# REVERSE - GRAPHOIDAL MAGIC STRENGTH

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***Abstract*** : The magic labelling $f$ of $G$, there is a constant $c(f)$ such that $f(x) + f(y) + f(xy)$, , for every edge $xy \in E(G)$. The magic strength of $G$ is defined as the minimum of all $c(f)$, and is denoted by $m(G)$. Let $G = (V, E)$ be a graph and let $\psi$ be a graphoidal cover of $G$. In this paper we determine reverse process of magic graphoidal strength called reverse- graphoidal magic strength and also proved reverse- graphoidal magic strength of Path, Star, Comb, and $[P_n : S_1]$.

***Index Terms:***: Graphoidal Constant, Magic Graphoidal, Magic Srength, reverse- magic graphoidal, reverse-grahoidal magic strength.

## 1.INTRODUCTION

A graph $G$ is said to be magic if there exist a bijection $f: V \cup E \rightarrow \{1,2,3 \dots \dots m + n\}$; where '$n$' is the number of vertices and '$m$' is the number of edges of a graph. Such that for all edges $xy$, $f(x) + f(y) + f(xy)$ is a constant. Such a bijection is called a magic labeling of $G$. Let $P$ be a path $\{v_1, v_2, \dots \dots, v_n\}$ in $\psi$ with $f^*(P) = f(v_1) + f(v_n) + \sum_{i=1}^{n-1} f(v_i v_{i+1}) = k$ is a constant, where $f^*$ is the induced labeling on $\psi$. Then, we say that G admits $\psi$- magic graphoidal total labeling of G. A graph G is called magic graphoidal if there exists a minimum graphoidal cover $\psi$ of G such that G admits $\psi$- magic graphoidal total labelling of *G*.

B.D. Acharya and E. Sampath Kumar [1] defined graphoidal covering of graph. Selvam,Vasuki, Jeyanthi [ 9 ] introduced the concept of magic strength of a graph .

Here we introduced a new concept (ie. Reverse) process of magic strength of a graphoidal is called ***reverse- graphoidal magic strength***.

### Definition 1.1

The ***Path graph*** $P_n$ is the $n -$ vertex graph with $(n-1)$ edges, all on a single path.

### Definition1.2

A complete bipartite graph $K_{1,n}$ is called a ***star*** and it has $(n+1)$ vertices and $n$ edges

### Definition 1.3

The ***Trivial graph*** $K_1$ or $P_1$ is the graph with one vertex and no edges.

### Definition1.4

Let $P_n \Theta K_1$ be the ***Comb*** which is the graph obtained from a path $P_n$ by attaching pendant edge at each vertex of the path .

### Definition1.5

Let $S_1 = (v_0, v_1)$ be a star and let $[P_n : S_1]$ be the graph obtained from $n$ copies of $S_1$ and the path $P_n = (u_1, u_2, u_3, \dots \dots \dots \dots, u_n)$ by joining $u_j$ with the vertex $v_0$ of the $j^{th}$ copy of $S_1$ by means of an edge, for $1 \le j \le n$

## II.MAIN RESULTS

### Definition 2. 1

A reverse magic graphoidal labeling of a graph $G$ is one-to-one map $f$ from $V(G) \cup E(G) \rightarrow \{1,2,3, \dots \dots \dots, m + n\}$where '*n*' is the number of vertices of a graph and '*m*' is the number of the edges of a graph, with the property that , there is an integer constant '$\mu'_{rmgc}$ such that

$f^*(P) = \sum_{i=1}^{n-1} f(v_i\, v_{i+1}) - \{f(v_1) + f(v_n)\} = \mu_{rmgc}$, is a contant

Then the reverse methodology of magic graphoidal labeling is called reverse- magic graphoidal labeling (rmgl). Reverse process of magic graphoidal of a graph is called reverse- magic graphoidal graph.(rmgg).

