



BLIND ADAPTIVE EQUALIZER BASED ON PDF MATCHING FOR A RAYLEIGH TIME VARYING CHANNELS

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Abstract: In this project, We propose a new cost function for adaptive blind equalization based on Quadratic PDF matching for Rayleigh varying channel. The proposed approach is based on fitting the probability density function (PDF) of the equalizer output to a target PDF. The underlying PDF at the equalizer output is estimated by Parzen window technique. The cost function of the proposed technique is minimized by an algorithm belonging to the Stochastic-Gradient Descent family. The performance of the proposed adaptation strategy is benchmarked against CMA blind algorithm for QPSK modulation scheme in terms of ISI reduction.

Keywords- Adaptive Filtering, Blind Equalization, CMA, Quadratic distance, Probability Density Function, Cost function, Parzen Window Estimation.

I. INTRODUCTION

In Digital Communications the affect of Inter Symbol Interference (ISI) is predominant. Amplitude and Delay distortions of the wireless channels as well as multipath propagation of the transmitted signal give rise to ISI. The presence of ISI makes the communication unreliable by causing high error rates at the receiver. Equalization is defined as any signal processing technique that can counter ISI. Equalization plays a key role in digital communication systems. Typically, the physical channel introduces a distortion to the transmitted signal that can make it difficult to recover the original data. Conventional equalization requires transmitting a training sequence that is known at the receiver. This sequence allows the adaptation of the equalizer parameters to minimize some error measurement (typically the mean square error) between the actual equalizer output and the desired response (the training sequence). When a linear filter is used to implement the equalizer, the filter coefficients can be adapted by many adaptive algorithms that can be used to adapt the filter, which minimizes the expectation of square error.

Equalization can be classified into Supervised/Trained and Unsupervised/Blind Equalization. In supervised equalizers a particular training sequence that is available both at the transmitter and receiver in proper synchronism is transmitted for the purpose of initial training of the equalizer's weights. The initial training of the equalizer's weights is accomplished with the help of the Least Mean Square (LMS) algorithm. In blind equalization training signal transmission is not feasible. Resorting to the training sequences results in superior performance of the supervised equalizer. The performance determining is there convergence rate. And this is because of the absence of the training period. In ideal transmission channel ISI doesn't exist. An ideal channel's impulse response is given by

$$h(k) = A\delta(k-n) \quad (1)$$

When a training sequence is not available at the receiver, the problem at hand is named blind equalization. Blind equalization has received a great amount of attention during the last years because of its importance in communication systems. Without a reference sequence, the only knowledge about the transmitted sequence is limited to its probabilistic or statistical properties. Blind equalization algorithms minimize a cost function that is able to indirectly extract the higher order statistics of the signal or the current level of ISI at the equalizer output. Typically, the cost function is minimized by means of stochastic gradient algorithms..

II. STOCHASTIC GRADIENT APPROACH

The stochastic gradient approach uses a tapped delay line filter, or a transversal filter for implementing a linear adaptive equalizer to reduce a cost function of interest, In general a MSE cost function at a given instant k can be expressed as

$$J(k) = E \{e(k)\}^2 \quad (2)$$

Where E represents the statistical expectation operator and $e(k)$ denotes the error at instant k given by

$$e(k) = z(k) - s(k) \quad (3)$$

Where $s(k)$ is the reference signal and $z(k)$ is the equalizer output given by

$$z(k) = w(k)^H x(k) \quad (4)$$

Superscript H represents conjugate transpose (Hermitian). If $x(k), x(k-1), \dots$ represents the channel output at t instants $k, k-1, \dots$ then the equalizer's input vector can be represented as $s(k) = [x(k), \dots, x(k-N+1)]^T$ the equalizer's tap weight vector is given by $w(k) = [w_a(k), w_1(k), \dots, w_{N-1}(k)]^T$, where N gives the number of tap weights and the superscript T represents transpose, the weights update equation of the stochastic gradient approach is given by

$$w(k+1) = w(k) - \alpha x(k) e(k)^* \quad (5)$$

where α is a real valued parameter called the adaptation step size and the subscript * represents operation of complex conjugation. The error given in (3) is the general form of calculating error in stochastic gradient approach. In this family of algorithms each algorithm has its own definition for calculating the error, even then, Eq's. (4) and (5) remain the same except for some minor changes.

