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ON FUZZY SOFT STRONGLY BAIRE SPACES

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Abstract

In this present paper ,we have introduced and studied the concept of FSSNWDS , FSSFCS, FSSSCS, FSSRS, in FSTS . The notion of FSSBS is defined and sundry characterizations and properties are probe in this work . Finally we give an some related examples in this paper.

Keywords:

Fuzzy soft dense sets[FSDS], Fuzzy soft strongly nowhere dense sets[FSSNWDS], Fuzzy soft simply open sets[FSSOPS],FSSBS.

1.Introduction

The concept of fuzzy topological space was introduced by **C.L. Chang** [4] in 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Several Mathematicians have tried all the pivotal concepts of general topology for extension to the fuzzy setting. In 1965, **Lotfi A.Zadeh** introduced the concept of fuzzy sets as a new approach for modeling uncertainties[19].In 1899, **Rene Louis Baire** [2] introduced the concepts of first and second category sets in his doctoral thesis. In classical topology, Baire space, named in honor of Rene Louis Baire, was first introduced in **Bourbaki's** [**3**] Topologie generale Chapter IX. The concepts of Baire spaces have been studied extensively in classical topology in [**5,12,20,21**]. The concept of Baire spaces in fuzzy setting was introduced and studied by **G.Thangaraj and S.Anjalmose** in [15].**Maji,P.K.Roy**,[10] further studied the theory of soft sets and used this theory to solve some decision making problems. The concept of fuzzy soft set is introduced and studied [6-9,12] a more generalized concept which is a combination of fuzzy set and fuzzy soft set. The notion of fuzzy simply open sets by means of fuzzy nowhere denseness of fuzzy boundary sets in fuzzy topological spaces is introduced and studied by **G.Thangaraj and K.Dinakaran** in [**16**]. In this paper the concept of fuzzy soft strongly Baire spaces in FSTS are introduced and studied.

2.Preliminaries

In section 2 we have given some basic definitions and notion are self-contained.

Definition 2.1[6]:

The fuzzy soft set $F_\phi \in FS(U,E)$ is said to be null fuzzy soft set and it is denoted by ϕ, if for all $e\in E$, $F(e)$ is the null fuzzy soft set $\bar{0}$ of U, where $\bar{0}(x) = 0$ for all $x\in U$.

Definition 2.2 [6]:

Let $FE\in FS(U, \mathrm{E})$ and $FE(e) = \bar{1}$ all $e\in E$, where $\bar{1}(x) = 1$ for all $x\in U$.Then F_E is called absolute fuzzy soft set. It is denoted by $\bar{E}$.

Definition 2.3 [6]:

A fuzzy soft set F_A is said to be a fuzzy soft subset of a fuzzy soft set G_B over a common universe U if $A\subseteq B$ and $F_A(e) \subseteq G_B(e)$ for all $e\in A$, *i.e.*, if $\mu^e F_A(x) \leq \mu^e G_B(x)$ for all $x\in U$ and for all $e\in E$ and denoted by $F_A \tilde{\subseteq} G_B$.

Definition 2.4[6]:

Two fuzzy soft sets F_A and G_B over a common universe U are said to be fuzzy soft equal if F_A is a fuzzy soft subset of G_B of G_B is a fuzzy soft subset of F_A.

Definition 2.5[10]:

The union of two fuzzy soft sets F_A and G_B over the common universe U is the fuzzy soft set HC, defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cup \mu^e_{G_B}$ for all $e \in E$, where $C = A \cup B$. Here we write $H_C = F_A \tilde{\vee} G_B$

Definition 2.6[6]:

Let F_A and G_B be two fuzzy soft set, then the intersection of F_A and G_B is a fuzzy soft set H_C, defined by $H(e) = \mu^e_{F_C} = \mu^e_{F_A} \cap \mu^e_{G_B}$ for all $e \in E$, where $C = A \cap B$. Here we write $H_C = F_A \tilde{\wedge} G_B$.

Lemma 2.1[1]:

For a family A = $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X, $\vee(\text{cl}(\lambda_\alpha)) \leq \text{cl}(\vee(\{\lambda_\alpha\})$. In case A is a finite set, $\vee(\text{cl}(\lambda_\alpha)) = \text{cl}(\vee((\lambda_\alpha))$. Also $\vee(\text{int}(\lambda_\alpha)) \leq \text{int}(\vee(\lambda_\alpha))$.