Selvam and Vasuki [9] made a note, Let $f$ be a magic labeling of $G$ with constant $c\ (f)$.Then adding all the constant obtained at each edge. We have

$$\varepsilon\, c(f) = \sum_{v \in V} d(v) f(v) + \sum_{e \in E} f(e)$$

From the above equation we introduce the concept of reverse process of graphoidal magic strength and it is called ***reverse - graphoidal magic strength*** and it is denoted as $rgms(G)$, is defined as the minimum of all $\mu_{rmgc}$ where the minimum is taken over all reverse magic graphoidal total labeling $f$ of $(G)$.

ie, $\boldsymbol{rgms\ (G) = min\ \{\mu_{rmgc}((f)): f}$ is a reverse- magic graphoidal labeling of $G$ }

To proceed further, we make the following equation.

**Note 1.** Let $f$ be a reverse magic graphoidal labeling of $G$ with the constant $\mu_{rmgc}$. Then ,adding all constant obtained at each edge, we get

$$\mu_{rmgc}(f) = \sum_{e \in E} f(e) - \sum_{v \in V} d(v) f(v)$$

**Theorem 2.1**

$$rgms(P_n) = \frac{3n^2 - 9n + 2}{2}$$

**Proof:** Let $(v_1, v_2, \ldots\ldots, v_n)$ are the vertices and $\{(v_1v_2), (v_2v_3), \ldots\ldots, (v_{n-1}v_n)\}$ are the edges of $P_n$.

Define $f: V \cup E \rightarrow \{1, 2, \ldots\ldots, m+n\}$ by

$f(v_1) = 1$

$f(v_n) = m + n = 2n - 1$

$f(v_1v_2) = n$

$f(v_2v_3) = n + 1$

$f(v_3v_4) = n + 2$

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$f(v_{n-1}v_n) = 2n - 2$

Let $\psi = \{P = (v_1v_2), (v_2v_3), (v_3v_4), \ldots\ldots, (v_{n-1}v_n)\}$

And we have the equation,

$$\mu_{rmgc}(f) = \sum_{e \in E} f(e) - \sum_{v \in V} d(v) f(v)$$

The equation becomes,

$$\mu_{rmgc} f(P) = f(v_1v_2) + f(v_2v_3) + \cdots \ldots . f(v_{n-1}v_n) - \{1 \times f(v_1) + 1 \times f(v_n)\}$$

$$= n + n + 1 + n + 2 + \cdots \ldots + 2n - 2 + \{1 \times 1 + (2n-1) \times 1\}$$

$$= n(n-1) + \frac{(n-1)(n-2)}{2} - 2n$$

$$= n^2 - n + \frac{3n^2 - 3n + 2}{2} - 2n$$

$$= \frac{3n^2 - 9n + 2}{2} \qquad \underline{\hspace{6cm}}(1)$$

From the equation (1), we conclude that

$$\mu_{rmgc}(P_n) = \frac{3n^2 - 9n + 2}{2}$$

$$\therefore \quad rgms\ (P_n) = \frac{3n^2 - 9n + 2}{2}$$

**Theorem 2.2**

$$rgms\ (K_{1,n}) = -(n+1)$$

**Proof:**

Let $(v, v_1, v_2, \ldots \ldots \ldots \ldots \ldots \ldots . . , v_n)$ are the vertices and $\{(vv_1), (vv_2), (vv_3), \ldots \ldots . . , (vv_n)\}$ are the edges of $K_{1,n}$.

Define $f: V \cup E \rightarrow \{1, 2, \ldots . . . , m + n\}$ by

$$f(v) = m + n = 2n + 1$$

$$f(v_1) = 1, \quad f(v_2) = 2, \quad f(v_3) = 3, \ldots \ldots \ldots \ldots \ldots \ldots ., f(v_n) = n$$

$$f(vv_1) = n + 1, \ f(vv_2) = n + 2, f(vv_3) = n + 3, \ldots \ldots \ldots \ldots , f(vv_n) = 2n$$

