PROPERTIES OF FUZZY DISTANCE MEASURE

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Abstract:

In this paper we investigate the properties of some recently proposed fuzzy distance measure. We find out some short comings for these distances and then the obtained results are illustrated by solving several examples and compared with the other fuzzy distances.

Keywords:

Fuzzy distance measure,trapezoidal fuzzy numbers,triangular fuzzy numbers.

I.Introduction:

Applications of fuzzy numbers for indicating uncertain and vague information in decision making linguistic controllers expert systems,data mining, pattern recognition. Recently several authors have attempted compute the fuzzy distance between Fuzzy numbers.In (2) voxman proposed novel fuzzy distance .In(4) a new fuzzy distance is proposed based on the interval difference and also the metric properties are studied this is the starting point to proposed some new result on the fuzzy distance measure. Consequently Guha and Chakraborty in (2) improved the previous fuzzy distance measure by applying absolute value of fuzzy numbers which is obtained through the extension principle.

In sec 2 some basic concepts are provided. In sec 3 some fuzzy distance Measures are considered and then new results are given in sec 4 .some illustrative Examples are given.

II. PRELIMINARIES:

2.1 Definition:

A generalized fuzzy number $\breve{A}$ represented by $\breve{A} = (a_1 , a_2, \beta , \gamma)$ is a normalized convex fuzzy set on the real line R if

(i) Supp $\breve{A} = x ,\mu_{\breve{A}} (x) > 0$ is a closed and bounded interval

(ii) $\mu_{\breve{A}}$ is an upper semi continuous function

(iii) $a_1 - \beta < a_1 \leq a_2 < a_2 + \gamma$

(iv) Membership function is of the following form

$$\mu_{\breve{A}}(x) = \begin{cases} f(x) & ,x \in [a_1 - \beta, a_1] \\ 1 & x \in [a_1, a_2] \\ h(x) & x \in [a_2, a_2 + \gamma] \end{cases}$$

where f and h are the monotonic increasing and decreasing functions in $[a_1 - \beta, a_1]$ and $[a_2, a_2 + \gamma]$

2.2 Definition :

If A is a fuzzy number with r-cut representation $[L_A^{-1}(r), R_A^{-1}(r)]$ And S is a reducing function then the value of A is defined as

$$\text{Val}(A) = \int_0^1 S(r)[L_A^{-1}(r) + R_A^{-1}(r)]\,dr$$

2.3 Definition:

If A is a fuzzy number with r-cut representation $[L_A^{-1}(r), R_A^{-1}(r)]$ and S is a reducing function then the ambiguity of A is defined as

$$\text{Amb}(A) = \int_0^1 S(r)[R_A^{-1}(r) - L_A^{-1}(r)]\,dr$$

III .SOME FUZZY DISTANCE MEASURES:

3.1 Voxman's fuzzy distance measure:

We briefly describe the fuzzy distance measure by Voxman

The fuzzy distance function of F $,\Delta : F \times F \to F$

$$\Delta(A,B)(Z) = \text{Sup}_{|x-y|=z} \min\{\mu_A(x), \mu_B(y)\}$$

For each pair of fuzzy number A,B. let $\Delta_{A,B}$ denoted the fuzzy number $\Delta(A,B)$.

If the r cut representation of $\Delta_{AB} = (L(r), R(r))$ is given by

$$L(r) = \begin{cases} \text{Max}(\mu_B(r) - \overline{\mu}_A(r), 0) \text{ if } (\mu_A(1) + \overline{\mu}_A(1) \le \mu_B(1) + \overline{\mu}_B(1) \\ \text{Max}(\mu_A(r) - \overline{\mu}_B(r), 0) \text{ if } (\mu_B(1) + \overline{\mu}_B(1) \le \mu_A(1) + \overline{\mu}_A(1) \end{cases}$$

And $R(r) = \max\{\overline{\mu}_A(r) - \underline{\mu}_B(r), \overline{\mu}_B(r) - \underline{\mu}_A(r)\}$

3.1 THEOREM:

Let us consider

$$\frac{\underline{\mu}_A(1) + \overline{\mu}_A(1)}{2} = \frac{\underline{\mu}_B(1) + \overline{\mu}_A(1)}{2}$$

Then $L(r) = \max \{ \underline{\mu}_B(r) - \overline{\mu}_A(r) \ ,0\}$

$$= \max \{ \underline{\mu}_A(r) - \overline{\mu}_B(r) \ ,0\} = 0 \ \ \forall\, r \in [0,1]$$

Proof:

