# THE NEIGHBORHOOD TOTAL EDGE DOMINATION NUMBER OF A JUMP GRAPH

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ABSTRACT

Let J(G)=( V, E ) be a jump graph without isolated vertices and isolated edges. An edge dominating set F of J(G) is called a neighborhood total edge dominating set if the edge induced sub graph < N – F > has no isolated edges. The neighborhood total edge domination number √’$_{nt}$(J(G)) of jump graph J(G) is the minimum cardinality of neighborhood total edge dominating set of J(G). In this paper, we initiate a study of this new parameter.

**Key words:** edge domination, connected edge domination total edge domination, neighborhood total edge domination**.**

Mathematics subject classification 05C69

## 1. INTRODUCTION:

All graphs considered here are finite, undirected without loops and multiple edges unless and otherwise stated, the graph J(G)= ( V, E ) considered here have p= |V| vertices and q=|E| edges .Any undefined terms in this paper may found in kulli [2].

A set D of vertices in a graph J(G) is called a dominating set if every vertex in V-D is adjacent to some vertex in D. The domination number √(J(G)) of J(G) is minimum cardinality of a dominating set of J(G) {Anupama S.B.et. al.,]

A set E of edges in ajump graph J(G) is called an edge dominating set if every edge in E-F is adjacent to at least one edge in F. The edge domination number √’(J(G)) of J(G). The concept of edge domination was introduced by Mitchell and Hedetiniemi in [21] and was studied by several authors.

An edge dominating set F of jump graph J(G) is a connected edge dominating set if the edge induced sub graph < F > is connected. The connected edge domination number √’$_{c}$(J(G)) of J(G) is minmum cardinality of a connected edge dominating set of J(G). The concept of connected edge domination was introduced by kulli and Sigarkanti [16] and was studied [17]. A set F of edges in a graph J(G)= ( v, E ) is called totl edge dominating set o9f J(G) if every edge in E is adjacent to at least one edge in F. The total edge domination number √’$_{t}$(J(G)) of J(G) is the minimum cardinality of total edge dominating set of J(G). this concept of edge domination number introduced by Kulli and Ptwari [15].

The vertices and edges of J(G) are called elements of J(G). A set X of elements of J(G) is an entire dominating set if every element not in X is either adjacent or incident to at least one element in X. The entire domination number √ε (J(G)) of J(G) is the minimum cardinality of entire dominating set of J(G). A set X of elements in J(G) is a total entire dominating set if every element in a total entire dominating set if every element in J(G) is either adjacent or incident to at least one element in X. The total entire domination number √ε (J(G) of J(G) is the minimum cardinality of a total dominating set of J(G). For any vertex v∈ V(J(G)), the open neighborhood of v the set N(v)= { v∈ V(J(G)), : uv ∈ E } and closed neighborhood of v is the set N[v] = N(v) ∪ {v}. For a set S ⊂ V(J(G)), he open neighborhood N(s) of S is defined by

$N(S)= \bigcup_{v \in S} N(v)$ for all vertices v ∈ S and closed neighborhood of s is

N[S] = N(S) ∪ S. Let S be the set of vertices and let u ∈ S. The private neighbor set of u with respect to S is the set pn[ u, S] = { v: N[S] ∩ S = {u} }. For any edge e ∈ E the open neighborhood of e is N(e) and the closed neighborhood of e is

N[e] = N(e) ∪{e}. If F ⊆ E and $e_{1=}$ ∈ F, $N(F) = \bigcup_{e \in S} N(e) and N[F]$ ∪ F if F ⊆ E and

$e_1$∈F, then private neighbor of $e_1$ with respect to F is the set

pn[ $e_1$ F ] = {$e_2$ : N[ $e_2$] ∩ F = { $e_1$} } . The degree of an edge uv is defined by deg u + deg v – 2. An edge uv is called an isolated edge if deg uv=0 Let Δ ‘(J(G)) denotes the maximum degree among the edges of J(G).

In the cycle $C_9$ = { $e_1$, $e_2$,…..$e_9$}, $F_1$ = {$e_1$, $e_4$, $e_7$} and $F_{2==}$ { $e_2$, $e_4$, $e_6$, $e_8$} are edge dominating set of J($C_9$). The induced sub graph < N($F_1$)> has no isolated edges and the induced sub gaph < N($F_2$)> has isolated edges . We introduced the concept of neighborhood total edge domination number and study some parameters.

**2.Results;**

We assume throughout that J(G) in a jump gasph without isolated vertices and without isolated edges

**Definition 1**. An edge dominating set F of a jump graph J(G) is called a neighborhood total edge dominating set if the induced sub graph < N – F > contains no isolated edges. The neighborhood total edge domination number $\surd'_{nt}$(J(G)) of J(G) is the minimum cardinality of a neighborhood total edge dominating set of J(G).

**Definition 2.** A neighborhood total edge dominating set is minimal if no proper subset of F is a neighborhood total edge dominating set.

