On Strongly $sg\alpha$-Irresolute Functions in Topological Spaces

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Abstract:
In this paper, we introduce and investigate the notion of strongly $sg\alpha$-irresolute functions. We obtain fundamental properties and characterization of strongly $sg\alpha$-irresolute functions and discuss the relationships between strongly $sg\alpha$-irresolute functions and other related functions.

Keywords: strongly $sg\alpha$-irresolute, strongly $\alpha$-irresolute, strongly $\beta$-$sg\alpha$ irresolute.

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1. Introduction


In 1996, Dontchev [12] introduced a new class of functions called contra-continuous functions. A new weaker form of this class of functions called contra semi-continuous function is introduced and investigated by Dontchev and Noiri [13].

In this paper, the notion of $sg\alpha$-closed sets [8] and contra $sg\alpha$-continuous Space[9]and characterization of contra $sg\alpha$-continuous functions in topological spaces [10] is applied to introduce and study a new class of functions called on strongly $sg\alpha$-Irresolute functions in topological spaces, as a new generalization of strongly $\alpha$-irresolute functions and strongly $\beta$-$sg\alpha$-irresolute functions, to obtain some of their characterizations and properties. Also the relationships with some other functions are discussed.

2. PRELIMINARIES

Throughout this paper $(X, \tau)$, $(Y, \sigma)$ and $(Z, \eta)$ always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and the interior of A are denoted by cl (A) and int (A) respectively. $(X, \tau)$ will be replaced by X if there is no chance of confusion. Let us recall the following definitions as pre requests.

A subset A of a topological space X is said to be
1. semi open [15] if $A \subseteq \text{cl} (\text{int} (A))$ and semi closed if $\text{int} (\text{cl} (A)) \subseteq A$.
2. $\alpha$-open [23] if $A \subseteq \text{int} (\text{cl} (\text{int} (A)))$ and $\alpha$-closed if $\text{cl} (\text{int} (\text{cl} (A))) \subseteq A$.
3. $\beta$-open or semi pre-open [1] if $A \subseteq \text{cl} (\text{int} (\text{cl} (A)))$ and $\beta$-closed or semi pre-closed if $\text{int} (\text{cl} (\text{int} (A))) \subseteq A$.
4. pre-open [11] if $A \subseteq \text{int} (\text{cl} (A))$ and pre-closed if $\text{cl} (\text{int} (A)) \subseteq A$.

The complement of a semi-open (resp.pre-open, $\alpha$-open, $\beta$-open) set is called semi-closed (resp.pre-closed, $\alpha$-closed, $\beta$-closed). The intersection of all semi-closed (resp.pre-closed, $\alpha$-closed, $\beta$-closed) sets
Definitions 2.2: A subset A of a space (X, τ) is called
1. g-closed[16] if cl (A) ⊆ U, whenever A ⊆ U and U is open in (X, τ).
2. gs-closed set[7] if scl (A)⊆U, whenever A⊆U and U is open in (X, τ).
4. αg-closed[17] if α cl (A)⊆U, whenever A⊆U and U is open in (X, τ).
5. ga-closed[18] if α cl (A)⊆U, whenever A⊆U and U is α-open in (X, τ).
6. gp-closed[19] if pcl (A)⊆U, whenever A⊆U and U is open in (X, τ).

Definition 2.3: Let X and Y be topological spaces. A function f: X → Y is said to be

1. continuous [14] if for each open set V of Y the set f −1 (V) is an open subset of X .
2. α-continuous [23] if f −1(V) is a α-closed set of (X, τ ) for every closed set V of (Y, σ).
3. β-continuous [1] if f −1(V) is a β-closed set of (X, τ ) for every closed set V of (Y, σ).
4. pre-continuous [21] if f −1(V) is a pre-closed set of (X, τ ) for every closed set V of (Y, σ).
5. semi-continuous [15] if f −1(V) is a semi-closed set of (X, τ ) for every closed set V of (Y, σ).