Let $\psi = \{P = (vv_1), (vv_2), (vv_3), \ldots \ldots (vv_n)\}$

And we have the equation,

$$\mu_{rmgc}(f) = \sum_{e \in E} f(e) - \sum_{v \in V} d(v) f(v)$$

Then the equation becomes,

$$\mu_{rmgc} f(P_{(1)}) = f(vv_1) - \{1 \times f(v) + 1 \times f(v_1)\}$$

$$= n + 1 - \{1 \times 1 + (2n+1) \times 1\}$$

$$= n + 1 - \{1 + 2n + 1\}$$

$$= -(n+1)\underline{\hspace{6cm}}(1)$$

$$\mu_{rmgc} f(P_{(2)}) = f(vv_2) - \{1 \times f(v) + 1 \times f(v_2)\}$$

$$= n + 2 - \{1 \times 2 + 1 \times (2n+1)\}$$

$$= -(n+1) \ \underline{\hspace{6cm}} \quad (2)$$

Continuing this process,

$\mu_{rmgc} f(P_{(k)}) = f(vv_n) - \{1 \times f(v) + 1 \times f(v_n)\}$

$= 2n - \{1 \times (2n+1) + 1 \times n\}$

$= 2n - \{2n + 1 + n\} = 2n - \{3n - 1\}$

$= -(n+1)$ ______________________________(3)

From(1), (2) and (3) we conclude that

$$\mu_{rmgc}(K_{1,n}) = -(n+1)$$

$$\therefore \quad rgms(K_{1,n}) = -(n+1)$$

**Theorem 2.3**

$rgms\ (P_n \theta K_1) = 3 \qquad for\ n > 2$

**Proof :**

Let $\{v_1, v_2, \dots, v_n, u_1, u_2, \dots \dots, u_n\}$ be the vertex set of $(P_n \theta K_1)$ and $\{(v_1u_1), (v_2u_2), \dots \dots, (v_nu_n), (v_1v_2), (v_2v_3), \dots \dots, (v_{n-1}v_n)\}$ are the edge set of $(P_n \theta K_1)$

Here $m + n = 4n - 1$

Define $f : V \cup E \to \{1,2, \dots \dots, m+n\}$ by

$f(u_1) = 1, f(u_2) = 4n - 1, f(u_3) = 4n - 2, f(u_4) = 4n - 3, \dots \dots \dots, f(u_n) = 3n + 1$

$f(v_2) = 2, f(v_3) = 3, \dots \dots \dots \dots \dots \dots, f(v_{n-1}) = n - 1$

$f(v_1u_1) = n, f(v_1v_2) = n + 1, f(v_2v_3) = 2n, \dots \dots \dots \dots \dots, f(v_{n-1}v_n) = n + 3$

$f(v_2u_2) = 2n + 2, f(v_2u_3) = 2n + 3, \dots \dots \dots \dots \dots \dots, f(v_nu_n) = 3n$

Let $\psi = \{P_1 = (u_1v_1u_2v_2),$

$P_2 = (v_2v_3u_3), (v_3v_4u_4), \dots \dots \dots, (v_{n-1}v_nu_n)\}$

And we have the equation,

$$\mu_{rmgc}(f) = \sum_{e \in E} f(e) - \sum_{v \in V} d(v) f(v)$$

Then the equation becomes,

$\mu_{rmgc} f(P_1) = f(u_1v_1) + f(v_1v_2) + f(v_2u_2) - \{1 \times f(u_1) + 1 \times f(u_2)\}$

$= n + n + 1 + 2n + 2 - \{1 \times 1 + 1 \times (4n - 1)\}$

$= 4n + 3 - \{1 + 4n - 1\}$

$= 3$ __________________________ (1)

$\mu_{rmgc} f(P_{(2(1))}) = f(v_2v_3) + f(v_3u_3) - \{1 \times f(v_2) + 1 \times f(u_3)\}$

$= 2n + 2n + 3 - \{1 \times 2 + 1 \times (4n - 2)\}$

$= 4n + 3 - \{2 + 4n - 2\}$

$= 3$ ______________________________ $(2)$

Continuing this process,

$\mu_{rmgc} f(P_{(2(k)}) = f(v_{n-1}u_n) + f(v_n u_n) - \{1 \times f(v_{n-1}) + 1 \times f(u_n)\}$