**Poposition 3**; For a jump graph J(G)

$$\surd'(J(G)) \leq \surd'_{nt}(J(G)) \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

**Proof;** Every neighborhood total edge dominating set is an edge dominating set. Thus (1) holds.

**Theorem 4:** If $P_n$ is a path with n ≥ 4 vertices then $\surd'(P_n) = \lfloor \frac{n}{2} \rfloor$.

**Proof:** let $P_n$ = ($V_1$, $V_2$ ……….$V_n$) be a path eith n ≥ 4 vertices If n≡ r (mod 4) r= 0 0r 3 then F = { $e_i$ : i= 4k-2, 4k-1, k=1,2,…..} is a neighborhood total edge dominating set of $P_n$ in n ≡ 2 (mod 4) then F ∪ { $e_{n-2}$} is a neighborhood total edge dominating set of $P_n$ .

Thus $\surd'(P_n) = \lfloor \frac{n}{2} \rfloor$.

Further if n ≡ 2 (mod 4), then for any $\surd'_t$-set F of $P_n$ < N(F)> has at least one isolated edge. Thus $\surd'_{nt}(P_n) \geq \lceil \frac{n}{2} \rceil \geq \lfloor \frac{n}{2} \rfloor$. Hence the result.

**Corollary 5.** If $P_n$ is a path with n ≥ 4 vertices then $\surd'_{nt}(J(P_n)) = \surd'_t(J(P_n))$ if and only if n is even or n ≡ 1 (mod 4)

**Proof:** Since $\surd'_t(J(P_n)) = \frac{n}{2}$ if n is even

$= \lfloor \frac{n}{2} \rfloor$., if $n \equiv 1 \pmod 4$, the result follows.

**Theorem 6:** If $C_n$ is a cycle with $n \geq 3$ vertices then

$$\surd'_{nt}(J(C_n)) = \lceil \frac{n}{3} \rceil + 1 \text{ if } n \equiv 2 \pmod 3$$

$$= \lceil \frac{n}{3} \rceil \text{ otherwise.}$$

**Proof:** Let $C_n = \{ v_1, v_2, \ldots\ldots\ldots v_n, v_1 \}$ be a cycle with $n \geq 3$ vertices if $n \equiv r \pmod 3$, r= 0 or 1, then F= { $e_i$: i= 3k-2, k=1,2,…} is a neighborhood total edge dominating set of $C_n$ if $n \equiv 2 \pmod 3$, then $F \cup \{e_n\}$ is a neighborhood total edge dominating set of $C_n$.

Then $$\surd'_{nt}(J(Cn))) = \begin{cases} \lceil \frac{n}{3} \rceil + 1 & \text{if } p \equiv 2 \pmod 3 \\ \lceil \frac{n}{3} \rceil & \text{otherwise} \end{cases}$$

We have $\surd'_{nt}(J(C_n)) \geq \surd\,'(J(C_n)) = \lceil \frac{n}{3} \rceil$ if $n \equiv 2 \pmod 3$then for any √'-set of F of $J(C_n)$, <N(F)> has at least one isolated edge. Thus $\surd'_{nt}(J(Cn))) \geq \lceil \frac{n}{3} \rceil + 1$ Hence the result.

**Corollary 7:** If $C_n$ is a cycle with $n \geq 3$ vertices then

$\surd'_{nt}(J(C_n)) = \surd'_t(J(C_n))$ if and only if n=4,5 or 8

$\surd'_{nt}(J(C_n)) = \surd'_c(J(C_n))$ if and only if n=3, 4 or 5

$\surd'_{nt}(J(C_n)) = \surd'(J(C_n))$ if and only if $n \equiv 0$ (mod 30 OR $n \equiv 1 \pmod 3$)

**Proof:** Since $\surd'_t(J(C_n)) = \frac{n}{2}$ if $n \equiv 0 \pmod 4$

$= \lfloor \frac{n}{2} \rfloor$ if $n \equiv 1 \pmod 4$ or $n \equiv 3 \pmod 4$

$= \lfloor \frac{n}{2} \rfloor + 1$ if $n \equiv 2 \pmod 4$

$\surd'_c(J(C_n)) = n-2$

$\surd'(C_n) = \lceil \frac{n}{3} \rceil$ the result follows

**Theorem 8:** If $K_{m,n}$ is a complete bipartite graph with $2 \leq m \leq n$ then

$$\surd'_{nt}(J(K_{m,n})) = m$$

**Proof:** In $K_{m,n}$ v is a vertex such that deg v = m Let F be the set of all edges incident with a vertex v. It is easy to see that f is an edge dominating set and the induced sub graph ,N(F)> is connected and does not contain an isolated edge.

Hence f is a neighborhood total edge dominating set.

Then $\surd'_{nt}(J(K_{m,n})) \leq |F| = \deg v = m$ since

$\surd'_{nt}(J(K_{m,n})) = m$

The theorem follows.