III. CONSTANT MODULUS ALGORITHM

Godard's CMA was the very first blind equalization algorithm that could be used in two dimensional digital communication systems. The reference signal in CMA is known as constant modulus. The constant modulus is denoted by R_2 and is given by

$$R_2 = \frac{E[|a(k)|^4]}{E[|a(k)|^2]} \quad (6)$$

Where $a(k)$ represents the transmitted symbol constellation.

The error signal $e_{cma}(k)$ is defined as

$$e_{cma} = z(k) \left[|z(k)| - \frac{R_2}{|z(k)|} \right] \quad (7)$$

Where $z(k)$ is the equalizer output. The cost function is defined as

$$J_{CMA}(k) = E\{[|z(k)| - R_2]^2\} \quad (8)$$

From (7) it is evident that the error function is nonlinear, and Godard has employed a non-convex cost function given by (8). The idea behind CMA is to penalize deviations of the output $z(k)$ from the constant modulus R_2 . The weight update can be obtained by substituting in given below

$$w_{CMA}(k+1) = w_{CMA}(k) - \alpha_{CMA} x(k) e_{CMA}(k)^* \quad (9)$$

From (6), it is obvious that CMA requires knowing the statistics of the transmitted signal constellation prior to equalization.

IV. RAYLEIGH FADING CHANNEL

Rayleigh fading model: Rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium (also called a communications channel) will vary randomly, or fade, according to a Rayleigh distribution.

The Rayleigh fading model is ideally suited to situations where there are large numbers of signal paths and reflections. Rayleigh fading is a model that can be used to describe the form of fading that occurs when multipath propagation exists.

In matlab, a Rayleigh fading channel can be used to create a Rayleigh fading time varying channel.

V. PARZEN WINDOW ESTIMATION

Parzen window method is widely used non-parametric approach to estimate the probability density function $p(x)$ for a specific point $p(x)$ from a sample $p(x_n)$ that doesn't require any knowledge or assumption about the undelined distribution. For Non-parametric estimation of density function, we do not assume any form for density function.

The basic idea for estimating unknown density function is based on the fact that the probability P that a vector x belongs to a region R [1]:

$$P = \int_R p(x) dx \quad (10)$$

It can be rewritten as

$$\int_R p(x) dx (x) V \approx \frac{k}{n}, \quad (11)$$

if we assume a small local region R , a large number of samples n , and k of n falling in R . Suppose that the region R is a d -dimensional hypercube around $x_i \in \mathbb{R}^n$, and let the volume V_n :

$$V_n = h_n^d \quad (12)$$

Where h_n is the length of an edge. Then the window function for this hypercube can be defined by

$$\varphi(u) = \begin{cases} 1, & |u_j| \leq \frac{1}{2} j = 1, 2, \dots, d \\ 0, & \text{else} \end{cases} \quad (13)$$

We simply shift this window function for x_i to determine if x_i belongs to the volume $V_n, \varphi(\frac{x-x_i}{h_n})$, and can compute the number of samples k_n falling in this volume using it:

$$k_n = \sum_{i=1}^n \varphi(\frac{x-x_i}{h_n}) \quad (14)$$

In Parzen window method, therefore, the estimate for density $p_n(x)$ is

$$p_n(x) = \frac{k_n/n}{V_n} = \frac{1}{n} \sum_{i=1}^n \frac{1}{V_n} \varphi(\frac{x-x_i}{h_n}) \quad (15)$$

In order to check how window length effects on $p_n(x)$ we define $\delta_n(x - x_i)$ by $\frac{1}{V_n} \varphi(\frac{x-x_i}{h_n})$ as an approximation of a unit impulse centered at x_i [2] and write $p_n(x)$ [1] by:

$$p_n(x) = \frac{1}{n} \sum_{i=1}^n \delta_n(x - x_i), \quad (16)$$

From this observation, we can infer the relationship between h_n and $p_n(x)$. If h_n is very large, the amplitude of δ_n is relatively small because $V_n = h_n^d$.