Definition 2.4: A function f: (X, τ ) → (Y, σ) is said to be

1. g-continuous [16] if f −1(V) is a g-closed set of (X, τ ) for every closed set V of (Y, σ).
2. gs-continuous[7] if f −1(V) is a gs-closed set of (X, τ ) for every closed set V of (Y, σ).
3. sg-continuous [6] if f −1(V) is a sg-closed set of (X, τ ) for every closed set V of (Y, σ).
4. αg-continuous [17] if f −1(V) is a αg-closed set of (X, τ ) for every closed set V of (Y, σ).
5. ga-continuous [18] if f −1(V) is a ga-closed set of (X, τ ) for every closed set V of (Y, σ).
6. gp-continuous [19] if f −1(V) is a gp-closed set of (X, τ ) for every closed set V of (Y, σ).

Definitions 2.5[22]: A function f: (X, τ ) →(Y, σ) is said to be almost continuous if for every open set V of Y , f −1(V)is regular open in X.
Definitions 2.6[8]: A subset A of space (X, τ) is called sga-closed if scl (A) ⊆ U , whenever A⊆U and U is α-open in X . The family of all sga-closed subsets of the space X is denoted by SGaC (X ).
Definitions 2.7[8]: The intersection of all sga-closed sets containing a set A is called sga-closure of A and is denoted by sga-cl(A). A set A is sga-closed set if and only if sga C l(A) = A.
Definitions 2.8[8]: A subset A in X is called sga-open in X if A c is sga-closed in X. The family of a sga-open sets is denoted by SGaO(X ).
Definitions 2.9[8]: The union of all sg α-open sets containing a set A is called sga-interior of A and is denoted by sga-I nt(A), A set A is sga-open set if and only if sga I nt (A) = A.
Definition 2.10[9]: A function f :(X, τ )→(Y, σ) is called sga-continuous if f −1(V) is sga-closed in (X, τ ) for every closed set V of (Y, σ).
**Definition 2.11** [9]: A function \( f: X \to Y \) is said to be Contra s\( g \alpha \)-Continuous if \( f^{-1}(V) \) is s\( g \alpha \)-closed in \( X \) for each open set \( V \) of \( Y \).

**Definition 2.12** [9]: A space \( X \) is called locally s\( g \alpha \)-indiscrete if every s\( g \alpha \)-open set is closed in \( X \).

**Definition 2.13** [9]: If a function \( f: X \to Y \) is called almost s\( g \alpha \)-continuous if for each \( x \in X \) and each open set \( V \) of \( Y \) containing \( f(x) \), there exists \( U \in SG_{\alpha}(X, x) \) such that \( f(U) \subset \text{cl}(V) \).

**Definition 2.14** [9]: If a function \( f: X \to Y \) is called quasi s\( g \alpha \)-open if image of every s\( g \alpha \)-open set of \( X \) is open set in \( Y \).

**Definition 2.15** [9]: If a function \( f: X \to Y \) is called weakly s\( g \alpha \)-continuous if for each \( x \in X \) and each open set \( V \) of \( Y \) containing \( f(x) \), there exists \( U \in SG_{\alpha}(X, x) \) such that \( f(U) \subset \text{cl}(V) \).

**Theorem 3.4** [9]: Let \( A \) be a subset of \( X \). Then s\( g \alpha \)-C \( l \) \((A)\)-s\( g \alpha \)-I \( nt \) \((A)\) is called s\( g \alpha \)-frontier of \( A \) and is denoted by s\( g \alpha \)-F \( r \) \((A)\).

**Theorem 3.3** [10]: A subset \( A \) of a topological space \( X \) is said to be s\( g \alpha \)-dense in \( Y \) if s\( g \alpha \) \( − \) Cl \((A)\) = \( Y \).

**Definition 2.16** [9]: Let \( X \) be a subset of \( X \). Then s\( g \alpha \)-C \( l \) \((A)\)-s\( g \alpha \)-I \( nt \) \((A)\) is called s\( g \alpha \)-connected provided that \( X \) is not the union of two disjoint non-empty s\( g \alpha \)-open sets.

**Definition 2.17** [10]: A subset \( A \) of a topological space \( X \) is said to be s\( g \alpha \)-connected provided that \( X \) is not the union of two disjoint non-empty s\( g \alpha \)-open sets.

**Definition 2.18** [10]: A subset \( A \) of a space \((x, \tau)\) is said to be s\( g \alpha \)-clopenn if \( A \) is both s\( g \alpha \)-open and s\( g \alpha \)-closed.

