8-10	5	17	22	17	22	0	0	0
9-10	7	10	17	15	22	5	5	0

Critical Path = 1-3-5-7-8-10

$$\begin{aligned} \text{Duration} &= 1+6+8+2+5 \\ &= 22 \end{aligned}$$

Where EST is the Earliest Starting Time, LST is the Latest Finishing Time

EFT is the Earliest Finishing Time ( $T_{EF}$ )

$T_{EF} = E + t_{ij}$  where E is the Earliest Starting Time and  $t_{ij}$  is the activity duration.

LST is the Latest Starting Time ( $T_{LS}$ )

$T_{LS} = L - t_{ij}$  where L is the Latest Finishing Time and  $t_{ij}$  is the activity duration.

Total Float (TF):

$$\begin{aligned} \text{TF} &= (L-E) - t_{ij} \\ &= T_{LS} - E \end{aligned}$$

Free Float (FF):

$$\begin{aligned} \text{FF} &= \text{TF} - \text{Head event slack} \\ &= \text{TF} - (L-E) \end{aligned}$$

Independent Float (IF):

$$\begin{aligned} \text{IF} &= \text{FF} - \text{Tail event slack} \\ &= \text{FF} - (L-E) \end{aligned}$$

## Program Evaluation and Review Technique (PERT)

A Visual tool used in project planning is commonly known as PERT or Software Assessment Analysis Method.[2] PERT helps the project managers to define the start and the end dates, as well as the timelines needed in the interim. The data is showed in chart form as a network.

**Optimistic Time (or) Least Time (or)  $t_o$  :**

Optimistic time is a term used in the assessment and analysis technique (PERT) of the software. It is the shortest estimated time span during which mission is likely to be accomplished and is used in the preparation of programs.

**Pessimistic Time (or) Greatest Time (or)  $t_p$  :**

Pessimistic time is a term used in the assessment and analysis technique (PERT) of the software. It is the longest estimated time span during which mission is likely to be accomplished and is used in the preparation of programs.

**Most Likely Time (or)  $t_m$  :**

Most Likely Time represents an estimation of time to complete an operation which is neither Optimistic nor Pessimistic, assuming that things can go in a normal direction. The Most Likely Time is expressed as 'tm' and if the activity is repeated multiple times, the time is indicated by 't' in most cases.

**General Formulas:****1. Expected duration:**

$$t_e = \frac{t_o + t_p + 4t_m}{6}$$

**2. Standard deviation ( $\sigma$ ):**

$$SD = \sqrt{\left(\frac{t_p - t_o}{6}\right)^2}$$

**3. Variance:**

$$\text{Variance} = \left(\frac{t_p - t_o}{6}\right)^2$$

**4. Standard Normal Distribution:**

$$Z = \frac{x - \mu}{\sigma}$$

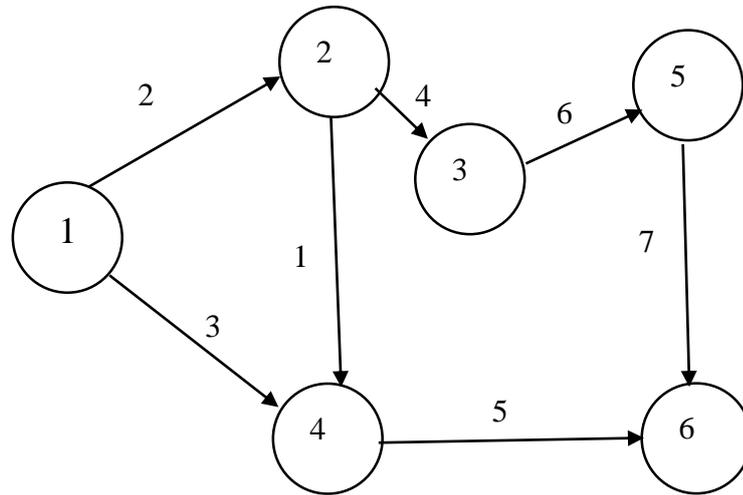
**Problem:**

A small project is composed of 7 activities whose time estimate are given below [4]

Activity	I	1	1	2	2	3	4	5
	J	4	2	3	4	5	6	6
Estimated duration	$t_o$	1	1	2	1	2	2	3
	$t_p$	7	7	8	1	14	8	15
	$t_m$	1	4	2	1	5	5	6

- Draw a project network.
- Find expected duration and variance of each activity.
- Calculate SD of project length.
- If the project due date is 21<sup>st</sup> week what is the probability of due date?

i.



ii.

Activity	$t_o$	$t_m$	$t_p$	$t_e$	variance
<b>1-2</b>	1	1	7	2	<b>1</b>
<b>2-3</b>	1	4	7	4	<b>1</b>
1-4	2	2	8	3	1
2-4	1	1	1	1	0
<b>3-5</b>	5	14	6	6	<b>1/36</b>
4-6	5	8	5	5	0
<b>5-6</b>	6	15	7	7	<b>1/36</b>

Critical Path	Duration
<b>1-2-3-5-6</b>	<b>19</b>
1-2-4-6	8
1-4-6	8

$$\text{Variance} = 1+1+1/36+1/36 = 2.05$$

iii.