**Theorem9:** If $K_p$ is a complete graph with $p \geq 3$ vertices then $\surd'_{nt}(J(K_p)) = \lfloor \frac{p}{2} \rfloor$

**Proof;** Let F be a minimum matching J( $K_p$)) . clearly F is an edge dominating set and also ,N(F)> is connected and does not contain an isolated edge. hence f is a neighborhood total edge dominating set. Then

$\surd'_{nt}(J(K_{m,n})) \leq |F| = \lfloor \frac{p}{2} \rfloor$

since $\surd'(J(K_p)) = \lfloor \frac{p}{2} \rfloor$ the result follows.

**Theorem 10:** A super set of a neighborhood edge total dominating set is a neighborhood total edge dominating set.

**Proof;** Let F be a neighborhood total edge dominating set of a jump graph J(G). Let $F_1 = F \cup \{e\}$ where $e \in E - F$. Then $e \in N(F_1)$ and $F_1$ is an edge dominating set of J(G). Suppose the induced subgraph $< N(F_1)>$ contains an isolated edge $e_1$. Then $N(e_1) \subseteq F - N(F)$. Then $e_1$ is an isolated edgein $<N(F)>$ which is a contradiction. Thus $<N(F_1)>$has no isolatedvertices.Therefore $F_1$ is a neighborhood total edge dominating set.

We establish a characterization of minimumneighborhod total edge dominating set.

**Theorem 11:** A neighborhood total edge dominating set F of a jump graph J(G) is minimal if and only if for every $e \in F$. one of the following holds

(i) $Pn[ e, F ] \neq \emptyset$

(ii)There exists an edge $e_1 \in N(F - \{e\})$ such that $N(e_1) \cap N(F - \{e\}) = \emptyset$

**Proof:** Let F be a minimal neighborhood total edge dominating set of J(G). let $e \in F$. Then either $F - \{e\}$ is an edge dominating set. And the induced sub graph $<N(F - \{e\})>$ contains an isolated vertex. Suppose $F - \{e\}$ is not an edge dominating set. Then $Pn[ e, F ] \neq \emptyset$. Suppose $F - \{e\}$ is an edge dominating set and $e_{1=} \in N( F - \{e\})$ is an isolated edge in $<N(F - \{e\})>$. Then $N(e_1) \cap N(F - \{e\}) = \emptyset$

Conersly suppose f is a neighborhood total edge dominating set of J(G) satisfying the conditions (i) and (ii). Then F is a minimal neighborhood total edge dominating set. Thus by theorem 10 the result follows.

**Theorem 12.** Let T be a tree. Then $\surd'_{nt}(T) =1$ if and only if $T = K_{1,p}$ $p \geq 3$ or S=m,n= $2\leq m \leq$ **n**

**Proof:** Let $T = p_3$ or $p_{4=}$ then clearly $\surd'_{nt}(J(T)) =2$ Thus $T \neq P_3$ or $P_4$. Let $\surd'_{nt}(J(T)) =1$. Let F= {e} be the $\surd'_{nt}$-set of J(T). Let e=uv, since $T \neq P_3$

deg $v \geq 3$ suppose deg u =2. Then <N(F)> has two components in which one component is an isolated edge. which is a contradiction. This implies that deg u = 1 or deg $u \geq 3$ If deg u =1 then $\surd'_{nt}(J(T)) = 1$ and $J(T) = K_{1,p}$

$p \geq 3$ If deg $u \geq 3$ $\surd'_{nt}(J(T)) = 1$ and J(T) = S=m,n $2 \leq m \leq n$.

Converse is obvious.

**Proposition 13 ;** If J(G) is a connected jump gaph with $\Delta' < q - 1$ then $\surd'_{nt}(J(G)) = q - \Delta'$.

**Proof:** Let e be an edge of a connected jump graphJ(G) and deg e $=\Delta'$ Since $\Delta' < q - 1$, there exists two adjacentedge $e_1$ and $e_2$ such that

$e_1 \neq e_2$ , $e_1 \in N(e)$ and and $e_2 \notin N9e)$. Let $F = ( N9e) - e_1) \cup \{ e_{2=} \}$. Then $|F| = \Delta'$. Further it is easy to see that E – f is a neighborhood total edge dominating set of J(G). thus $\surd'_{nt}(J(G)) \leq | E - F | = q - \Delta'$.

**Theorem 15:** for any graph J(G) $\surd'_{nt}(J(G)) = q$ if and only if $J(G) = mP_3$.

**Proof:** suppose $\surd'_{nt}(J(G)) = q$ on the contrary assume $G \neq mP_3$. Then J(G) has at least one component $G_1$ which is not $P_3$. Clearly all edges of $G_1$ are not in a neighborhood total edge dominating set. Hence $\surd'_{nt}(J(G)) \neq q$ which is a contradiction. Hence $J(G) = mP_3$.

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