VI. QUADRATIC PDF MATCHING METHOD OF EQUALIZATION

In case of CMA the equalizer pushes the pdf of the random variable $y = |y_k|^2$ at its output to $\delta(z - R_2)$ i.e.,

$$f_z(z) = \delta(z - R_2) \quad (17)$$

For $R_2 \neq 0$ the equalizer removes ISI

In case of Quadratic PDF matching method the random variable at the output $y = |y_k|^2$ with unknown PDF $f_z(z)$, and a desired random variable $Q = R_2 + N$, where R_2 is a constant and N is a random variable which accounts for the noise at the output of the equalizer and is assumed to be Gaussian. Because of this the target random variable is a zero mean Gaussian random variable with variance $\sigma^2 = q - R_2$ and is represented by

$$f_N = G_\sigma(q - R_2) \quad (18)$$

As the name implies this method tries to reduce the quadratic or Euclidean distance between the pdf of the random variable at equalizer output and target pdf. Thus, the cost function is

$$D_{QD}(Z||Q) = \int (f_z(z) - G_\sigma(z - R_p))^2 dz \quad (19)$$

where $R_2 = R_p$ for $p=2$

The previous equation after substituting Parzen window estimate of $f_z(z)$, developing the square and integrating modifies into cost function of the form:

$$J_{QD}^p(w) = \frac{1}{N^2} \sum_{j=k+1-N}^k \sum_{i=k+1-N}^k G_\sigma(|y_j|^p - |y_i|^p) - \frac{2}{N} \sum_{j=k+1-N}^k G_\sigma(|y_j|^p - R_p) \quad (20)$$

where $p=2$. Here N indicates the number of samples considered in the window of Parzen window estimation method, for simplicity $N=2$ i.e., only current and past sample are considered.

The minimization of above cost function using an SGD approach yields the following algorithm

$$w_{k+1} = w_k - \mu * (F_1(y_k, y_{k-1}) + F_2(y_k, y_{k-1})) \quad (21)$$

Where μ is step size of the algorithm and

$$F_1 = \frac{1}{2} e^{-\frac{(|y_k|^2 - |y_{k+1}|^2)^2}{2\sigma^2}} (|y_k|^2 - |y_{k+1}|^2) (y_{k-1} x_{k-1}^* - y_k x_k^*) \quad (22)$$

$$F_2 = \sum_{j=k-1}^k e^{-\frac{(|y_k|^2 - |y_{k+1}|^2)^2}{2\sigma^2}} (|y_j|^p - R_p) (y_j x_j^*) \quad (23)$$

VII. IMPLEMENTATION AND RESULTS

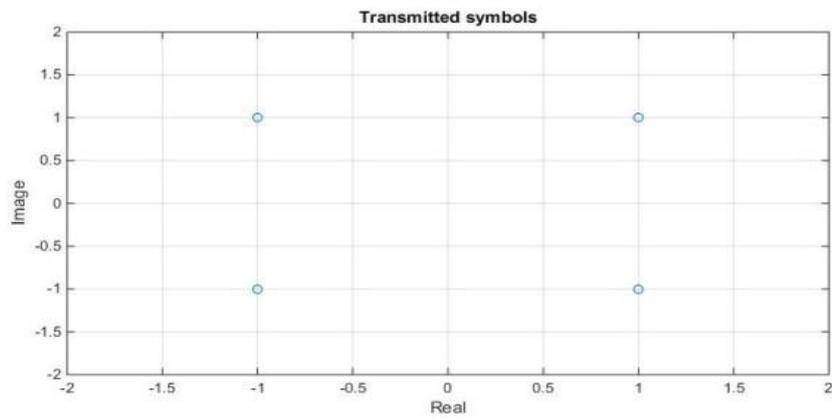


Fig 1 QAM Transmitted symbols

For implementation, We have considered a constellation of 4 symbols (QPSK modulated) and is transmitted as input to the channel as given in figure 1.2. If we consider the transmitted data as 5000, it means we are transmitting each symbol 1250 times.

