Triple Connected Complementary Tree Domination Number Of a Graph

A. Sophiya, Mr. M. Ramesh Kumar
Department of Mathematics,
Prist University,Tanjore(Puducherry branch).India.

***Abstract**:* In this paper we introduced in triple connected domination number and the triple connected complementary tree domination number of a graph.The minimum cardinality taken over all triple connected complementary tree dominating set is called the triple connected complementary tree domination number of G and is denoted by tct(G).And also we discuss some special graphs in triple connected complementary tree.

Introduction

A complete bipartite graph is a special kind of bipartite graph with partitions of order $|V_1| = m$ and $|V_2| = n$, is denoted K_{mn}. A star denoted by $K_{1,p-1}$ is a tree with one root vertex and p-1 pendent vertices. A bistar denoted by $B(m,n)$ is the graph obtained by joining of the stars $K_{1,m}$ and $K_{1,n}$.

The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating set in G. A dominating set S of a connected graph G is said to be a connected dominating set of G if the induced subgraph $< S >$ is connected. The minimum cardinality taken over all the connected dominating set is the connected domination number and is denoted by γ_c.

Different type of domination parameters to be introduced so many authors.Recently the concept of triple connected graphs has been introduced by Paulraj Joseph J, et.al.,. All paths, cycles, complete graphs and triple connected graphs. In [4] Mahadevan G. et.al.,

The minimum cardinality taken over all complementary tree dominating sets is called the complementary tree dominating sets is called the complementary tree domination number of G and is denoted by $\gamma_{tct}(G)$. In this paper, we use this idea to develop the triple connected domination number of graph and triple connected complementary tree domination number of a graph.

II. Triple connected complementary tree domination number

Definition 2.1

The minimum cardinality taken over all triple connected complementary tree dominating sets is called the triple connected complementary tree domination number of G and is denoted by $\gamma_{tct}(G)$. Any triple connected dominating with γ_{tct} vertices is called the γ_{tct}-set of G.

Example 2.2

For any graph G_1 in Figure 2.1, $S = \{v_1, v_2, v_3\}$ forms a γ_{tct}-set of G_1. Hence $\gamma_{tct}(G_1)$ = 3.

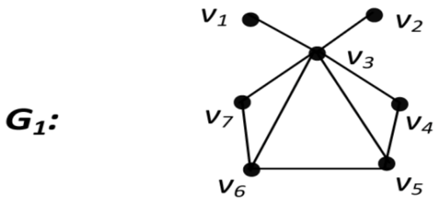

Fig 2.1 : Graph with $\gamma_{tct} = 3$

Observation 2.3

Triple connected dominating set (tcd-set) does not exist for all graphs and if exist for all graphs and if exists, then $\gamma_{tc}(G) \geq 3$.

Example 2.4

For the graph G_2 in Figure 2.2, any minimum dominating set must contain all the supports and any connected
sub graph containing these supports is not triple connected and hence γ_{tc} does not exist.

G_2 :

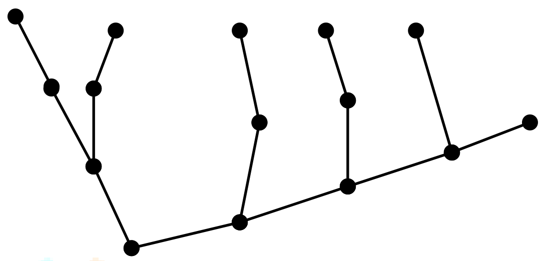

Fig 2.2 : Graph with no tcd set

Observation 2.5

For any connected graph G,$\gamma(G) \leq \gamma_c(G) \leq \gamma_{tc}(G)$ and the bounds are sharp.

Example 2.6

For the graph G_3 in Figure 2.3,$\gamma(G_3) = 4, \gamma_c(G_3) = 6, \gamma_{tc}(G_3) = 8$.

