Simulation of the buckling of an arch using finite element analysis

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Abstract: This paper illustrates how to set up a model within ELFEN to compute the buckling load for an arch subject to pressure loading. The work consists of a simple arch which is pin-jointed in the middle at both ends. The solution is obtained by carrying out a dynamic analysis using the ELFEN explicit code. Instructions are provided on how to create a 2D model of the arch. The analysis is completed using ELFEN Version 3.0.4(a finite element program for Microsoft Windows NT). The program allows pre-processing, analysis and post-processing stages to be completed within a single application. The program can be used to model a large number of situations including buckling, plastic deformation, forming and stress analysis.

In this paper it is shown that the shape of the buckling depend on buckling load. Also calculated the load factor.

Key words: Finite element analysis, critical load, buckling.

1. introduction

In structural engineering, buckling is the sudden change in shape (deformation) of a structural Colum or the wrinkling of a plate under shear[1].

- Elastic buckling – The result of compressive loading on a prismatic structure that occurs when critical load is within the elastic range (below the load to cause initial yielding).

- Elastic-plastic buckling – The result of compressive loading on a prismatic structure that occurs when the critical load is above the load that produces initial yielding[1].

- Lateral buckling – The result of compressive loading on a prismatic structure that occurs when the critical loads is within the elastic range and which involves deformations in the lateral (transverse) direction, usually associated with prismatic sections such as columns[1].
In this paper the finite element result of buckling load and arch load factor presented. In this paper, the buckling analysis of simple arch which is pin-jointed in the middle at both ends and subjected to an external pressure loading. The arch is model in 2D which removes any twisting buckling modes[2]. Linear Eigen value analysis shows that the lowest in-plane buckling mode is non-ax symmetric, therefore an initial perturbation is applied to the geometry is carried out using an analytical approach as well as the finite element results presented.

2. Geometry

The geometry consists of a simple arch which is pin-jointed in the middle at both ends and subjected to an external pressure loading as shown in fig 1.

3. Theoretical Solution

The theoretical buckling load $P_{cr}$ [1] using the parameters indicated in Fig 1.

$$P_{cr} = \frac{EI}{R^3} \left[\frac{4\pi^2}{a^2} - 1\right]^2 = 4.042 \text{ N/mm}$$

Where

- $E =$ Young’s Modulus
- $I =$ Moment of Inertia = $bd^3/12$
- $R =$ Radius of arch
- $a =$ Angle of arch in radians
- $b =$ thickness of arch (= 1.0m)
- $d =$ depth of arch

And finally, we can rearrange the above equation to solve for $P$, which will be a critical load that describes the onset of buckling for an arch with a pin joined in the middle instead of the load we're interested in the stress, we will divide this load by the cross sectional area of the arch.

$$\sigma_c = \frac{P_{cr}}{A} \quad \ldots (2)$$
5. Finite Element Model
The finite element model is to be input using ELEFEN PROGRAM as shown in fig 2.

![Fig 2 Model of Arch](image)

6. Material Properties
The material properties [3,4] used is shown in the Table 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>2.1 x 105 N/mm²</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Density</td>
<td>7.8 x 10⁻⁶ kg/mm³</td>
</tr>
</tbody>
</table>

![Table 1 Material Properties](image)

7. Finite Element Mesh
A structured mesh of linear elements is shown in fig 3 below:

![Fig 5 Finite Element Mesh](image)

8. Finite Element Results
Figure 4 shows the deformed meshes just before buckling takes place at a critical load of 3.863N/mm² and at various stages after buckling occurred. Figure 5 shows the graph of vertical displacement obtained from the high resolution history, where the buckling point can be clearly seen.

![Fig 4 Finite Element Results](image)

(a) Arc-load factor = 3.765 (Pre-Buckling)
(b) Arc-load factor = 3.726 (Post-Buckling)

(c) Arc-load factor = 2.476

(d) Arc-load factor = -0.916

(e) Arc-load factor = 8

Fig 4 Deformed Shapes at Various Stages of Buckling

Fig 5 Graph Showing Vertical Displacement of Centre of Arch
9. Dissociation

The results of analytical solution are compared and verified with the FEM calculations. The critical buckling loads obtained by the finite element and the analytical methods are in good agreement with each other.

Finally, the deformation shapes are various with stages of buckling also displacement any cares when we move away from the center of the arc.

10. References

3- Mechanics of material by R. C. Hibbler.
4- Mechanics of materials by Bear and Johnston.