**Definition 2.20** [10]: A topological space \( X \) is said to be s\( g \alpha \)-T\( 1 \)-space if for any pair of disjoint points \( x \) and \( y \), there exist disjoint s\( g \alpha \)-open sets \( G \) and \( H \) such that \( x \in G \) and \( y \in H \).

**Definition 2.21** [10]: A topological space \( X \) is said to be s\( g \alpha \)-T\( 2 \)-space if for any pair of disjoint points \( x \) and \( y \), there exist disjoint s\( g \alpha \)-open sets \( G \) and \( H \) such that \( x \in G \) and \( y \in H \).

**Definition 2.22** [10]: A space \((X, \tau)\) is said to be s\( g \alpha \)-T\( 1/2 \)-space if every s\( g \alpha \)-closed set is semi-closed. The notion of s\( g \alpha \)-T\( 1/2 \)-spaces and T\( 1/2 \) spaces are independent of each other.

**Definition 2.23** [10]: A topological space \( X \) is said to be s\( g \alpha \)-Normal if each pair of disjoint closed sets can be separated by disjoint s\( g \alpha \)-open sets.

**Definition 2.24** [10]: A function \( f: X \to Y \) is said to be strongly s\( g \alpha \)-open (resp. strongly s\( g \alpha \)-closed) if image of every s\( g \alpha \)-open (resp. s\( g \alpha \)-closed) set of \( X \) is s\( g \alpha \)-open (resp. s\( g \alpha \)-closed) set in \( Y \).

**Definition 2.25** [10]: A space \( X \) is said to be

1. s\( g \alpha \)-closed compact if every s\( g \alpha \)-closed cover of \( X \) has a finite subcover.
2. Countably s\( g \alpha \)-closed compact if every countable cover of \( X \) by s\( g \alpha \)-closed sets has a finite subcover.
3. s\( g \alpha \)-Lindelöf if every s\( g \alpha \)-closed cover of \( X \) has countable subcover.

### 3. Strongly s\( g \alpha \)-Irresolute functions.

**Definition 3.1**: A function \( f: X \to Y \) is said to be strongly s\( g \alpha \)-irresolute if \( f^{-1}(V) \) is open in \( X \) for every s\( g \alpha \)-open set \( V \) of \( Y \).

**Definition 3.2**: A function \( f: X \to Y \) is said to be strongly s\( g \alpha \)-irresolute if \( f^{-1}(V) \) is open in \( X \) for every s\( g \alpha \)-open set \( V \) of \( Y \).

**Theorem 3.3**: If \( f: X \to Y \) is a strongly s\( g \alpha \)-irresolute, then \( f \) is strongly s\( g \alpha \)-irresolute.

**Proof**: Let \( V \) be s\( g \alpha \)-open in \( Y \) and hence \( V \) is s\( g \alpha \)-open in \( Y \). Since \( f \) is strongly s\( g \alpha \)-irresolute, then \( f^{-1}(V) \) is open in \( X \). Therefore \( f^{-1}(V) \) is open in \( X \) for every s\( g \alpha \)-open set \( V \) in \( Y \). Hence \( f \) is strongly s\( g \alpha \)-irresolute.

**Theorem 3.4**: If \( f: X \to Y \) is a continuous and \( Y \) is a s\( g \alpha \)-T\( 1/2 \)-space, then \( f \) is strongly s\( g \alpha \)-irresolute.

**Proof**: Let \( V \) be s\( g \alpha \)-open in \( Y \). Since \( Y \) is s\( g \alpha \)-T\( 1/2 \)-space, \( V \) is s\( g \alpha \)-open in \( Y \) and hence open in \( Y \). Since \( f \) is continuous, \( f^{-1}(V) \) is open in \( X \). Thus, \( f^{-1}(V) \) is open in \( X \) for every s\( g \alpha \)-open set \( V \) in \( Y \). Hence \( f \) is strongly s\( g \alpha \)-irresolute.
Theorem: 3.5

If \( f: X \to Y \) is a sga-irresolute, \( X \) is a sga-\(-T_{1/2}\)-space, then \( f \) is strongly sga-irresolute.

**Proof:** Let \( V \) be sga-open in \( Y \). Since \( f \) is sga-irresolute, \( f^{-1}(V) \) is sga-open in \( X \). Since \( X \) is a sga-\(-T_{1/2}\)-space, \( f^{-1}(V) \) is \( \alpha \)-open in \( X \) and hence open in \( X \). Thus, \( f^{-1}(V) \) is open in \( X \) for every sga-open set \( V \) in \( Y \). Hence \( f \) is strongly sga-irresolute.

