APPLICATIONS OF QUEUEING THEORY

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Abstract

There are many situations in our daily life when a queue is formed. Queuing theory is the mathematical study of waiting lines and waiting time. Queuing theory applies not only in day to day life but also in sequence of computer system, networks, medical field, banking sectors etc. In this paper, we analyze the basic application of queuing theory in our daily life.

Keywords

Waiting time,waiting cost,waiting line,system time,queue length,service process

Introduction

The main objective of this research work is to analyse the behaviour of single server queueing system.The behavior of queueing models accurately describes the operation of many telecommunication networks. Mostly retrial queues have been considered as an interesting problem in tele-traffic theory and telephone networks where subscribers redial after receiving a busy signal with the time dependent solutions. In this research work, our main objectives are,

- To obtain some important system performance measures.
- To analyse the performance measures with numerical illustration.
- To analyse the performance measure through graphical illustration.

The generating function technique has been adopted to discuss about the behaviour of single server queueing systems. Closed form solutions of steady state distributions have been derived for the transient solutions using Final value theorem on Laplace transform. For all models Numerical & Graphical studies have been done for various values of the system parameters.

Queue length

Probability distribution of queue length can be obtained with the help of the given Probability distribution of the arrival and service process.A small queue indicates excess of service facilities.

Waiting time in Queue

It refers to the time spent by the customer in the queueing system before of his service.

Waiting time in system

It involves study of a system's behavior overtime.

Motive of the study

The basic ideas of some important concepts and applications of queueing theory in the following field motivates the study of these system.This paper aims to study of the applications of telephone system,Retail shopping queue & Random access protocols in digital communication networks,Banking,computer system.

Telephone systems

A telephone subscriber who obtains a busy signal repeats the call until the demand is satisfied. As a result, the total flow of calls circulating in a telephone network consists of the superposition of the flow of primary calls, which reflects the real wishes of the use of auto repeat facilities increases the effect of the repeated demands in modern telephone systems.

Everybody knows from his/her own experience that a telephone subscriber who obtains a busy signal repeats the call until the required connection is made.

Retail shopping queue

In a shop, a customer, who finds a queue too long may wish to do something else and return later on with the hope that the queue dissolves. Similar behaviour may demonstrate some impatient customers who entered the waiting line but then discovered that the residual waiting time is too long.

The aim is to increase both the number of customers that can be served and customer satisfaction with the entire queue experience.

Random access protocols in digital communication networks

Consider a communication line with slotted time which is shared by several stations. The duration of the slot equals the transmission time of a single packet of data. If two or more stations are transmitting packets simultaneously then a collision takes places, i.e. all packets are destroyed and must be retransmitted. If the stations involved in the conflict would try to retransmit destroyed packets in the nearest slot, then a collision occurs with certainty. To avoid this, each station independently of other stations, transmits the packet with probability p and delays actions until the next slot with probability 1−p, or equivalently, each station introduces a random delay before next attempt to transmit the packet

Banking

Most banks used standard queuing models. It is useful to avoid standing in a queue for a long time. Bank is an example of unlimited queue length and bank customers' arrival time randomly and the service time between three different services: open an account, transaction, and balance, with different period of time for each service.

Computer Systems

Queues are very common in computer systems. Computer systems interact with one another to answer different queries related to the inquiries of the queue (Allen, 1977). In computer systems, queuing system helps in calculating the service facilities with one and more than one servers. It also helps in waiting room or buffer of infinite and finite capacity. Different people in different populations try to get some kind of service by entering in the queuing system. In addition, the word customer in a computer system can be used for a computer system program, packet in a communication network, job in a computer system or any kind of inquiry or request in the system (Inria, 2004).

Once a particular customer is served in the queuing system, he leaves. If service centers are busy then customer directly goes into the waiting system of computers (Inria, 2004). D. G Kendell devised the method and applied it to the theory of queues (Blum, n.d). A notation known as Kendall's notation describes the queuing system of computer systems (Inria, 2004): (A/B/c/K)Interarrival time distribution is described through A in this notation. B in this notion is the service time distribution, c is the number of servers and K is the size of the system capacity that includes the servers (Inria, 2004).