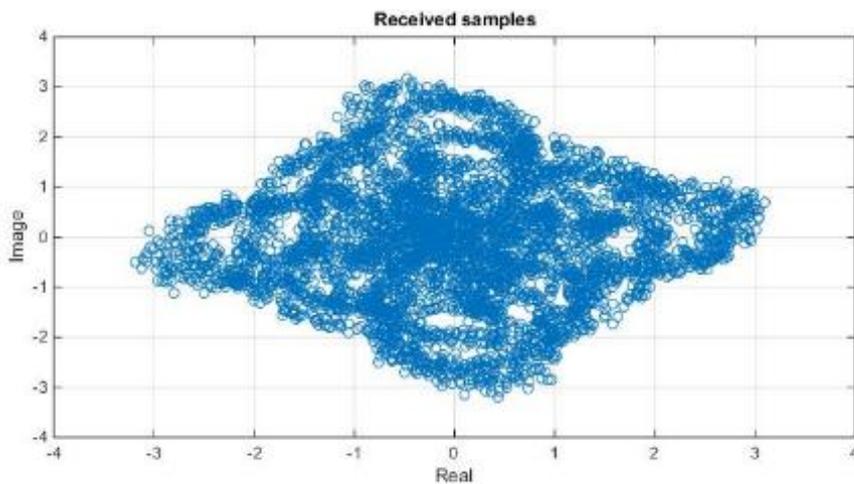


Fig.2 Received samples from Rayleigh channel

The transmitted symbols are sent via a fast time varying Rayleigh channel that results as in fig 1.3. these received symbols are then given as input to the cascaded equaliser.

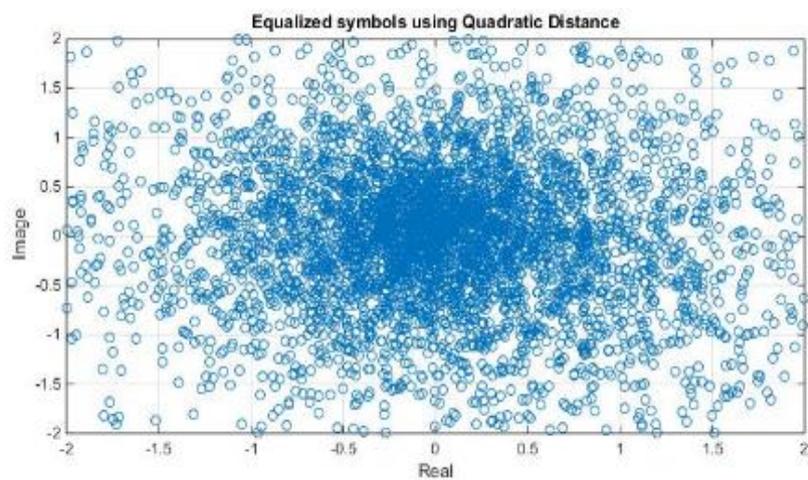


Fig 3 Equalised symbols after quadratic distance.