Definition 2.7[18]:

Let $F_A \in FS(U, E)$ be a fuzzy soft set. Then the complement of F_A, denoted by F_A^C, defined by

$$F_A^C(e) = \begin{cases} \bar{1} - \mu^e_{F_A}, & if \ e \in A \\ \bar{1}, & if\, e \notin A \end{cases}$$

Definition 2.8[14]:

Let ψ be the collection of fuzzy soft sets over U. Then ψ is called a fuzzy soft topology on U if ψ satisfies the following axioms:

(i) $\phi, \bar{E}$ belong to ψ.

(ii) The union of any number of fuzzy soft sets in ψ belongs to ψ.

(iii) The intersection of any two fuzzy soft sets ψ belongs to ψ. The triplet (U, E, ψ) is called a fuzzy soft topological space over U. The members of ψ are called fuzzy soft open sets in U and complements of them are called fuzzy soft closed sets in U.

Definition 2.9[14]:

The union of all fuzzy soft open subsets of F_A over (U, E, ψ) is called the interior of F_A and is denoted by $int^{fs}(F_A)$.

Proposition 2.1[14]:

$int^{fs}(F_A \tilde{\wedge} G_B) = int^{fs}(F_A) \tilde{\wedge} int^{fs}(G_B)$.

Definition 2.10 [14]:

Let $F_A \in FS(U,E)$ be a fuzzy soft set. Then the intersection of all closed sets, each containing F_A, is called the closure of F_A and is denoted by $cl^{fs}(F_A)$.

Remarks 2.11 [14]:

(1) For any fuzzy soft set F_A in a fuzzy soft topological space (U, E, ψ), it is easy to see that $((F_A))^c = int^{fs}(F_A^C)$ and $(int^{fs}(F_A))^c = cl^{fs}(F_A^C)$.

(2) For any fuzzy soft F_A subset of a fuzzy soft topological space (U, E, ψ) we define the fuzzy soft subspace topology on F_A by $K_D \epsilon \psi_{F_A}$ if $K_D = F_A \tilde{\wedge} G_B$ for some $G_B \epsilon \psi$.

(3) For any fuzzy soft H_C in F_A fuzzy soft subspace of a fuzzy soft topological space, we denote to the interior and closure of H_C in F_A by $int^{fs}_{F_A}(H_C)$ and $cl^{fs}_{F_A}(H_C)$, respectively.

Definition2.12[13]:

A fuzzy soft set F_A in a FSTS (U, E, ψ) is called a FSNWD set if there exist no non-zero fuzzy soft open set G_B in (U, E, ψ) such that $G_B < cl^{fs}(F_A)$. ie), $int^{fs} cl^{fs}(F_A) = 0$.

Definition 2.13[13]

A fuzzy soft set F_A in a FSTS (U, E, ψ) is called fuzzy soft dense if there exist no fuzzy soft closed set G_B in (U, E, ψ) such that $F_A < G_B < 1$. ie) $cl^{fs}(F_A) = 1$.

Definition 2.14[13]:

Let (U, E, ψ) be a fuzzy soft topology . A fuzzy soft set F_A in (U, E, ψ) is called fuzzy soft first category . If $F_A = \vee_{i=1}^{\infty}(F_{A_i})$ where (F_{A_i})'s are fuzzy soft nowhere dense sets in (U, E, ψ). Any other fuzzy soft set in (U, E, ψ) is said to be of fuzzy soft second category.

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Definition3.1:

A FSS F_A defined on U is called a FSSNWDS in FSTS (U, E,ψ).If $F_A \wedge (1-F_A)$ is a FSNWDS in (U,E,ψ). ie) F_A is a FSSNWDS in (U,E,ψ).If $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 0$ in (U,E,ψ).