$= n + 3 + 3n - \{1 \times (n-1) + 1 \times (3n+1)\}$

$= 4n + 3 - \{n - 1 + 3n + 1\}$

$= 3$ ______________________________ $(3)$

from $(1), (2)$ and $(3)$ we conclude that

$$\mu_{rmgc}(P_n \theta K_1) = 3$$

$\therefore\ rgms\ (P_n \theta K_1) = 3$

**Theorem 2.4**

$rgms[P_n : S_1] = 4n + 8, \quad for\ n > 2$

**Proof :**

Let $\{v_1, v_2, \dots \dots, v_n, w_1, w_2, \dots \dots \dots, w_n, u_1, u_2, \dots \dots, u_n\}$ be the vertex set and $\{(v_1w_1), (v_2w_2), \dots \dots, (v_n w_n), (w_1u_1), (w_2u_2), \dots \dots, (w_n u_n), (v_1v_2), (v_2v_3), \dots \dots, (v_{n-1} v_n)\}$ be the edge set of $[P_n : S_1]$

Here $m + n = 6n - 1$

Define $f : V \cup E \rightarrow \{1, 2, \dots \dots, m + n\}$ by

$f(u_1) = 1, f(u_2) = 6n - 1, f(u_3) = 6n - 2, \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots, f(u_n) = 5n + 1$

$f(v_2) = 2, f(v_3) = 3, f(v_4) = 4, \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots, f(v_{n-1}) = n - 1$

$f(u_1w_1) = n, \quad f(w_1v_1) = n + 1, \quad f(v_1v_2) = n + 2,$

$f(v_2w_2) = 3n + 3, f(v_3w_3) = 8n + 4, f(v_4w_4) = 3n + 5, \dots \dots \dots, f(v_nw_n) = 4n + 1$

$f(w_2u_2) = 4n + 2, f(w_3u_3) = 4n + 3, f(w_4u_4) = 4n + 4, \dots \dots \dots, f(w_nu_n) = 5n$

$f(v_2v_3) = 3n + 1, \quad f(v_3v_4 = 3n - 1, \dots \dots \dots \dots \dots, f(v_{n-1}v_n) = n + 7$

Let $\psi = \{P_1 = (u_1w_1v_1v_2w_2u_2),$

$P_2 = (v_2v_3w_3u_3), \dots \dots \dots \dots \dots, (v_{n-1}v_nw_nu_n)\}$

And we have the equation,

$$\mu_{rmgc}(f) = \sum_{e \in E} f(e) - \sum_{v \in V} d(v) f(v)$$

Then the equation becomes,

$$\mu_{rmgc} f(P_1) = f(u_1w_1) + f(w_1v_1) + f(v_1v_2) + f(v_2w_2) + f(w_2u_2) - \{1 \times f(u_1) + 1 \times f(u_2)\}$$

$= n + n + 1 + n + 2 + 3n + 3 + 4n + 2 - \{1 \times 1 + 1 \times (6n - 1)$

$= 10n + 8 - \{1 + 6n - 1\}$

$= 4n + 8$ ______________________________ $(1)$

$\mu_{rmgc}(f(P_{2(1)}) = f(v_2v_3) + f(v_3w_3) + f(w_3u_3) - \{1 \times f(v_2) + 1 \times f(u_3)\}$

$$= 3n + 1 + 3n + 4 + 4n + 3 - \{1 \times 2 + 1 \times (6n - 2)\}$$

$$= 4n + 8 \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (2)$$

Continuing this process,

$$\mu_{rmgc} f(P_{2(k)}) = f(v_{n-1} v_n) + f(v_n w_n) + f(w_n u_n) - \{1 \times f(v_{n-1}) + 1 \times f(u_n)\}$$

$$= n + 7 + 4n + 1 + 5n - \{1 \times (n - 1) + 1 \times (5n + 1)\}$$

$$= 4n + 8 \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (3)$$

From(1), (2) and (3) we conclude that

$$\mu_{rmgc}[P_n : S_1] = 4n + 8$$

$\therefore \; rgms\, [P_n : S_1] = 4n + 8$

## III. CONCLUSION

The magic strength of a graph is one the most interesting area in graph theory. As all the graphs reverse techniques of magic strength is very interesting to investigate graphs or graph families which admit reverse- graphoidal magic strength. Here we reporting reverse- graphoidal magic strength of various graphs

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