Theorem: 3.6

Let \( f: X \to Y \) and \( g: Y \to Z \) be any functions. Then

(i) \( g \circ f: X \to Z \) is sga-irresolute if \( f \) is sga-continuous and \( g \) is strongly sga-irresolute.

(ii) \( g \circ f: X \to Z \) is strongly sga-irresolute if \( f \) is strongly sga-irresolute and \( g \) is sga-irresolute.

**Proof:**

(i) Let \( V \) be a sga-open set in \( Z \). Since \( g \) is strongly sga-irresolute, \( g^{-1}(V) \) is open in \( Y \). Since \( f \) is sga-continuous, \( f^{-1}(g^{-1}(V)) \) is sga-open in \( X \).

\[ \Rightarrow (g \circ f)^{-1}(V) \text{ is sga-open in } X \text{ for every sga-open set } V \text{ in } Z. \]

\[ \Rightarrow (g \circ f) \text{ is sga-irresolute.} \]

(ii) Let \( V \) be a sga-open set in \( Z \). Since \( g \) is sga-irresolute, \( g^{-1}(V) \) is sga-open in \( Y \). Since \( f \) is strongly sga-irresolute, \( f^{-1}(g^{-1}(V)) \) is open in \( X \).

\[ \Rightarrow (g \circ f)^{-1}(V) \text{ is open in } X \text{ for every sga-open set } V \text{ in } Z. \]

\[ \Rightarrow (g \circ f) \text{ is strongly sga-irresolute.} \]

Theorem: 3.7

The following are equivalent for a function \( f: X \to Y \):

(i) \( f \) is strongly sga-irresolute.

(ii) For each \( x \in X \) and each sga-open set \( V \) of \( Y \) containing \( f(x) \), there exists an open set \( U \) in \( X \) containing \( x \) such that \( f(U) \subseteq V \).

(iii) \( f^{-1}(V) \subseteq \text{int}(f^{-1}(V)) \) for each sga-open set \( V \) of \( Y \).

(iv) \( f^{-1}(F) \) is closed in \( X \) for every sga-closed set \( F \) of \( Y \).

**Proof:** (i) \( \Rightarrow \) (ii):

Let \( x \in X \) and \( V \) be a sga-open set in \( Y \) containing \( f(x) \). By hypothesis, \( f^{-1}(V) \) is open in \( X \) and contains \( x \).

Set \( U = f^{-1}(V) \). Then \( U \) is open in \( X \) and \( f(U) \subseteq V \).

(ii) \( \Rightarrow \) (iii):

Let \( V \) be a sga-open set in \( Y \) and \( x \in f^{-1}(V) \).

By assumption, there exists an open set \( U \) in \( X \) containing \( x \), such that \( f(U) \subseteq V \).

Then \( x \in U \subseteq \text{int}(U) \)

\[ \subseteq \text{int}(f^{-1}(V)). \]

Then \( f^{-1}(V) \subseteq \text{int}(f^{-1}(V)) \)

(iii) \( \Rightarrow \) (iv):

Let \( F \) be a sga-closed set in \( Y \). Set \( V = Y - F \). Then \( V \) is sga-open in \( Y \).
By (iii), \( f^{-1}(V) \subseteq \text{int}(f^{-1}(V)) \).

Hence \( f^{-1}(F) \) is closed in \( X \).

(iv) \( \Rightarrow \) (i):

Let \( V \) be \( \text{sg} \alpha \)-open set in \( Y \). Let \( F = Y - V \). That is \( F \) is \( \text{sg} \alpha \)-closed set in \( Y \). Then \( f^{-1}(F) \) is closed in \( X \), (by (iv)). Then \( f^{-1}(V) \) is open in \( X \). Hence \( f \) is strongly \( \text{sg} \alpha \)-irresolute.

**Theorem:3.8**

A function \( f: X \rightarrow Y \) is strongly \( \text{sg} \alpha \)-irresolute if \( A \) is open in \( X \), then \( f/A: A \rightarrow Y \) is strongly \( \text{sg} \alpha \)-irresolute.