Example3.1:

Let U = {a,b,c }. The fuzzy soft sets F_A, G_B & H_C are defined on U as follows:

F_A: U ⟶ [0,1] is defined as $F_A(a) = 0.4$, $F_B(b) = 0.5$, $F_B(c) = 0.3$

G_B : U ⟶ [0,1] is defined as $G_B(a) = 0.5$, $G_B(b) = 0.5$, $G_B(C) = 0.3$

H_C :U ⟶ [0,1] is defined as $H_C(a) = 0.4$, $H_C(b) = 0.4$, $H_C(c) = 0.4$

Clearly $\psi = \{0, F_A, G_B, H_C, G_B \vee H_C, H_C \vee F_A, G_B \wedge H_C, 1\}$ is a FST on U

Hence $int^{fs}cl^{fs}(F_A) = int^{fs}(1-G_B \vee H_C) = 0$ in (U,E,ψ) and hence (U,E,ψ) is a FSSNWDS .

Proposition 3.1:

If F_A is a FSNWDS in a FSTS (U,E,ψ), after that F_A is a FSSNWDS in (U,E,ψ).

Proof:

Let F_A be a FSNWDS in (U,E,ψ). Subsequently $int^{fs}cl^{fs}(F_A) = 0$, in (U,E,ψ). since $[F_A \wedge (1-F_A)] \leq F_A$, in (U,E,$\psi$), $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq int^{fs}cl^{fs}(F_A)$ and hence $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq 0$ .ie), $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 0$. Hence F_A is a FSSNWDS in (U,E,ψ).

Remark 3.1:

A FSSNWDS in a FSTS (U,E,ψ) be a not required FSNWDS in (U,E,ψ). For in example 3.1, H_C is a FSSNWDS , but not a FSNWDS in (U,E,ψ).

Proposition 3.2:

If $int^{fs}cl^{fs}(F_A)$ is a fuzzy soft dense set, for a fuzzy soft set F_A defined on U, in a FSTS (U,E,ψ), then F_A is a FSSNWDS in (U,E,ψ).

Proof:

Suppose that $int^{fs}(F_A)$ is a fuzzy soft dense set in (U,E,ψ). Then $cl^{fs}[int^{fs}(F_A)] = 1$ in (U,E,ψ) and $1 - cl^{fs}[int^{fs}(F_A)] = 0$. ⇒ $int^{fs}cl^{fs}(1- F_A) = 0$ in (U,E,ψ). since $F_A \wedge (1-F_A) \leq (1-F_A)$, $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq int^{fs}cl^{fs}(1- F_A)$ and hence $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq 0$. i.e)$int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 0$. Hence F_A is a FSSNWDS in (U,E,ψ).

Proposition 3.3:

If $1-F_A$ is a FSNWDS in a FSTS (U,E,ψ), then F_A is a FSSNWDS in (U,E,ψ).

Proof:

Suppose that $1-F_A$ is a FSNWDS in (U,E,ψ). Then, $int^{fs}[cl^{fs}(1-F_A)] = 0$ in (U,E,ψ).Since $F_A \wedge (1-F_A) \leq 1-F_A$, $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq int^{fs}[cl^{fs}(1- F_A)]$ and hence $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq 0$. ie) $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 0$ in (U,E,ψ) and hence F_A is a FSSNWDS in (U,E,ψ).

Proposition 3.4:

If $cl^{fs}[int^{fs}(1- F_A)] = 1$, for a fuzzy soft set F_A defined on U in a FSTS (U,E,ψ), then F_A is a FSSNWDS in (U,E,ψ).

Proof:

Suppose that $cl^{fs}[int^{fs}(1-F_A)] = 1$ in (U,E,ψ). Then $1- cl^{fs}[int^{fs}(1-F_A)] = 0$ and $1-\{1-int^{fs}[cl^{fs}(F_A)]\} = 0$ ⇒ $int^{fs}[cl^{fs}(F_A)] = 0$ in (U,E,ψ). Thus F_A is a FSNWDS in (U,E,ψ). Then by proposition 3.1, F_A is a FSSNWDS in (U,E,ψ).

Proposition 3.5:

If F_A is a FSSNWDS in a FSTS(U,E,ψ), then $1- F_A$ is also a FSSNWDS in (U,E,ψ).

