IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Coloring of Bull Graphs and related graphs

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ABSTRACT: The Bull graph is a graph with 5 vertices and 5 edges consisting of a triangle with two disjoint pendant edges. In this paper, we investigate Proper vertex colorings of Bull graph and some of its related graphs such as Middle graph, Total graph, Splitting graph, Degree splitting graph, Shadow graph and Litact graph of the Bull graph.

Key Words: Graph, Proper Coloring, Bull graph

1. INTRODUCTION

Graph coloring (Proper Coloring) take a major stage in Graph Theory since the advent of the famous four color conjecture. Several variations of graph coloring were investigated [5] and still new types of coloring are available such as σ −(Sigma) coloring [4] and Roman coloring [6, 7].

The Bull graph is a planar undirected graph with five vertices and five edges in the form of a triangle with two disjoint pendant edges. There are three variants of Bull graph (Figure.1). The Bull graph was introduced by Weisstein [2]. Although the Bull-free graphs were studied [2] and Labeling of Bull graphs was also studied [3]. But there are no study related to coloring of Bull graph and related graphs. Suresh Kumar and Preethi K Pillai [8, 9] initiated a study on the sigma colorings and Roman colorings of these graphs. In this Paper, we initiate a study on the Proper (vertex) coloring of Bull graph and its related graphs. We recall the definitions of the Bull graph related graphs.

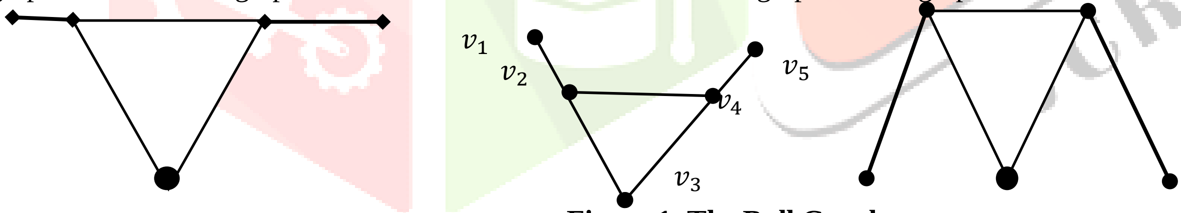

Figure.1. The Bull Graph

In this Paper, we consider the proper coloring for the Bull graph and related graphs. By a graph here we mean a finite undirected graph without loops and parallel edges. For the terms and definitions not explicitly defined here, reader may refer [10].

Definition. 1.1. A (proper) vertex coloring of a graph G with vertex set V(G) and edge set E(G) is a an assignment of colors to V(G) such that adjacent vertices have distinct colors The least positive integer k for which G has a k colouring is called the chromatic number of G and is denoted by $\chi(G)$.

Definition. 1.2. The Middle graph M(G) of a graph G is the graph whose vertex set is V(G)∪E(G) and in which two vertices of M(G) are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge of G incident to it.

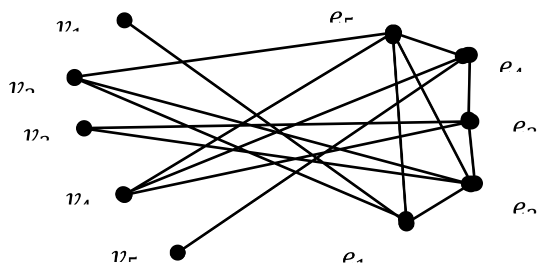

Figure.2. Middle graph of the Bull Graph

Definition. 1.3. The Total graph T(G) of a graph G is the graph whose vertex set is V(G)∪E(G) and in which two vertices of T(G) are adjacent whenever they are adjacent or incident in G..

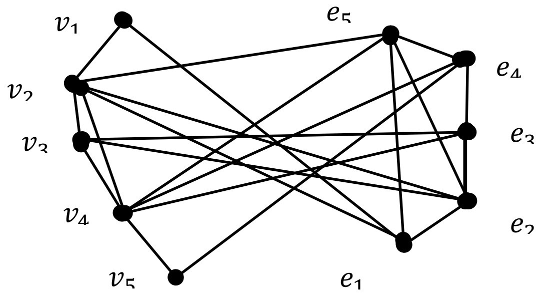

Figure. 3. Total graph of the Bull graph

Definition.1.4. The Splitting graph S(G) of a graph G is obtained by adding a new vertex v′ corresponding to each vertex v of G such that N(v)=N(v′) where N(v) and N(v′) are the neighbourhood set of v and v′ respectively.