**Proof:** Let \( V \) be a \( \text{sg} \alpha \)-open set in \( Y \). By hypothesis, \( f^{-1}(V) \) is open in \( X \). But \( (f/A)^{-1}(V) = A \neq f^{-1}(V) \) is open in \( A \) and hence \( f/A \) is strongly \( \text{sg} \alpha \)-irresolute.

**Theorem:3.9**

Let \( f: X \rightarrow Y \) be a function and \( \{A_i: i \in \Lambda\} \) be a cover of \( X \) by open sets of \( (X, \tau) \). Then \( f \) is strongly \( \text{sg} \alpha \)-irresolute if \( f/A_i: (A_i, \tau/A_i) \rightarrow (Y, \sigma) \) is strongly \( \text{sg} \alpha \)-irresolute for each \( i \in \Lambda \).

**Proof:** Let \( V \) be a \( \text{sg} \alpha \)-open set in \( Y \). By hypothesis, \( (f/A_i)^{-1}(V) \) is open in \( A_i \). Since \( A_i \) is open in \( X \), \( (f/A_i)^{-1}(V) \) is open in \( X \) for every \( i \in \Lambda \).

\[
f^{-1}(V) = X \cap f^{-1}(V) = \bigcup \{A_i \cap f^{-1}(V): i \in \Lambda\}
\]

Hence \( f \) is strongly \( \text{sg} \alpha \)-irresolute.

**Theorem:3.10**

Let \( f: X \rightarrow Y \) be a strongly \( \text{sg} \alpha \)-irresolute surjective function. If \( X \) is compact, then \( Y \) is \( \text{SG} \alpha \text{O}\)-compact.

**Proof:** Let \( \{A_i: i \in \Lambda\} \) be a cover of \( \text{sg} \alpha \)-open sets of \( Y \). Since \( f \) is strongly \( \text{sg} \alpha \)-irresolute and \( X \) is compact, we get \( X \subseteq \bigcup \{f^{-1}(A_i): i \in \Lambda\} \). Since \( f \) is surjective, \( Y = f(X) \subseteq \bigcup \{A_i: i \in \Lambda\} \). Hence \( Y \) is \( \text{SG} \alpha \text{O}\)-compact.

**Theorem:3.11**

If \( f: X \rightarrow Y \) is strongly \( \text{sg} \alpha \)-irresolute and a subset \( B \) of \( X \) is compact relative to \( X \), then \( f(B) \) is \( \text{SG} \alpha \text{O}\)-compact relative to \( Y \).

**Proof:** Obvious.

**Definition: 3.12**

A function \( f: X \rightarrow Y \) is said to be

(i) a strongly \( \alpha \)-\( \text{sg} \alpha \)-irresolute function if \( f^{-1}(V) \) is \( \alpha \)-open in \( X \) for every \( \text{sg} \alpha \)-open set \( V \) in \( Y \).

(ii) a strongly \( \beta \)-\( \text{sg} \alpha \)-irresolute function if \( f^{-1}(V) \) is \( \beta \)-open in \( X \) for every \( \text{sg} \alpha \)-open set \( V \) in \( Y \).
Theorem: 3.13

(i) If \( f: X \rightarrow Y \) is strongly \( \alpha \)-s\( \gamma \alpha \)-irresolute, then \( f \) is strongly s\( \gamma \alpha \)-irresolute.

(ii) If \( f: X \rightarrow Y \) is strongly \( \alpha \)-s\( \gamma \alpha \)-irresolute, then \( f \) is strongly \( \beta \)-s\( \gamma \alpha \)-irresolute.

**Proof:**

(i) Let \( f \) be a strongly \( \alpha \)-s\( \gamma \alpha \)-irresolute function and let \( V \) be a s\( \gamma \alpha \)-open set in \( Y \). Then \( f^{-1}(V) \) is \( \alpha \)-open in \( X \) and hence open in \( X \).

\[
\Rightarrow f^{-1}(V) \text{ is open in } X \text{ for every s}\( \gamma \alpha \)-open set } V \text{ in } Y.
\]

Hence \( f \) is strongly \( \alpha \)-s\( \gamma \alpha \)-irresolute.