Proof:

Let F_A be a FSSNWDS in (U,E,ψ). Then $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 0$ in (U,E,ψ). Now $int^{fs}cl^{fs}\{(1-F_A) \wedge [1-(1-F_A)]\} = int^{fs}cl^{fs}([(1-F_A) \wedge F_A]$ and hence $int^{fs}cl^{fs}\{(1- F_A) \wedge [1- (1-F_A)]\} = 0$. ⇒ $1- F_A$ is a FSSNWDS in (U,E,ψ).

Proposition 3.6:

If F_A is a FSSNWDS in a FSTS(U,E,ψ), then 1- F_A is a FSSNWDS in (U,E,ψ).

Proof:

Let $F_A$be a FSNWDS in (U,E,ψ). Then, by proposition 3.1, F_A is a FSSNWDS in (U,E,ψ).and by proposition 3.5, 1- F_A is a FSSNWDS in (U,E,ψ).

Proposition 3.7:

If F_A is a FSSNWDS in a FSTS (U,E,ψ), then $cl^{fs}(F_A) \vee cl^{fs}(1-F_A) = 1$, in (U,E,$\psi$).

Proof:

Let $F_A$be a FSSNWDS in (U,E,ψ). Then $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 0$ in (U,E,ψ). Now $1\text{-}int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 1$ and hence $cl^{fs}int^{fs}[\,1\text{-}\{F_A \wedge (1-F_A)\}] = 1$, in (U,E,ψ). But , $cl^{fs}int^{fs}[1\text{-}\{F_A \wedge (1-F_A)\}] \leq cl^{fs}[\,1\text{-}\{F_A \wedge (1-F_A)\}]$ implies that $1 \leq cl^{fs}[\,1\text{-}\{F_A \wedge (1-F_A)\}]$. Thus , $cl^{fs}[\,1\text{-}\{F_A \wedge (1-F_A)\}] = 1$, this implies that $cl^{fs}\,[(1-F_A\,) \vee F_A] = 1$ in (U,E,ψ). But by lemma 2.1, $cl^{fs}\,[(1-F_A) \vee F_A] = cl^{fs}(1-F_A\,) \vee cl^{fs}(F_A)$. Hence $cl^{fs}(F_A) \vee cl^{fs}(1-F_A\,) = 1$, in (U,E,ψ).

Proposition3.8:

If F_A is a FSS[fuzzy soft set FSS] defined on U such that $int^{fs}[fsbd(F_A)] = 0$ in a FSTS (U,E,ψ), then F_A is a FSSNWDS in(U,E,ψ).

Proof:

Let $F_A$be a FSS defined on U .such that $int^{fs}[fsbd(F_A)] = 0$ in (U,E,ψ). since $fsbd(F_A) = cl^{fs}(F_A) \wedge cl^{fs}(1-F_A)$ and $cl^{fs}(F_A) \wedge cl^{fs}(1-F_A) \geq cl^{fs}[F_A \wedge (1-F_A)]$, we have $fsbd(F_A) \geq cl^{fs}[F_A \wedge (1-F_A)]$ and hence $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq int^{fs}[fsbd(F_A)]$ in (U,E,ψ). then $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] \leq 0$. ie) $int^{fs}cl^{fs}[F_A \wedge (1-F_A)] = 0$ and hence F_A is a FSSNWDS in (U,E,ψ).

Proposition 3.9:

If F_A is a FS simply open set in a FSTS (U,E,ψ), then F_A is a FSSNWDS in(U,E,ψ).

Proof:

Let F_A be a FS simply open set in (U,E,ψ). Then $int^{fs}cl^{fs}\,[fsbd(F_A)] = 0$, in (U,E,$\psi$). But $int^{fs}[fsbd(F_A)] \leq int^{fs}cl^{fs}[fsbd(F_A)] \Rightarrow int^{fs}[fsbd(F_A)] = 0$ in (U,E,ψ). Then, by proposition 3.8, F_A is a FSSNWDS in(U,E,ψ).

Proposition 3.10:

If F_A is a FSCS with $int^{fs}(F_A) = 0$ in a FSTS (U,E,ψ), then F_A is aFSSNWDS in(U,E,ψ).