The symbol of A is often replaced by the symbol of M because it represents the exponential distribution presented through Markov (Bolch & Greiner, 2006). On the other hand, D represents deterministic distribution and G or GI for general distribution. A lot of computer system applications produce different classes of customers preferential treatment, which means that service is provided in a queue and the customers, which are of priority, are served first They also work within the framework of two basic priority policies, which are n on-preemptive priority policy and preemptive priority policy. Even simplest computer systems have multiple resource systems. Thus, there are multiple of queues associated with each of these multiple computer systems (Inria, 2004).

State of the queueing sysem

The steady state of a queueing system is the state where the probability of the number of customers in the system is independent of time.

Components of the queueing system

A queueing system can be completely described by
1.The input or arrival pattern of customers
2.The service mechanism or service pattern of servers.
3.Queue discipline.
4.System capacity.
5.Number of service channels .
6.Number of service stages.

The input or arrival pattern of customers

The input pattern means the manner in which the arrivals occur.It is specified by the inter-arrival time between any two consecutive arrivals.It is also necessary to know whether customer can arrive simultaneously(batch or bulk arrival),and if so,the probability distribution describing the size of the batch.It is also necessary to know the reaction of a customer upon entering the system.A customer may decide to wait no matter how long the queue becomes,or,on the other hand,if the queue is too long,the customer may decide not to enter the system.If a customer decides not to enter the queue upon aqrrival,the customer is said to have balked.A customer may enter the queue,but after a time,lose patience and decide to leave.In this case,the customer is said to have reneged.In the event that there are two or more parallel waiting lines,customers may switch from one to another,that is,jockey for position.These three situations are all examples of queues with impatient customer.One final factor to be considered regarding the arrival pattern is the manner in which the pattern changes with time.An arrival pattern that does not change with time(i.e. the probability distribution describing the input process is time-independent)is called a stationary arrival pattern.One that is not time-independent is called non-stationary.

The service mechanism (or service pattern)

Most important,a probability distribution is needed to describe the sequence opf customers service times.Service may also be single or batch.The service process may depend on the number of customers waiting for service.A server may work faster if the queue is building up or on the contrary,may get frustrated and become less efficient.The situation in which service depends on the number of customers waiting is referred to as state-dependent service.The important difference between the arrival and service processes is that the former process is always going on but there can be no service unless there are no customers to serve.Even if the service rate is high,it is very likely that service to some of the customers will be delayed by waiting line.In general,customers arrive and depart at irregular intervals.Hence the queue lengthwill assume no definitive pattern unless arrivals and services are deterministic.Thus it follows that a probability distribution for queue lengths will be the result of two separate processes-arrivals and services which are generally,throughout universally assumed mutually independent.Service patterns may be deterministic or stochastic.Here the service time refers to length of the time a customer spends in the service facility.The availability of service,the capacity of the system and the service time of the individual customer constitute the service mechanism of a qeueing system.

The queue discipline

The queue discipline is the rule determining the formation of the queue,the manner of the customer’s behavior while waiting,and the manner in which they are chosen for service.The simplest discipline is “First come,First served”(FCFS) according to which the customers are served in the order of their arrival.For example,such type of queue discipline is observed at a ration shop,at cinema ticket windows,at railway stations.If the order is reversed,we have ‘Last come,first served(LCFS)discipline,as in the case of a big godown the items which come last are taken out first.An extremely difficult queue discipline to handle might be “service in random order”(SIRO)or”might is right”.Some of the queue service discipline are:

System capacity

A system may have an infinite capacity-That is,the queue in front of the server may grow to any length. According to whether the capacity is infinite or finite.If finite,it will have to be specified by the number of places available for the queue as well as for the one being served,if any.In some queueing processes there is a physical limitation to the amount of waiting room,so that when the line reaches a certain length,no further customers are allowed to as finite queueing situation.