It is sufficient to show that the maximum value of $\underline{\mu}_B(r) - \overline{\mu}_A(r)$ and $\{ \underline{\mu}_A(r) - \mu_B(r) \}$ is less than (or) equal to 0 for all $\forall\, r \in [0,1]$

$$\underline{\mu}_B(r) - \overline{\mu}_A(r) \le 0$$

$$\underline{\mu}_A(r) - \overline{\mu}_B(r) \le 0$$

It is obvious that for all $r \in [0,1]$

$\underline{\mu}_B(r) - \overline{\mu}_A(r) \le \underline{\mu}_B(1) - \overline{\mu}_A(1)$

$\underline{\mu}_A(r) - \overline{\mu}_B(r) \le \underline{\mu}_A(1) - \overline{\mu}_B(1)$

now let us assume that

$\underline{\mu}_B(1) - \overline{\mu}_A(1) > 0$

then

$\underline{\mu}_B(r) > \overline{\mu}_A(r)$

$\underline{\mu}_B(1) > \overline{\mu}_A(1) \geq \underline{\mu}_A(1)$

$\Rightarrow \underline{\mu}_B(1) > \underline{\mu}_A(1)$

By the assumption of the theorem

$\underline{\mu}_A(1) - \overline{\mu}_B(1) = \underline{\mu}_B(1) - \overline{\mu}_A(1) > 0$

$\underline{\mu}_A(1) > \overline{\mu}_B(1) \geq \underline{\mu}_B(1)$

$\Rightarrow \underline{\mu}_A(1) > \underline{\mu}_B(1)$

So,

$\Rightarrow \underline{\mu}_A(1) = \underline{\mu}_B(1)$

We get $\overline{\mu}_B(r) - \overline{\mu}_A(r) > 0$ which leads to obtain that

$\underline{\mu}_B(r) - \underline{\mu}_A(r) > 0$

we get $\Rightarrow \underline{\mu}_A(1) > \underline{\mu}_B(1)$

moreover applying the assumption of the theorem

$\mu_A(1) - \overline{\mu}_B(1) = \mu_B(1) - \overline{\mu}_A(1) > 0$

which is contradiction

so the assumption

$\mu_B(1) - \overline{\mu}_A(1) > 0$ is wrong and should be $\mu_B(1) - \overline{\mu}_A(1) \leq 0$

hence we have

$\mu_B(1) - \overline{\mu}_A(1) \leq \mu_B(1) - \overline{\mu}_A(1) \leq 0$

hence proved.

3.2CHAKRABORTY ET AL FUZZY DISTANCE MEASURE:

3.2.1 Definition:

Consider two generalized fuzzy numbers as $A_1=(\alpha_1, \alpha_2, \beta_1, \gamma_1)$

$A_2=(b_1, b_2, \beta_2, \gamma_2)$

Therefore r cut of A_1 and A_2 represents two respectively $[A_1]_r = [A_1^L(r) - A_1^R(r)]$

And $[A_2]_r = [A_2^L(r) - A_2^R(r)]$ to formulate the fuzzy distance measure between A_1 and A_2

(i) $[A_1]_r - [A_2]_r$ if $\dfrac{A_1^L(1)+A_1^R(1)}{2} \geq \dfrac{A_2^L(1)+A_2^R(1)}{2}$

(ii) $[A_2]_r - [A_1]_r$ if $\dfrac{A_1^L(1)+A_1^R(1)}{2} < \dfrac{A_2^L(1)+A_2^R(1)}{2}$

$$\text{Where } \lambda = \begin{cases} 1 & \text{if } \dfrac{A_1^L(1)+A_1^R(1)}{2} \geq \dfrac{A_2^L(1)+A_2^R(1)}{2} \\ 0 & \text{if } \dfrac{A_1^L(1)+A_1^R(1)}{2} < \dfrac{A_2^L(1)+A_2^R(1)}{2} \end{cases}$$

The fuzzy distance measure between A_1 and A_2 is defined by

$d(x_1, x_2) = (d_{r=1}^L, d_{r=1}^R, \theta, \sigma)$

where $\theta = d_{r=1}^L - \max\{\int_0^1 d_r^L\, dr, 0, \sigma\}$

$= \int_0^1 d_r^R\, dr - d_r^R$

3.2.1 THEOREM:

Let Ă$_1$ and Ă$_2$ be two arbitrary fuzzy numbers which fuzzy distance applied.

If $\frac{A_1^L(1)+A_1^R(1)}{2} = \frac{A_2^L(1)+A_2^R(1)}{2}$ then $\theta = 0$ if both Ă$_1$ and Ă$_2$ are two triangular fuzzy numbers and $\theta < 0$ if atleast one of Ă$_1$ or Ă$_2$ are non triangular.