Proof:

Let $F_A$be a FS closed set with $int^{fs}(F_A) = 0$ in (U,E,ψ).Then,$int^{fs}cl^{fs}[cl^{fs}(F_A) \wedge cl^{fs}(1-F_A)\,] = int^{fs}cl^{fs}\,[F_A \wedge (1-int^{fs}\,F_A)] = int^{fs}cl^{fs}\,[F_A \wedge 1] = int^{fs}cl^{fs}\,(F_A) = int^{fs}(F_A) = 0$, and hence F_A is a fuzzy soft simply open set in (U,E,ψ).By prop 3.9,F_A is a FSNWDS in (U,E,ψ).

Proposition 3.11:

If F_A is a FS open and fuzzy soft dense set in a FSTS (U,E,ψ), then F_A is a is a FSSNWDS in (U,E,ψ).

Proof:

Let $F_A$be a fuzzy soft open and fuzzy soft dense set in (U,E,ψ). Then 1-$F_A$is a fuzzy soft closed set with $int^{fs}\,(1\text{-}F_A) = 1\text{-}cl^{fs}(F_A) = 1\text{-}1 = 0$ in (U,E,ψ). Then ,by proposition 3.10, 1-$F_A$is a FSSNWDS in (U,E,ψ) and by proposition 3.5,1-(1-F_A) is a FSSNWDS in (U,E,ψ) and thus F_A is a FSSNWDS in (U,E,ψ).

Theorem 3.1[17]:

If F_A is a FSNWDS in a FSTS (U,E,ψ) after that $cl^{fs}\,(F_A)$ is a FSSNWDS in (U,E,ψ).

Proposition 3.12:

If F_A is a FSNWDS in a FSTS (U,E,ψ), then $cl^{fs}(F_A)$ is a FSSOPS in (U,E,ψ).

Proof:

Let $F_A$be a FSNWDS in (U,E,ψ). Then by theorem 3.1, $cl^{fs}(F_A)$ is a fuzzy soft simply open set in (U,E,ψ) and then proposition 3.9, $cl^{fs}(F_A)$ is a FSSNWDS in (U,E,ψ).

Definition 3.2:

A FSS F_A in a FSTS (U,E,ψ) is called a FSS first category set if $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ). Any other FSS in (U,E,ψ) is said to be a FSS second category set in (U,E,ψ).

Definition 3.3:

If F_A is a FSS first category set in a FSTS (U,E,ψ), then $1-F_A$ is a fuzzy soft strongly residual set in (U,E,ψ).

Definition 3.4:

A FSTS (U,E,ψ) is called a FSS first category set, if the FSS 1_x is a FSS first category set in (U,E,ψ). ie), $1_x = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ). otherwise (U,E,ψ) will be called a FSS second category set.

Proposition 3.13

If F_A is a fuzzy soft first category set in a FSTS (U,E,ψ). Then F_A is a FSS first category set in (U,E,ψ).

Proof:

Let F_A be a fuzzy soft first category set in (U,E,ψ). Then $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSNWDS in (U,E,ψ). By proposition 3.1, the FSNWDS. Where (F_{A_i})'s are FSSNWDS in (U,E,ψ) and hence $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ). $\Rightarrow F_A$ is a FSS first category set in (U,E,ψ).

Proposition 3.14

If $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft closed set with $int^{fs}(F_{A_i}) = 0$ in (U,E,ψ), then F_A is a FSS first category set in (U,E,ψ).

Proof:

Suppose that $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where $1-F_{A_i} \in T$ and $int^{fs}(F_{A_i}) = 0$ in (U,E,ψ). Now by proposition 3.10, the fuzzy soft closed sets (F_{A_i})'s with $int^{fs}(F_{A_i}) = 0$, are FSSNWDS in (U,E,ψ) and hence $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ),$\Rightarrow F_A$ is a FSS first category set in (U,E,ψ).

Proposition 3.15

If F_A is a fuzzy soft F_{σ} – set such that $int^{fs}(F_A) = 0$ in (U,E,ψ), then F_A is a FSSFCS in (U,E,ψ).