(ii) Let \( f \) be a strongly \( \alpha \)-s\( \gamma \alpha \)-irresolute function and let \( V \) be a s\( \gamma \alpha \)-open set in \( Y \). Then

\[
f^{-1}(V) \text{ is } \alpha\text{-open in } X \text{ and hence open in } X.
\]

\[
\Rightarrow f^{-1}(V) \text{ is open in } X \text{ for every s}\( \gamma \alpha \)-open set } V \text{ in } Y.
\]

\[
\Rightarrow f^{-1}(V) \text{ is } \beta\text{-open in } X \text{ for every s}\( \gamma \alpha \)-open set } V \text{ in } Y.
\]

Hence \( f \) is strongly \( \beta \)-s\( \gamma \alpha \)-irresolute.

**Remark: 3.14**

Converse of the above need not be true as seen in the following examples.

**Example: 3.15**

(i) Let \( X = Y = \{a,b,c\} \), \( \tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}\} \) and \( \sigma = \{\phi, Y, \{a\}, \{b\}, \{a,b\}\} \).

Let \( f: X \rightarrow Y \) be an identity map. Here for every s\( \gamma \alpha \)-open set \( V \) in \( Y \), \( f^{-1}(V) \) is s\( \gamma \alpha \)-open and \( \beta\)-open in \( X \). Hence \( f \) is strongly s\( \gamma \alpha \)-irresolute and strongly \( \beta \)-s\( \gamma \alpha \)-irresolute.

But for every s\( \gamma \alpha \)-open set \( V \) in \( Y \), \( f^{-1}(V) \) is not \( \alpha \)-open in \( X \). Thus, \( f \) is not strongly \( \alpha \)-s\( \gamma \alpha \)-irresolute. Hence strongly s\( \gamma \alpha \)-irresolute function need not be strongly \( \alpha \)-s\( \gamma \alpha \)-irresolute function and strongly \( \beta \)-s\( \gamma \alpha \)-irresolute function.

**Theorem: 3.16**

If \( f: X \rightarrow Y \) and \( g: Y \rightarrow Z \), then \( g \circ f: X \rightarrow Z \) is

(i) strongly s\( \gamma \alpha \)-irresolute if \( f \) is strongly \( \alpha \)-s\( \gamma \alpha \)-irresolute and \( g \) is s\( \gamma \alpha \)-irresolute.

(ii) strongly \( \beta \)-s\( \gamma \alpha \)-irresolute if \( f \) is strongly s\( \gamma \alpha \)-irresolute and \( g \) is s\( \gamma \alpha \)-irresolute.

**Proof:** Let \( V \) be an s\( \gamma \alpha \)-open set in \( Z \). Since \( g \) is s\( \gamma \alpha \)-irresolute, \( g^{-1}(V) \) is s\( \gamma \alpha \)-open in \( Y \). Since \( f \) is strongly \( \alpha \)-s\( \gamma \alpha \)-irresolute, \( f^{-1}(g^{-1}(V)) \) is \( \alpha \)-open in \( X \).

\[
\Rightarrow (g \circ f)^{-1}(V) \text{ is regular open in } X \text{ and hence open in } X.
\]

Hence \( (g \circ f)^{-1}(V) \) is strongly s\( \gamma \alpha \)-irresolute.

(i) Let \( V \) be an s\( \gamma \alpha \)-open set in \( Z \). Since \( g \) is s\( \gamma \alpha \)-irresolute, \( g^{-1}(V) \) is s\( \gamma \alpha \)-open in \( Y \). Since \( f \) is strongly s\( \gamma \alpha \)-irresolute, \( f^{-1}(g^{-1}(V)) \) is open in \( X \) and hence \( \beta \)-open in \( X \).

\[
\Rightarrow (g \circ f)^{-1}(V) \text{ is } \beta\text{-open in } X \text{ for every s}\( \gamma \alpha \)-open set } V \text{ in } Z.
\]
Hence \((g \circ f)\) is strongly \(\beta\)-\(\text{sg} \alpha\)- irresolute.

**Theorem: 3.17**

If \(f: X \to Y\) and \(g: Y \to Z\), then \(g \circ f: X \to Z\) is

(i) strongly \(\alpha\)-\(\text{sg} \alpha\)- irresolute if \(f\) is regular irresolute and \(g\) is strongly \(\alpha\)-\(\text{sg} \alpha\)- irresolute.

