IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

SOME PROPERTIES OF SEMI-PRECONNECTED SPACES

Ram Lakhan Sah

Dept. of Mathematics

RRM Campus(TU), Janakpurdham

Dhanusha, Province No. 2

Abstract: *The aim of this paper is to introduce and discuss concepts of semi-preconnected spaces via the notion of semi-preopen sets as well as to investigate the characteristics and relationship to some well-known classes of topological spaces.*

Index Terms: *Semi-open set, preopen set, semi-continuity, precontinuity, semi-connected space, preconnected space.*

1. Introduction:

The concept of semi-open set and semi-continuity was introduced by Norman Levin [3]. P Das [2] defined the concept of semi connectedness in a topological space. The idea of preopen sets and precontinuity in topological spaces was introduced by Mashhour et al, [4]. Many mathematicians such as AP Balan Dhana, K Rao Chandrasekhar [1] have extended various concepts of preopen sets and preconnectedness in topological spaces. Here, we present some properties of semi preopen sets and semi-precontinuity in topological spaces.

2. Preliminaries:

All topological spaces considered in this paper lack separation axioms unless otherwise stated. We shall denote throughout this paper topological spaces by X and Y. A topological space consisting of the two points {0,1} with discrete topology will be denoted by D_2. We shall denote closure of A in X by cl(A) and interior of A by int(A).

2.1 Definition: ([3])

A subset A of topological space X is called semi-open iff there exists an open set $O \subset A \subset cl(O)$.

A set A in X is called semi-closed in X if its complement is semi-open in X.

2.2 Definition: ([3])

A subset A of topological space X is called preopen if $A \subset int\big(cl(A)\big)$.

A set A in X is called preclosed in X if its complement is preopen in X.

2.3 Definition: ([1])

A subset A of a space X is said to be semi-preopen if there exists a preopen set U in X such that

$U \subset A \subset cl(U)$.

2.4 Definition:

A topological space X is said to be connected if X cannot be expressed as union of two non-empty disjoint open sets.

2.5 Definition: ([4])

A function $f: X \to Y$ is said to be semi-continuous (resp. pre-continuous) if inverse of every open set in Y is semi-open (resp. preopen) in X.

3. Semi-Preconnected Spaces:

3.1 Definition: ([5])

A space X is called semi-preconnected if it is not union of two non-empty disjoint semi-open sets. A subset A of X is called semi-preconnected if it is semi-preconnected as a subspace of X.

3.2 Theorem:

The following assertions are equivalent:

(i) Space X is semi-connected.
(ii) The only subsets of X both semi-preopen and semi-preclosed are ϕ and X.
(iii) No semi-precontinuous function $f: X \to D_2$ is surjective.

Proof: (i) ⇒ (ii)

$G \subset X$ is both semi-preopen and semi preclosed and $G \neq \phi, X$; then $X = G \cup G^c$.

Therefore, X is not semi-preconnected. By contrapositive, (i) ⇒ (ii).

(ii) ⇒ (iii)

If $f: X \to D_2$ were a semi-precontinuous surjective function,

then $f^{-1}(\{0\}) \neq \emptyset, X$.

But $\{0\}$ is both open and closed in D_2.

Hence, $f^{-1}(\{0\})$ is semi-preopen and semi-preclosed in X.

This contradicts D_2.

(iii)⇒(i)

If $X = A \cup B$, where A and B are disjoint non-empty semi-preopen sets,

then A and B are also semi-preclosed.

Let $\chi_A(x) = \begin{cases} 1 \text{ } if \text{ } x \in A \\ 0 \text{ } if \text{ } x \notin A \end{cases}$ be characteristic function on X.

Then, $\chi_A(x)$ is a semi-precontinuous surjective function.

3.3 Theorem:

The semi-precontinuous image of a semi-preconnected space is semi-preconnected.

Proof: The map $f: X \to f(X)$ is semi-precontinuous. If $f(X)$ is not semi-preconnected, then there would be a semi- precontinuous surjection $g: f(X) \to D_2$. But then $gof: X \to D_2$ would also be a semi-precontinuous surjective function, contradicting the semi-preconnectedness of X.

3.4 Theorem:

Let Y be any space. The union of any family of semi-preconnected subsets having at least one point in common is also semi-preconnected.

Proof: Let $C = \cup A_\alpha$, $y_0 \in \cap A_\alpha$ and $f: C \to D_2$ be semi-precontinuous.