Proof:

Let F_A be a fuzzy soft F_{σ} – set in (U,E,ψ). Then $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSCS in (U,E,ψ). By lemma 2.2, $\vee_{i=1}^{\infty} int^{fs}(F_{A_i}) \leq int^{fs}\{\vee_{i=1}^{\infty}(F_{A_i})\}$, in (U,E,$\psi$). $\Rightarrow \vee_{i=1}^{\infty} int^{fs}(F_{A_i}) \leq int^{fs}\{\vee_{i=1}^{\infty}(F_{A_i})\} = int^{fs}(F_A)$. since $int^{fs}(F_A)= 0$, $\vee_{i=1}^{\infty} int^{fs}(F_{A_i}) = 0$ in (U,E,ψ). $\Rightarrow int^{fs}(F_{A_i}) = 0$, thus (F_{A_i})'s are FSCS with $int^{fs}(F_{A_i}) = 0$, in (U,E,ψ). Then by prop 3.10, (F_{A_i})'s areFSSNWDS in (U,E,ψ). Hence $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ). $\Rightarrow F_A$ is a FSSFCS in (U,E,ψ).

Proposition 3.16:

If F_A is a FSσ –NWDS in a FSTS (U,E,ψ) then F_A is a FSSFCS in (U,E,ψ).

Proof:

Let F_A be a FS σ –NWDS in (U,E,ψ). Then F_A is a fuzzy soft F_{σ} – set in (U,E,ψ). such that $int^{fs}(F_A) = 0$. Then by proposition 3.15, F_A is a FSSFCS in (U,E,ψ).

Proposition 3.17:

If $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft simply open sets in a FSTS (U,E,ψ), then F_A is a FSSFCS in (U,E,ψ).

Proof:

Suppose that $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft simply open sets in (U,E,ψ). By proposition 3.9, the fuzzy soft simply open set (F_{A_i})'s are FSSNWDS in (U,E,ψ) and hence F_A is a FSSFCS in (U,E,ψ).

Proposition 3.18:

If $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where $int^{fs}[fsbd(F_{A_i})] = 0$ in (U,E,ψ), then F_A is a FSSFCS in (U,E,ψ).

Proof:

The proof follows from proposition 3.8.

Proposition 3.19:

If $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft open and fuzzy soft dense set in a FSTS (U,E,ψ), then F_A is a FSSFCS in (U,E,ψ).

Proof:

The proof follows from proposition 3.11.

Proposition 3.20:

If F_A is FSFCS in a FSTS (U,E,ψ), then there exist a FSSFCS G_B in (U,E,ψ) such that $cl^{fs}(F_A) \geq G_B$.

Proof:

Let F_A be a FSFCS in (U,E,ψ). Then $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft nowhere dense set in (U,E,ψ). By proposition 3.12,{ $cl^{fs}(F_{A_i})$}'s are FSSNWDS in (U,E,ψ). Then $\vee_{i=1}^{\infty} cl^{fs}(F_{A_i})$ is a FSSFCS in (U,E,ψ). Let $G_B = \vee_{i=1}^{\infty} cl^{fs}(F_{A_i})$. Now $\vee_{i=1}^{\infty} cl^{fs}(F_{A_i}) \leq cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] \Rightarrow G_B \leq cl^{fs}(F_A)$, in (U,E,ψ).

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Definition 4.1:

A FSTS (U,E,ψ) is called a FSSBS if $cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$ where (F_{A_i})'s are FSSNWDS in (U,E,ψ).

Example 4.1.1:

Let U = {a,b,c}. The fuzzy soft sets F_A, G_B & H_C are defined on U as follows:

F_A: U ⟶ [0,1] is defined as F_A (a) =0.2, F_B(b) = 0.5, F_B(c) = 0.4

G_B : U ⟶ [0,1] is defined as G_B(a) = 0.4, G_B(b) = 0.5, G_B(C) =0.2

H_C :U ⟶ [0,1] is defined as H_C(a) =0.5, H_C(b) = 0.3, H_C(c) = 0.3

Then T ={0, F_A, G_B, H_C, $F_A \vee G_B$, $G_B \vee H_C$, $H_C \vee F_A$, $F_A \wedge G_B$, $G_B \wedge H_C$, $H_C \wedge F_A$, 1} is a fuzzy topology on U.