(ii) strongly \(\alpha\)-\(\text{sg} \alpha\)- irresolute if \(f\) is \(\alpha\)- continuous and \(g\) is strongly \(\text{sg} \alpha\)- irresolute.

(iii) strongly \(\beta\)-\(\text{sg} \alpha\)- irresolute if \(f\) is continuous and \(g\) is strongly \(\text{sg} \alpha\)- irresolute.

**Proof:** Let \(V\) be a \(\text{sg} \alpha\)-open set in \(Z\). Since \(g\) is strongly \(\alpha\)-\(\text{sg} \alpha\)- irresolute, \(g^{-1}(V)\) is \(\alpha\)- open in \(Y\). Since \(f\) is \(\alpha\)- irresolute, \(f^{-1}(g^{-1}(V))\) is \(\alpha\)- open in \(X\).

\[\Rightarrow (g \circ f)^{-1}(V) \text{ is } \alpha\text{- open in } X.\]

Hence \((g \circ f)^{-1}(V)\) is \(\alpha\)-open in \(X\).

(i) Let \(V\) be an \(\text{sg} \alpha\)-open set in \(Z\). Since \(g\) is strongly \(\text{sg} \alpha\)- irresolute, \(g^{-1}(V)\) is open in \(Y\). Since \(f\) is \(\alpha\)- continuous, \(f^{-1}(g^{-1}(V))\) is \(\alpha\)-open in \(X\).

\[\Rightarrow (g \circ f)^{-1}(V) \text{ is } \alpha\text{- open in } X.\]

Hence \((g \circ f)^{-1}(V)\) is \(\alpha\)-open in \(X\).

(ii) Let \(V\) be an \(\text{sg} \alpha\)-open set in \(Z\). Since \(g\) is strongly \(\text{sg} \alpha\)- irresolute, \(g^{-1}(V)\) is open in \(Y\). Since \(f\) is continuous, \(f^{-1}(g^{-1}(V))\) is open in \(X\).

\[\Rightarrow (g \circ f)^{-1}(V) \text{ is open in } X \text{ and hence } \beta\text{- open in } X.\]

Hence \((g \circ f)^{-1}(V)\) is \(\beta\)-open in \(X\).

**Theorem: 3.18**

The following are equivalent for a function \(f: X \to Y\):

(i) \(f\) is strongly \(\alpha\)-\(\text{sg} \alpha\)- irresolute.

(ii) For each \(x \in X\) and each \(\text{sg} \alpha\)-open set \(V\) of \(Y\) containing \(f(x)\), there exists a \(\alpha\)- open set \(U\) in \(X\) containing \(x\) such that \(f(U) \subset V\).

(iii) \(f^{-1}(V) \subset \text{Cl}(\text{Int}(f^{-1}(V)))\) for each \(\text{sg} \alpha\)-open set \(V\) of \(Y\).

(iv) \(f^{-1}(F)\) is regular closed in \(X\) for every \(\text{sg} \alpha\)-closed set \(F\) of \(Y\).

**Proof:** Similar to that of Theorem 3.7

**Theorem: 3.19**

The following are equivalent for a function \(f: X \to Y\):

(i) \(f\) is strongly \(\beta\)-\(\text{sg} \alpha\)- irresolute.

(ii) For each \(x \in X\) and each \(\text{sg} \alpha\)-open set \(V\) of \(Y\) containing \(f(x)\), there exists a \(\beta\)- open set \(U\) in \(X\) containing \(x\) such that \(f(U) \subset V\).

(iii) \(f^{-1}(V) \subset \text{Cl}(\text{Int}(f^{-1}(V)))\) for each \(\text{sg} \alpha\)-open set \(V\) of \(Y\).

(iv) \(f^{-1}(F)\) is \(\beta\)-closed in \(X\) for every \(\text{sg} \alpha\)-closed set \(F\) of \(Y\).
**Proof:** Similar to that of Theorem 3.7.

**Lemma:** 3.20

If \( f: X \rightarrow Y \) is strongly \( \alpha \)-sg\( \alpha \)-irresolute and \( A \) is a \( \alpha \)-open subset of \( X \), then \( f/A : A \rightarrow Y \) is strongly \( \alpha \)-sg\( \alpha \)-irresolute.