Proof:

Suppse that Ă$_1$ and Ă$_2$ are triangular fuzzy numbers.

then for proving Ă$_1$ and Ă$_2$ θ =0 we show that $d_{r=1}^L = 0$ $\max\{\int_0^1 d_r^L\, dr, 0\} = 0$

Since $\frac{A_1^L(1)+A_1^R(1)}{2} = \frac{A_2^L(1)+A_2^R(1)}{2}$

For [Ă$_1$]$_r$ – [Ă$_2$]$_r$

We get [Ă$_1$]$_r$ – [Ă$_2$]$_r$ = ($(A_1^L(r) - A_2^R(r)$ $(A_1^R(r) - A_2^L(r))$)

$\Rightarrow d_r^L = A_1^L(r) - A_2^R(r)$

$d_r^R = A_1^R(r) - A_2^L(r)$

There fore $d_{r=1}^L = A_1^L(1) - A_2^R(1)$

By applying assumption of the theorem it is obvious that

$A_1^L(1) = A_1^R(1) = A_2^L(1) = A_2^R(1)$

So $d_{r=1}^L = A_1^L(1) - A_2^R(1) = 0$

Also we get

$d_r^L = A_1^L(r) - A_2^R(r) \le d_{r=1}^L = 0$

$\Rightarrow \int_0^1 d_r^L\, dr \le 0$

Therefore $\max\{\int_0^1 d_r^L\, dr, 0\} = 0$

Since $\frac{A_1^L(1)+A_1^R(1)}{2} = \frac{A_2^L(1)+A_2^R(1)}{2}$

We consider for [Ă$_1$]$_r$ – [Ă$_2$]$_r$

As follow it is obvious that $A_1^L(1) \neq A_2^R(1)$

Otherwise by using assumption of theorem we should have $A_2^L(1) = A_1^R(1)$

Now let us consider $d_{r=1}^L = A_1^L(1) - A_2^R(1) > 0$

$$A_1^R(1) \geq A_1^L(1) > A_2^R(1) \geq A_2^L(1)$$

$$A_1^R(1) > A_2^L(1)$$

By hypothesis of the theorem

And the assumption $A_1^L(1) - A_2^R(1) > 0$

We have

$$\frac{A_1^L(1)+A_1^R(1)}{2} = \frac{A_2^L(1)+A_2^R(1)}{2}$$

$$\Rightarrow A_1^R(1) < A_2^L(1)$$

Then by equation we get $\Rightarrow A_1^R(1) = A_2^L(1)$

Which contradicts

$A_1^L(1) \neq A_2^R(1)$

so $d_{r=1}^L = A_1^L(1) - A_2^R(1) < 0$

$$d_r^L = A_1^L(r) - A_2^R(r)$$

$$\leq d_{r=1}^L < 0$$

$$\Rightarrow \int_0^1 d_r^L \, dr < 0$$

So $\max\{\int_0^1 d_r^L \, dr, 0\} = 0$

Hence $\theta < 0$.

4.NUMERICAL EXAMPLES:

4.1 EXAMPLE:

Let us consider $[\breve{A}_1]_r = [1+r, 3-r]$ and $[\breve{A}_2]_r = [2r, 4-2r]$

We get

$[d_r^L\ d_r^R] = [-3+3r, 3-3r]$

$\Rightarrow \theta=0$, $\sigma=1.5$

$\Rightarrow \dot{d}_{\text{Chakraborty}}(\breve{A}_1, \breve{A}_2) = (0,0,0,1.5)$

4.2 EXAMPLE:

Let us consider fuzzy numbers $[\breve{A}_1]_r = [r, 1]$

$[\breve{A}_2]_r = [1, 2-r]$ and $[\breve{A}_3]_r = [1, 3-r]$

It is easily to verify that $\breve{A}_1$ and $\breve{A}_2$ are identical and equal to 1.consider interval difference $[\breve{A}_1]_r - [\breve{A}_2]_r = [2r-2, 0]$

$\dot{d}_{\text{Chakraborty}}(\breve{A}_1, \breve{A}_2) = (0,0,0,0)$

$\dot{d}_{\text{Chakraborty}}(\breve{A}_2, \breve{A}_3) = (0,1,0,0.5)$

$\dot{d}_{\text{Chakraborty}}(\breve{A}_1, \breve{A}_3) = (0,1,0,1)$

So we have

$\dot{d}_{\text{Chakraborty}}(\breve{A}_1, \breve{A}_3) \nleq \dot{d}_{\text{Chakraborty}}(\breve{A}_1, \breve{A}_2) + \dot{d}_{\text{Chakraborty}}(\breve{A}_2, \breve{A}_3)$

4.3 EXAMPLE:

Let $[\breve{A}_1]_r = [r, 3-r]$ and $[\breve{A}_2]_r = [r, 4-2r]$

Then we get $[\breve{A}_1]_r - [\breve{A}_2]_r = [3r-4, 3-2r]$

$\dot{d}_{\text{Chakraborty}}(\breve{A}_1, \breve{A}_2) = (-1,1,-1,1)$

It is not a fuzzy number.

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