The fuzzy strongly nowhere dense set are in (U,E,ψ) are{1- F_A,1- G_B , 1-H_C,1-$F_A \vee G_B$,1- $G_B \vee H_C$,1- $H_C \vee F_A$, 1-$F_A \wedge G_B$, 1-$G_B \wedge H_C$,1-$H_C \wedge F_A$}. $int^{fs}cl^{fs}(1-H_C \wedge F_A)$=$intcl^{fs}(\vee_{i=1}^{\infty}(1-H_C \wedge F_A) \vee (H_C \wedge F_A)$= $int^{fs}cl^{fs}(1-H_C \wedge F_A)$ =0 .Hence (U,E,ψ) is a fuzzy soft strongly Baire space.

Proposition 4.1:

Let (U,E,ψ) be a FSTS .Then, the following are equivalent.

(i)(U,E,ψ) is a FSSBS.

(ii)$cl^{fs}(F_A)$ = 1, for each FSSFCS F_A in (U,E,ψ).

(iii) $int^{fs}(G_B)$ = 0, for each fuzzy soft strongly residual set G_B in (U,E,ψ).

Proof:

(i)⇒(ii)

Let F_A be a FSSFCS in (U,E,ψ). Then $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft strongly nowhere dense set in (U,E,ψ). since (U,E,ψ) is a FSSBS , $cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$, and hence $cl^{fs}(F_A) = 1$, in (U,E,ψ).

(ii)⇒(iii)

Let G_B be a FSSRS in (U,E,ψ). Then 1- G_B is a FSSFCS in (U,E,ψ). By hypothesis , $cl^{fs}(1-G_B) = 1$ in (U,E,ψ). Then 1- $int^{fs}(G_B) = cl^{fs}(1-G_B)$ = 1 and hence $int(G_B) = 0$ in (U,E,ψ).

(iii)⇒(i)

Let F_A be a FSSFCS in (U,E,ψ). Then $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are FSSNWDS in (U,E,ψ). Since F_A is a FSSFCS in (U,E,ψ), 1-F_A is a FSS residual set in (U,E,ψ). By hypothesis , $int^{fs}(1-F_A) = 0$ in (U,E,ψ). Now $1-cl^{fs}(F_A) = int^{fs}(1-F_A) = 0 \Rightarrow cl^{fs}(F_A) = 1$ and then $cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$, where (F_{A_i})'s are FSSNWDS in $(U,E,\psi) \Rightarrow (U,E,\psi)$ is a FSSBS.

Proposition 4.2:

If (U,E,ψ) is a FSSBS, then

(i)$cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$, where $1- F_{A_i} \in T$ and $int^{fs}(F_{A_i}) = 0$ in (U,E,ψ).

(ii)$cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$,where (F_{A_i})'s are fuzzy soft simply open sets in (U,E,ψ).

(iii)$cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i}] = 1$, where $int^{fs}[fsbd(F_{A_i})] = 0$ in (U,E,ψ).

Proof:

(i)Let $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft closed sets with $int^{fs}(F_{A_i}) = 0$. Then , by proposition 3.14, F_A is a FSSFCS in (U,E,ψ). Since (U,E,ψ) is a FSSBS , by proposition 4.1, $cl^{fs}(F_A) = 1$ in (U,E,ψ). Thus $cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$, where $1- F_{A_i} \in T$ and $int^{fs}(F_{A_i}) = 0$.

(ii) Let $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft simply open sets in (U,E,ψ). Then , by prop 3.17, F_A is a FSSFCS in (U,E,ψ). since (U,E,ψ) is a FSSBS, by proposition 4.1, $cl^{fs}(F_A) = 1$ in (U,E,ψ). Thus $cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$, where (F_{A_i})'s are fuzzy soft simply open sets in a FSSBS (U,E,ψ).

(iii)Let $F_A = \vee_{i=1}^{\infty}(F_{A_i})$, where (F_{A_i})'s are fuzzy soft sets on (U) with $int^{fs}[fsbd(F_{A_i})] = 0$ in (U,E,ψ). Then by prop 3.18, F_A is a FSSFCS in (U,E,ψ). Since (U,E,ψ) is a FSSBS, by prop 4.1, $cl^{fs}(F_A) = 1$ in (U,E,ψ). Thus $cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$, where $int^{fs}[fsbd(F_{A_i})] = 0$ in (U,E,ψ).