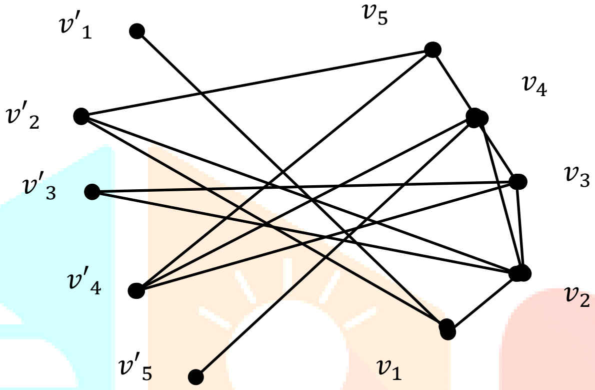

Figure.4. Splitting graph of the Bull graph

Definition. 1.5. Let G = (V, E) be a graph with V= S_1 ∪ S_2 ∪ …… ∪ S_t ∪ T, where each S_i is a set of vertices having at least two vertices and having the same degree and T=V-∪ S_i. The Degree Splitting graph DS(G) of a graph G is obtained from G by adding vertices $w_1, w_2,$ … …. . . w_t and joining $w_i,$ to each vertex of S_i (1≤ i≤ t)

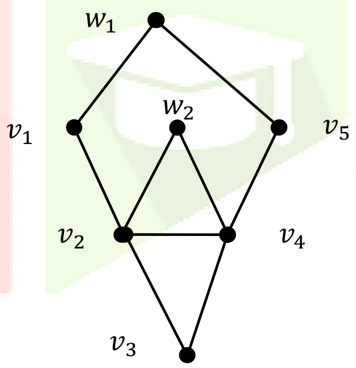

Figure.5. Degree Splitting graph of Bull graph

Definition.1.7. The Shadow graph, D_2(G) of a connected graph G is constructed by taking two copies of G, say G' and G" and joining each vertex u′ in G' to all the adjacent vertices of the corresponding vertex, u" in G".

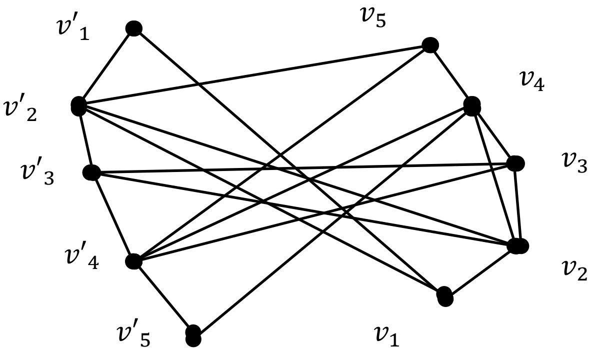

Figure.6. Shadow graph of Bull graph

Definition.1.8. Let c(G) denotes the set of all cut vertices of the Bull graph G. The Litact graph m(G) of a graph G is the graph whose vertex set is V(G)∪c(G) in which two vertices are adjacent if they correspond to adjacent edges of G or to adjacent cut-vertices of G or one corresponds to an edge e_i of G and the other corresponds to a cut-vertex c_j of G and e_i is incident with c_j.

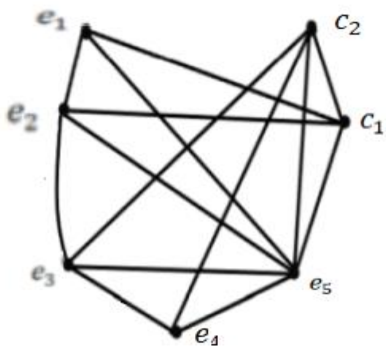

Figure.7. The Litact graph of Bull graph

2. MAIN RESULTS:

In this section, we discuss the Proper Coloring of the Bull Graphs and the related Graphs and will show that all these related graphs have the chromatic number three.

Theorem. 2.1 If G is the Bull graph, then, $\chi(G) = 3$

Proof. In the Bull graph, let the vertices be v_i for i = 1, 2, 3, 4, 5. Suppose that v_2, v_5 are colored with color 1, v_1, v_4 are colored with color 2 and v_3 is colored with color 3. Then no two adjacent vertices have the same color so that the coloring is a proper coloring of G with 3 colors. Since it has a triangle, at least 3 colors are needed for its coloring. Hence, $\chi(G) = 3$.