Since each A_α is semi-preconnected, no $f|_{A_\alpha}$ is surjective.

Again, since $y_0 \in A_\alpha$ for all α, $f(y) = f(y_\alpha)$ for all $y \in A_\alpha$.

Hence, f cannot be surjective and so C is semi-preconnected.

3.5 Theorem:

Let $f: X \to Y$ be continuous and open. If V be preopen in Y then $f^{-1}(V)$ is also preopen in X.

Proof: Let V be preopen set in Y.

Then, there exists an open set W in Y such that

$V \subset W \subset cl(V)$.

$\Rightarrow f^{-1}(V) \subset f^{-1}(W) \subset f^{-1}(cl(V)) \subset cl(f^{-1}(V))$, as f is open.

But $f^{-1}(W)$ is open as f is continuous.

Hence, $f^{-1}(V)$ is preopen in X.

3.6 Corollary:

Let $f: X \to Y$ be continuous and open. If V be semi-preopen in Y then $f^{-1}(V)$ is also semi-preopen in X.

Proof: Let V be semi-preopen set in Y.

Then, there exists preopen set U in Y such that

$U \subset V \subset cl(U)$.

$\Rightarrow f^{-1}(U) \subset f^{-1}(V) \subset f^{-1}(cl(U)) \subset cl(f^{-1}(U))$, as f is continuous and open.

But $f^{-1}(U)$ is open as f is preopen, by theorem 3.5

Hence, $f^{-1}(V)$ is semi-preopen in X.

3.7 Theorem:

Let $A \subset Y$ be semi-preconnected. Then any subset B satisfying $A \subset B \subset semi\ pre\ cl(A)$ is also connected.

In particular, the semi-preclosure of a semi-preconnected set is semi-preconnected.

Proof: Let $f: B \to D_2$ be semi-precontinuous.

Then, since A is semi-preconnected, $f|_A$ is not surjective.

But, $B = semi\ pre\ cl(A) \cap B = semi\ pre\ cl_B(A)$

Hence, $f(B) = f(semi\ precl_B(A))$

$\subset pre\ cl_B(f(A))$

$\subset semi\ pre\ cl(f(A))$, by precontinuity of f

$= f(A)$, because $f(A)$ is preconnected.

So, f cannot be surjective and B is semi-preconnected.

3.8 Theorem:

If $f: X \to Y$ is open and precontinuous surjective function and X is semi-preconnected, then Y is connected.

Proof: Suppose Y is not connected. Then,

$Y = V_1 \cup V_2$, Where $V_1 \cap V_2 = \emptyset$ and $V_1, V_2 \in \tau$.

So, $X = f^{-1}(V_1) \cup f^{-1}(V_2)$, $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$.

Since V_i is both open and closed, V_i is preopen.

Since f is open and preopen, $f^{-1}(V_i)$ is preopen, by theorem 3.5

Thus, X is not preconnected, a contradiction. Hence, Y must be connected.

3.9 Theorem:

Let A be preconnected subspace of X. If B be a subspace of X such that

A $\subset B \subset cl(A)$ then B is preconnected.

Proof: Let $B = G \cup H$, a disconnection by preclosed sets G and H in X.

Then, $G = B \cap E, H = B \cap F$ where E and F are preclosed in X.

Since A is preconnected, $A \subset G$ or $A \subset H$.

Suppose $A \subset G = B \cap E \subset E$. Then, $cl(A) \subset cl(E) = E$.

Hence, $B \subset E$.

Also, $B \subset B \cap E = G. \Rightarrow G \cup H \subset G$, which is impossible.

Therefore, B is preconnected.

4. Conclusion:

In this paper, we see that properties similar to semi-open sets and semi-continuity can also be derived for semi-preopen set and semi-precontinuity in topological spaces.

5. References:

[1] Balan Dhana, AP & Rao Chandrasekhar, K (2008), Semi-Preconnected spaces. *The Mathematics Education, Vol XLII (3).*
[2] Das, P (1973), Notes on some applications of semi open sets. *Progress of Math, BHU*, 7, p-33-44.
[3] Levine, N (2018), Semi-Open Sets and Semi-Continuity in Topological spaces. *The American Mathematical Monthly, 1* p-36-41
[4] Mashhour, AS (1983), On closure spaces. *Indian J. of Pure and Appl, Math*, 14,680-691.
[5] Tapi, UD & Deole, Bhagyashri A (2014), Semi connectedness and Preconnectedness in closure space. *IJMCAR,4(4), p-75-82.*