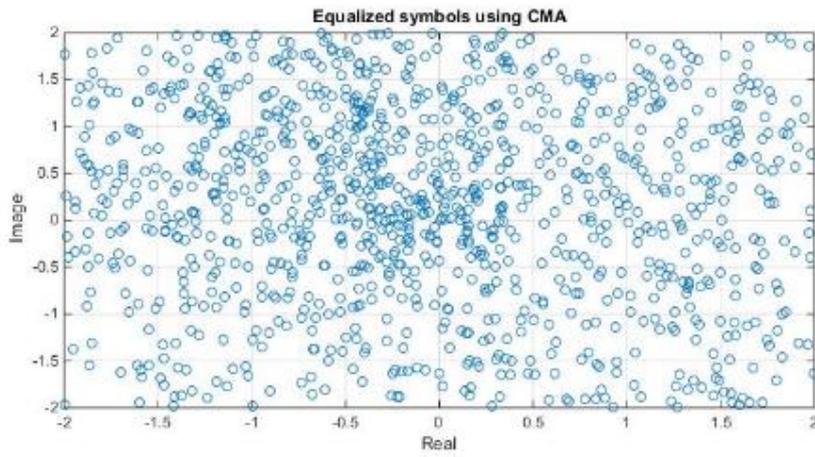


Fig 4 Equalised symbols after CMA

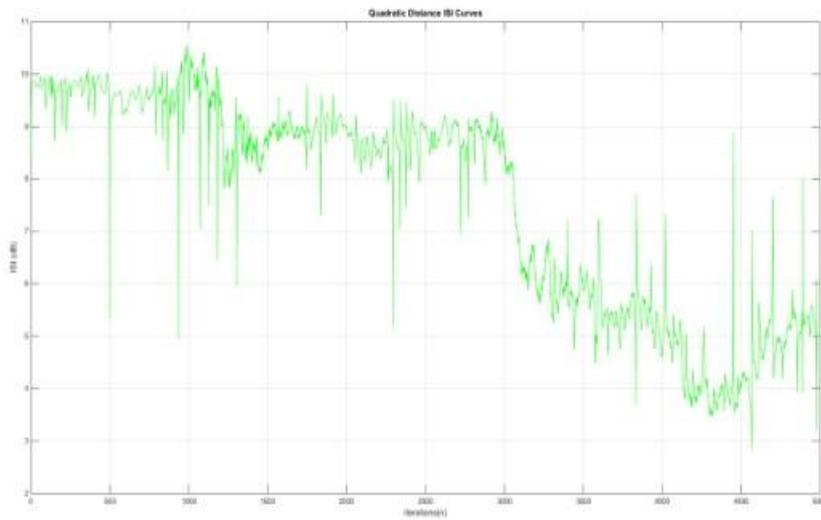


Fig5ISI curve for a Quadratic distance

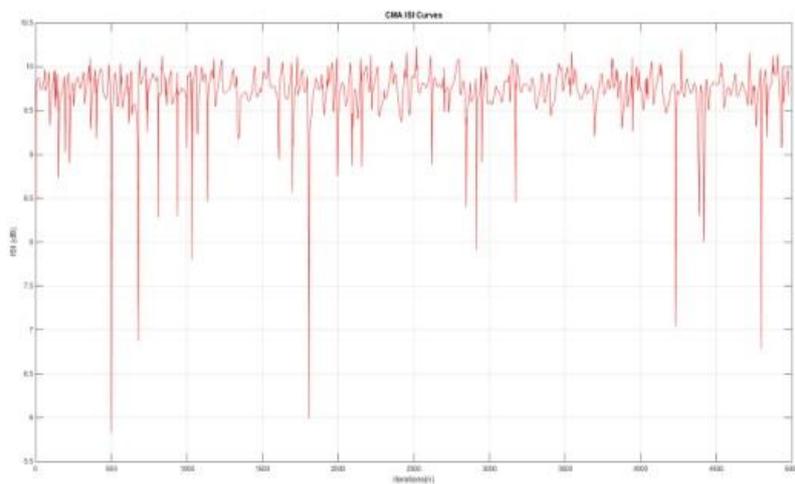


Fig.6 ISI curve for a CMA

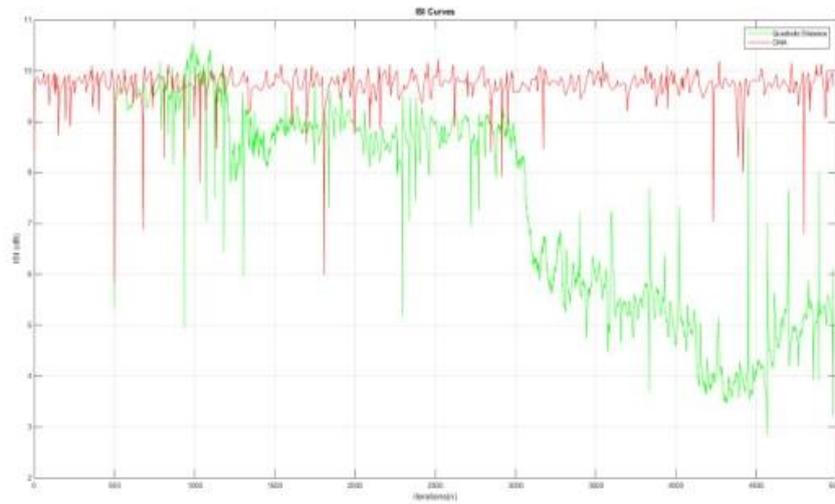


Fig.7 Comparison of ISI curves for a Quadratic distance and CMA

VIII. CONCLUSION

A new algorithm for blind adaptive equalization under fast time-varying channels has been reviewed. The new cost function of this method is based on minimizing the quadratic distance between pdf of equalizer output and the desired pdf, i.e., achieving a correlation between both pdf's. The proposed method shows lesser ISI compared to the classical CMA in constant environments. This approach needs to be improved in regards to convergence and can be thought of being extended to multi user communication.

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