Theorem. 2.2. For the Middle graph, MG (G), of Bull graph, G, $\chi(MG(\text{G})) = 4$

Proof. Let the vertices the Middle graph of Bull graph be $\{v_1, v_2, v_3, v_4, v_5\}$and $\{e_1, e_2, e_3, e_4, e_5\}$ A proper 4-coloring of MG(G) with 4 colors is shown in Figure.7. Since it has an induced subgraph, K_4 at least 4 colors are needed for its coloring So that $\chi(MG(\text{G})) = 4$.

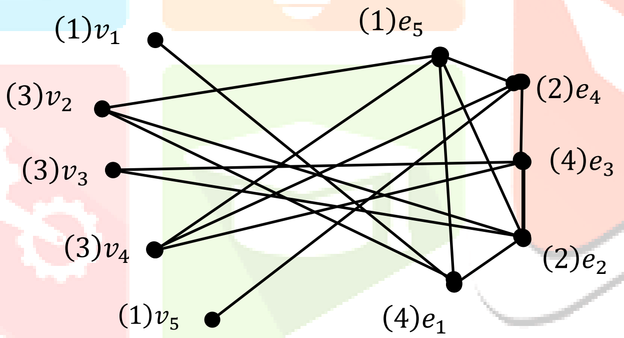

Figure 8

Theorem. 2.3. For the Total graph, $TG(G)$, of the Bull graph, G, $\chi(TG(\text{G})) = 4$.

Proof. Let the vertices of Total graph of Bull graph be $v_i, i = 1,2,3,4,5$ and $e_j, j = 1,2,3,4,5$. A proper coloring of $TG(G)$, with 4 colors is shown in Figure.8 (colors are shown in brackets). Since it has an induced subgraph, K_4 at least 4 colors are needed for any coloring. Hence, $\chi(TG(\text{G})) = 4$.

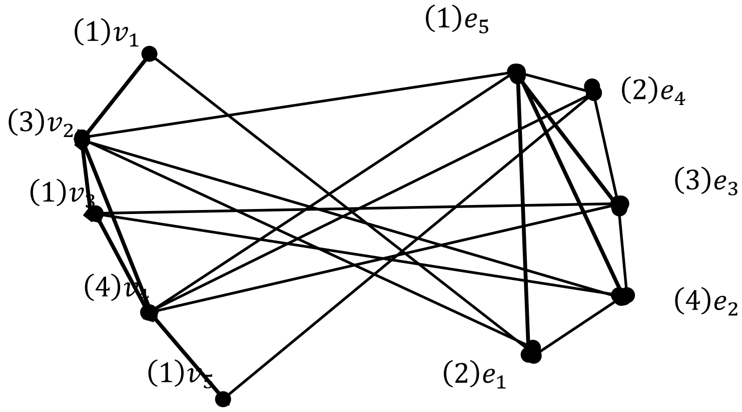

Figure 8

Theorem. 2.4. For the Splitting graph, $S(G)$ of the Bull graph, $\chi(S(\mathrm{G})) = 3$.

Proof. In the Splitting graph of Bull graph, let the vertices be v_i for I = 1 to 5 and $v'_j$for j= 1 to 5. Suppose that v_1, v_4 v'_1 ,v'_4 are colored with color 1, v_3 , v_5 ,v'_3 are colored with color 2 and v_2 v'_2 , v'_5 are colored with the color 3(Figure.9). Then no two adjacent vertices have the same color so that the coloring is a proper coloring of Splitting graph with 3 colors. Since it has a triangle at least 3 colors are needed for its coloring. Hence, $\chi(S(\mathrm{G})) = 3$.