Proposition 4.3:

If each fuzzy soft open sets is a FSDS in a FSTS (U,E,ψ),then (U,E,ψ) is a FSSBS.

Proof:

Let (F_{A_i})'s are fuzzy soft open sets in (U,E,ψ). By hypothesis (F_{A_i})'s are fuzzy soft dense sets in (U,E,ψ). Then (F_{A_i})'s are fuzzy soft open and fuzzy soft dense sets in (U,E,ψ). Then by prop 3.11, (F_{A_i})'s are FSSNWDS in (U,E,ψ). Let $F_A = \vee_{i=1}^{\infty}(F_{A_i})$. Then F_A is a FSSFCS in (U,E,ψ). Now $cl^{fs}(F_A) = cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] \geq \vee_{i=1}^{\infty} cl^{fs}(F_{A_i}) = \vee_{i=1}^{\infty}(1) = 1$ ie) $cl^{fs}(F_A) = 1$ in (U,E,ψ). Then, by prop 4.1, $(U,E,\psi)$is a FSSBS.

Proposition 4.4:

If F_A is a FSFCS in a FSSBS, then F_A is a FSDS in (U,E,ψ).

Proof:

Let F_A be a FSFCS in (U,E,ψ). By prop 3.17, there exist a FSSFCS G_B in (U,E,ψ). such that $cl^{fs}(F_A) \geq G_B$. Then $cl^{fs}[cl^{fs}(F_A)] \geq cl^{fs}(G_B)$. Since (U,E,ψ) is a FSSBS by prop 4.1, for the FSSFCS G_B in (U,E,ψ), $cl^{fs}(G_B) = 1$. Then $cl^{fs}(F_A) \geq 1$. Hence F_A is a FSDS in (U,E,ψ).

Proposition 4.5:

If (F_{A_i})'s (i=1to∞) are fuzzy soft simply open sets in a FSSBS, then (F_{A_i})'s are not FSDS in (U,E,ψ).

Proof:

Let (F_{A_i})'s (i=1to∞) be fuzzy soft simply open sets in (U,E,ψ).Suppose that $F_A = \vee_{i=1}^{\infty}(F_{A_i})$ .Then by prop 3.17, F_A is a FSSFCS in (U,E,ψ).Since (U,E,ψ) is a FSSBS, by prop 4.1, $cl^{fs}(F_A) = 1$ in (U,E,ψ). Then $cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] = 1$ in (U,E,ψ). But $\vee_{i=1}^{\infty} cl^{fs}(F_{A_i}) < cl^{fs}[\vee_{i=1}^{\infty}(F_{A_i})] \Rightarrow \vee_{i=1}^{\infty} cl^{fs}(F_{A_i}) \neq 1$ in (U,E,ψ) and hence $cl^{fs}(F_{A_i}) \neq 1$. Thus the fuzzy soft simply open sets (F_{A_i})'s are not FSDS in (U,E,ψ).

Proposition 4.6:

If $F_A$is a FSFCS in a FSSBS , then F_A is a FSDS in (U,E,ψ).

Proof:

Let F_A be a FSFCS in (U,E,ψ). By prop 3.20 , there exist a FSSFCS G_B in (U,E,ψ). Such that $cl^{fs}(F_A) \geq G_B$.Then $cl^{fs}[cl^{fs}(F_A)] \geq cl^{fs}(G_B)$. Since (U,E,ψ) is a FSSBS by prop 4.1, for the FSSFCS G_B in (U,E,ψ), $cl^{fs}(G_B)=1$. Then $cl^{fs}(F_A) \geq 1$. ie) $cl^{fs}(F_A) = 1$. Hence F_A is a FSDS in (U,E,ψ).

Proposition 4.7:

If H_C is a FSRS in a FSSBS , then $int^{fs}(H_E) = 0$ in (U,E,ψ).

Proof:

Let H_C be a FS residual set in (U,E,ψ). Then $1 – H_C$is a FSFCS in (U,E,ψ). since (U,E,ψ) is a FSSBS, by prop 4.6, $cl^{fs}(1-H_C) = 1$ in (U,E,ψ). Then $1 – int^{fs}(H_C) = 1$ and hence $int^{fs}(H_C) = 0$ in (U,E,ψ).

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