The number of service channels

The number of service channels refers to the number of parallel service stations which can service customers simultaneously.A system may have a single server or a number of parallel servers.An arrival who finds more than free server may choose at random any one of them for receiving service,If he finds all the servers busy,he joins a queue common to all the servers,The first customer from the common queue goes to the server who becomes free first.The kind of situation is common-for example,in bank or at a ticket counter.

The number of service stages

A queueing system may have only a single stage of service such as the barber shop and supermarket examples,or it may have several stages.An xample of a multistage queueing system would be a physical examination procedure,where each patient must proceed through several stages,such as medical history;ear,nose,and throat examination;blood tests;electrocardiogram;eye examination;and so on.In some multistage queueing processes recycling may occur.Recycling is common in manufacturing processes where quality control inspections are performed after certain stages,and parts that do not meet quality standards are sent back for reprocessing.

Kendall Notation for queueing system

A queue is described in shorthand notation by

(A/B/C):(D/E)

Where,

A:Describes the arrival process of the customers.

The codes used are:

a) **M**(Morkovian):Inter-arrival of customers are independently,identically and exponentiallydistributed.It corresponds to a poisson point process.

b) **D**(Degenerate):Inter-arrival of customers are constant and always the same.

c) **GI**(General Independent):Inter-arrival of customers have a general distribution

d) **G**(General):Inter-arrival of customers have a general distribution and can be dependent on each other.

B:Describes the distribution of service time of a customer.

The codes are the same as A.

C:Describes the distribution of service time of a customer.

D:The number of places in the system(in the queue)

The capacity is assumed to be unlimited,or infinite.

E:The service discipline

It is the way of customers are ordered to be served.The codes used are:

(a) FIFO(First InFirst Out),the customers are served in the order they arrived in.

(b) LIFO(Last In First Out),the customers are served in the reverse order to the order they arrived in.

(c) SIRO(Served in Random Order),the customers are served randomly.

(d) PNPN(Priority Service),the customers are served with regard to their priority.All the customers of the highest priority are served first,then the customers of lower priorities are served,and so on.The service may be pre-emptive or not.

(e) PS(Processor sharing),the customers are served equally.System capacity is shared between customers and they all effectively experience the same delay.

Notations and Symbols

The following symbols and terminology will be used henceforth in connection with the queueing models.

n = number of units in the system

$P_n(t)$= Transient state probability that exactly there are n customers in the

queueing system at time t.

p_n= Steady state probability of having n customers

λ = Mean arrival rate

μ = Mean service rate

σ = Retrial rate

ρ = Traffic intensity or Utilization factor for servers facility

L_s= Expected number of customers in the system

L_q= Expected number of customers in the queue

W_s= Expected waiting time per customer in the queueing system

W_q= Expected waiting time of customer in the queue

Little's formula

Little's law,Little's result,or Little's theorem is perhaps the most widely used formula in queueing theory was published by J.Little (1961).It is simple to state and intuitive,widely applicable,and depends only on weak assumption about the properties of the queueing system.It says that the average number of customers in the system is equal to the average arrival rate of customer to the system multiplied by the average system time per customer.

Letting,

L=average number of items in the queueing system

W=average waiting time

λ=average number of items arriving per unit time

The law is $L = \lambda W$
Further, the relationship between Ls, Lq, Ws, and Wq

1. $Ls = Lq + \frac{\lambda}{\mu}$

2. Ws = Ls

3. Wq = Lq

4. $Ws = Wq + \frac{1}{\mu}$

Conclusion

In conclusion, queuing theory has a much-diversified range of applications. It enjoys a very dominant place in the contemporary analytical techniques. Information is obtained on the basis of analysis done by the queuing systems. Queuing theory plays a very important role in the development of everyday social life. All the applications discussed above are practical and their worth is portrayed in the actions of everyday life.

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