**Proof:**

Let \( V \) be a sg\( \alpha \)-open in \( Y \). By hypothesis, \( f^{-1}(V) \) is \( \alpha \)-open in \( X \). But \((f/A)^{-1}(V) = A \cap f^{-1}(V)\) is regular open in \( A \). Hence \( f/A \) is strongly \( \alpha \)-sg\( \alpha \)-irresolute.

**Theorem:** 3.21

Let \( f: X \rightarrow Y \) and \( \{A_\lambda : \lambda \in \Lambda\} \) be a cover of \( X \) by \( \alpha \)-open set of \((X, \tau)\). Then \( f \) is a strongly \( \alpha \)-sg\( \alpha \)-irresolute function if \( f/A_\lambda : A_\lambda \rightarrow Y \) is strongly \( \alpha \)-sg\( \alpha \)-irresolute for each \( \lambda \in \Lambda \).

**Proof:** Let \( V \) be any sg\( \alpha \)-open set in \( Y \). By hypothesis, \((f/A_\lambda)^{-1}(V) \) is \( \alpha \)-open in \( A_\lambda \). Since \( A_\lambda \) is regular open in \( X \), it follows that \((f/A_\lambda)^{-1}(V) \) is sg\( \alpha \)-open in \( X \) for each \( \lambda \in \Lambda \).

\[
f^{-1}(V) = X \cap f^{-1}(V) = \bigcup \{A_\lambda \cap f^{-1}(V) : \lambda \in \Lambda\} = \bigcup \{(f/A_\lambda)^{-1}(V) : \lambda \in \Lambda\}
\]

Hence \( f \) is strongly \( \alpha \)-sg\( \alpha \)-irresolute.

**Lemma:** 3.22

If \( f: X \rightarrow Y \) is strongly \( \beta \)-sg\( \alpha \)-irresolute and \( A \) is a \( \alpha \)-open subset of \( X \), then \( f/A : A \rightarrow Y \) is strongly \( \beta \)-sg\( \alpha \)-irresolute.

**Proof:** Let \( V \) be a sg\( \alpha \)-open in \( Y \). By hypothesis, \( f^{-1}(V) \) is \( \beta \)-open in \( X \). But \((f/A)^{-1}(V) = A \cap f^{-1}(V)\) is \( \beta \)-open in \( A \). Hence \( f/A \) is strongly \( \beta \)-sg\( \alpha \)-irresolute.

**Theorem:** 3.23

Let \( f: X \rightarrow Y \) and \( \{A_\lambda : \lambda \in \Lambda\} \) be a cover of \( X \) by \( \beta \)-open sets of \((X, \tau)\). Then \( f \) is a strongly \( \beta \)-sg\( \alpha \)-irresolute function if \( f/A_\lambda : A_\lambda \rightarrow Y \) is strongly \( \beta \)-sg\( \alpha \)-irresolute for each \( \lambda \in \Lambda \).

**Proof:** Let \( V \) be any sg\( \alpha \)-open set in \( Y \). By hypothesis, \((f/A_\lambda)^{-1}(V) \) is \( \beta \)-open in \( A_\lambda \). Since \( A_\lambda \) is \( \beta \)-open in \( X \), it follows that \((f/A_\lambda)^{-1}(V) \) is \( \beta \)-open in \( X \) for each \( \lambda \in \Lambda \).

\[
f^{-1}(V) = X \cap f^{-1}(V) = \bigcup \{A_\lambda \cap f^{-1}(V) : \lambda \in \Lambda\} = \bigcup \{(f/A_\lambda)^{-1}(V) : \lambda \in \Lambda\}
\]

Hence \( f \) is strongly \( \beta \)-sg\( \alpha \)-irresolute.

**Theorem:** 3.24

If a function \( f: X \rightarrow Y \) is strongly \( \beta \)-sg\( \alpha \)-irresolute, then \( f^{-1}(B) \) is \( \beta \)-closed in \( X \) for any nowhere dense set \( B \) of \( Y \).

**Proof:** Let \( B \) be any nowhere dense subset of \( Y \). Then \( Y - B \) is regular in \( Y \) and hence sg\( \alpha \)-open in \( Y \). By hypothesis, \( f^{-1}(Y - B) \) is \( \beta \)-open in \( X \). Hence \( f^{-1}(B) \) is \( \beta \)-closed in \( X \).
REFERENCES