Standard Deviation (SD):

$$SD = \sqrt{2.05} = 1.43$$

$$\text{SD} = 1.4$$

iv.

$$\text{Normal Distribution (Z)} = \frac{x - \mu}{\sigma} = \frac{21 - 19}{1.4} = \frac{2}{1.4}$$

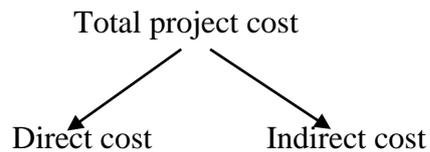
$$Z = 1.43$$

$$P(1.43) = 0.9236$$

$$\text{The probability if the due date is 21}^{\text{st}} \text{ week} = 0.9236$$

$$= 92.36\%$$

## Cost Analysis:



### Direct Cost:

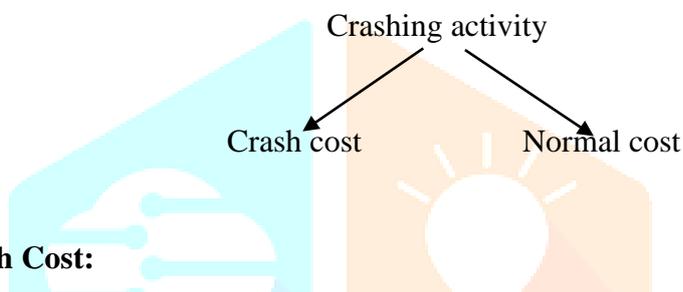
The direct cost directly depends upon the amount of resources involved in the execution of the individual activity. For eg. Cost of the material, Equipment and Labour.

### Indirect Cost:

The indirect cost are those expenditures which cannot be allocated to individual activities of the project.

### Crashing Activity:

Crashing activity is the process of reducing the duration of the activity by putting an extra effort.



### Crash Cost:

Crash cost is the cost associated with selecting the faster alternative to complete the effort (or) The activity cost corresponding to the crash time is called the crash cost which is equal to the minimum direct cost required to achieve the crash performance.

### Normal Cost:

Normal cost is the original cost estimate to complete the activity (or) The normal cost is equal to the absolute minimum of the direct cost required to perform an activity the corresponding activity duration is known as normal time.

### Cost Slope:

The term cost slope is defined as “Increase in the cost of the activity per unit decrease in the time”.

$$\text{Cost slope} = \frac{\text{cash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$$

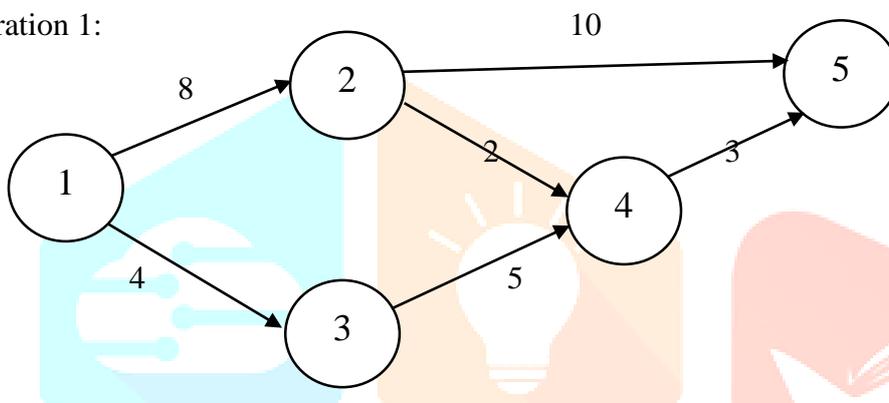
Problem:

Determine the optimum project duration and the cost for the following data.				
	normal		crash	
Act	time	cost	time	cost
1-2	8	100	6	200
1-3	4	150	2	350
2-4	2	50	1	90
2-5	10	100	5	400
3-4	5	100	1	200
4-5	3	80	1	100

- If the indirect cost is Rs 100/day. Find the optimum duration.
- Minimum duration.

Solution:

Iteration 1:



Critical path	Project duration
1-2-5	8+10 = 18
1-3-4-5	4+5+3 = 12
1-2-4-5	8+2+3 = 13

Critical path is **1-2-5**. ie)  $8+10 = 18$

Total cost = Normal cost + Indirect cost + Direct cost

$$= 580 + 100 (18) + 0$$

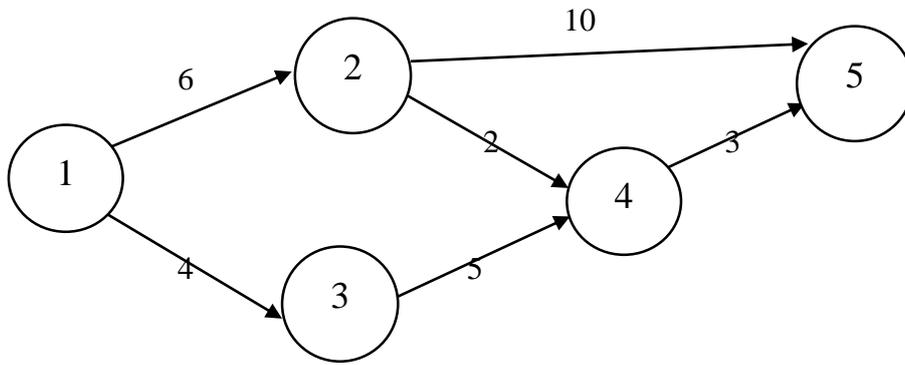
$$= 580 + 1800$$

$$= 2300$$

$$\text{Cost slope} = \frac{\text{cash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$$