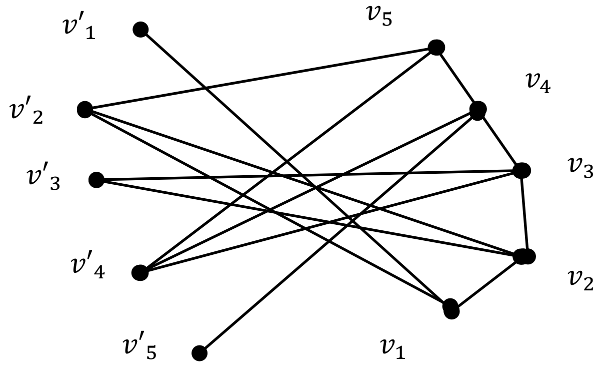

Figure 9

Theorem. 2.5. For the Degree Splitting graph, $DS(\mathrm{G})$ of the Bull graph, $\chi(DS(\mathrm{G})) = 3$.

Proof. In the Degree Splitting graph of Bull graph, let the vertices be v_i for I = 1 to 5 and $w_j$for j= 1 to 2. A proper coloring of Splitting graph with 3 colors is shown in Figure.10 (Colors are in brackets). Since it has a triangle at least 3 colors are needed for its coloring. Hence, $\chi(DS(\mathrm{G})) = 3$.

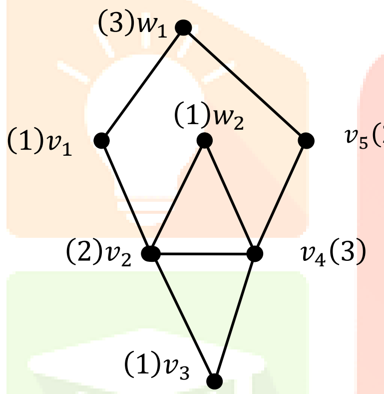

Figure. 10

Theorem. 2.6. Let $D_2(\mathrm{G})$ be the Shadow Graph of Bull graph, G. Then $\chi(D_2(\mathrm{G})) = 3$.

Proof. In the Shadow graph of Bull graph, let the vertices be v_i for I = 1 to 5 and $v'_j$for j= 1 to 5. Suppose that $v_2, v_{5,} v'_2, v'_5$ are colored with color 1, v_1, v_4 , v'_1 , v'_4 are colored with color 2, v_3, v'_3 are colored with color 3 (figure.11). Then no two adjacent vertices have the same color so that the coloring is a proper coloring of Splitting graph with 3 colors. Since it has a triangle at least 3 colors are needed for its coloring. Hence, $\chi(D_2(\mathrm{G})) = 3$.

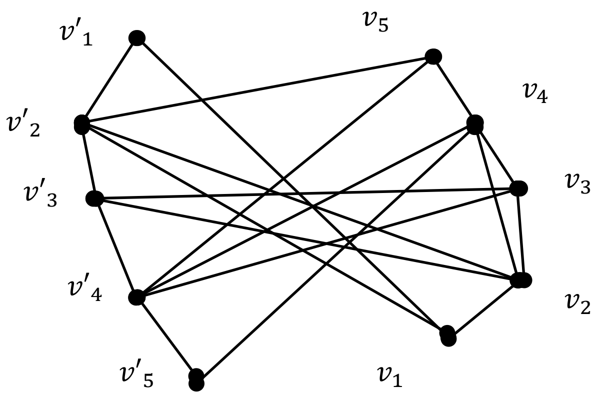

Figure.11.

Theorem. 2.7. For the Litact graph, m(G), of the Bull graph, G, $\chi(m(G)) = 4$.

Proof. Let the vertices of the Litact graph of Bull graph, be e_i, i = 1, 2, 3, 4, 5 and c_j, j = 1, 2, 3, 4, 5. Suppose that e_2, c_2 are colored with the color 1, c_1, e_4 are colored with the color 2, e_5 is colored with the color 3 and e_1, e_3 are colored with color 4. then no two adjacent vertices have the same color so that $\chi(m(G)) = 4$.

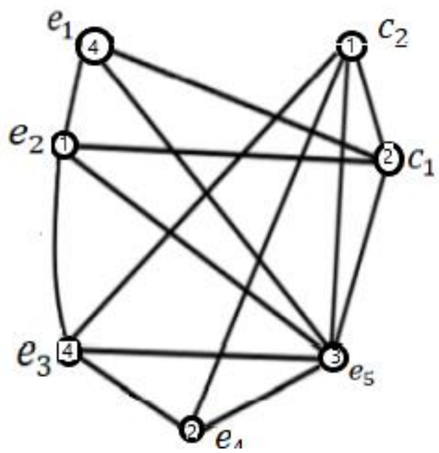

Figure. 12.

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