Activity	Cost slope
1-2	50
1-3	100
2-4	40
2-5	60
3-4	25
4-5	10

Iteration 2:



Critical path	Project duration
1-2-5	6+10 = 16
1-3-4-5	4+5+3 = 12
1-2-4-5	6+2+3 = 11

Critical path is **1-2-5**. ie)  $6+10 = 16$

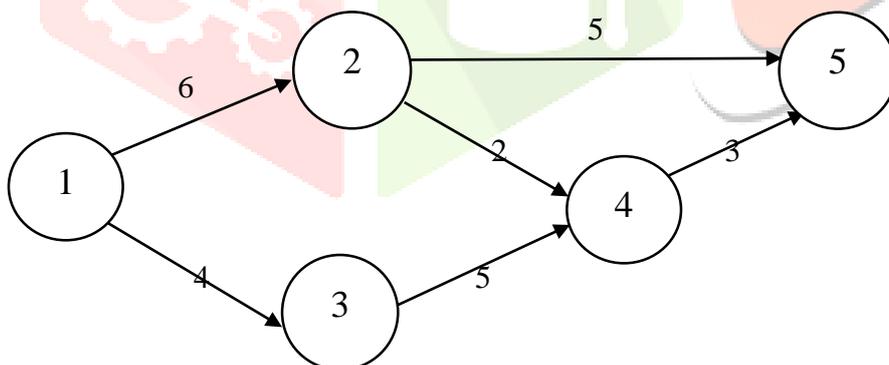
Total cost = Normal cost + Indirect cost + Direct cost

$$= 580 + 100 (16) + 2 (50)$$

$$= 580 + 1600 + 100$$

$$= 2280$$

Iteration 3:



Critical path	Project duration
1-2-5	6+5 = 11
1-3-4-5	4+5+3 = 12
1-2-4-5	6+2+3 = 11

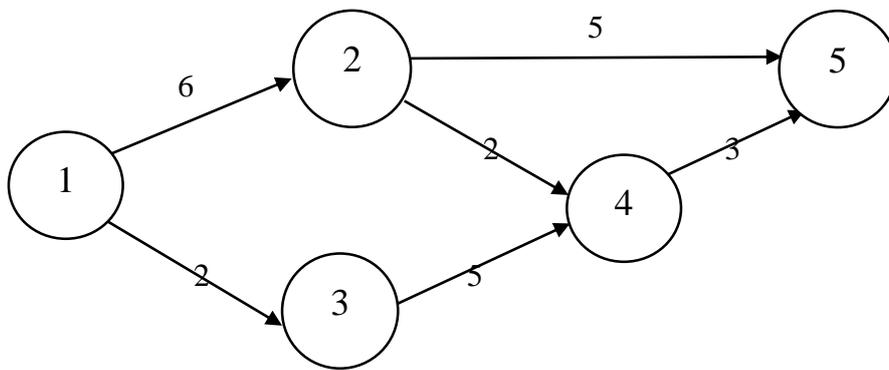
Total cost = Normal cost + Indirect cost + Direct cost

$$= 580 + 100 (12) + 5 (60)$$

$$= 580 + 1200 + 300$$

$$= 2080$$

Iteration 4:



Critical path	Project duration
1-2-5	6+5 = 11
1-3-4-5	5+2+3 = 10
1-2-4-5	6+2+3 = 11

Total cost = Normal cost + Indirect cost + Direct cost

$$= 580 + 100 (11) + 2 (100)$$

$$= 580 + 1100 + 200$$

$$= 1880$$

Least cost is obtained by crashing activity 1-3 by 2 weeks. If further crashing is continued then total cost increases. So optimal project duration is 11 weeks and optimal cost is Rs. 1880.

### Conclusion:

Usage of CPM and PERT can greatly optimize the project duration and helps project managers to identify the plan and schedule the projects. These methods help the project managers to analyze various ways in which the project can be done and select the optimal one where they can reduce the duration and increase the profit. Since employing these methods and having more accurate completion times for tasks, the length of these activities using CPM and PERT could have been at that point. For small-scale Research and Development projects, such operations are closely related to each other, such as the acquisition of critical materials and research activities. This sequence of events may be on a critical course. This fact points out that any delay in procurement could result in a lag not only in the subsequent R&D operations, but also in the completion date of the project. And also I have found the cost slope of the project by which a minimum project duration is obtained along with the minimalized cost which will help project manager to attain maximum profit within a short period of time.

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