G_3 :

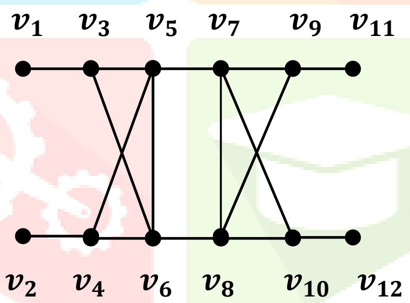

Fig 2.3

Observation 2.7

The complement of the triple connected complementary tree dominating set need not to be triple connected complementary tree dominating set.

Example 2..8

For the graph G_4 in figure 2.4, $S = \{v_1, v_2, v_3$ forms a triple connected complementary tree dominating set of G_4 . But the complement $V - S = \{v_4, v_5, v_6, v_7\}$ is not a triple connected complementary tree dominating set.

v_4 v_1 v_3 v_7

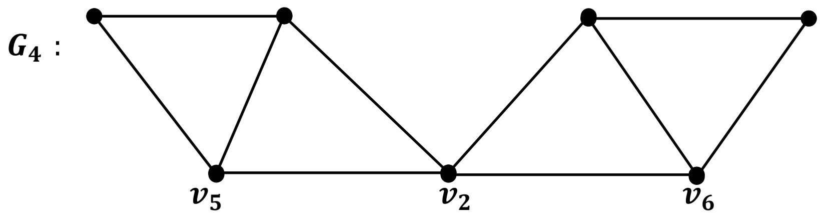

Fig 2.4

Theorem 2.9

For a connected graph G with $P > 5$ vertices, $\gamma_{tct}(G) = P - 2$ if and only if G is isomorphic to P_p or C_p.

proof Suppose G is isomorphic to P_p or C_p, then clearly $\gamma_{tct}(G) = P - 2$. Conversely, let G be a connected graph with $P > 5$ vertices and $\gamma_{tct}(G) = P - 2$. Let $S = \{v_1, v_2, \dots \dots . . v_{p-2}\}$ be a γ_{tct} set and let $V - S = V(G) - V(S) = \{v_{p-1}, v_p\}$. Then $(V - S) = K_2, \overline{K_2}$.

Claim $\langle S\rangle$ is a tree.

Suppose $\langle S\rangle$ is not a tree. Then, $\langle S\rangle$ contain a cycle. Without loss generality, let $C = \{v_1 v_2, \dots \dots v_q v_1\}, q \geq p - 2$ be a cycle of shortest length in $\langle S\rangle$. Now let $(V - S) = K_2 = v_{p-1}, v_p$. Since G is connected and is a γ_{tct} set of G , v_{p-1}or v_p are adjacent to a vertex v_k in $\langle S\rangle$.If v_k is in C, then $S = \{v_{p-1}, v_i, v_{i+1}, \dots . . v_{i-3}\} \cup \{x \in V(G) : X \notin C\}$ forms a γ_{tct} set of G so that $\gamma_{tct}(G) < p - 2$, Which is a contradiction. Suppose v_{p-1} or v_p is adjacent to a vertex v_i in $\langle S\rangle - C$, then we can construct a γ_{tct} set which is contain v_{p-1}, v_i with fewer elements than $p - 2$ Which is a contradiction. Similarly, if $\langle V - S\rangle = \overline{K_2}$, we can prove that no graph exists. Hence $\langle S\rangle$ is a tree. But $\langle S\rangle$ is a triple connected dominating set. Therefore by observation 2.5, we have $\langle S\rangle \cong P_{p-2}$.

Case i $(V - S) = K_2 = v_{p-1}, v_p$

Since G is connected and S is a γ_{tct} set of G, then there exists a vertex, say, v_i in P_{p-2} which is adjacent to a vertex, say, v_{p-1} in K_2. If $v_i = v_1 \, or \, v_{p-2}$, then $G \cong P_p$. If $v_i = v_1$ is adjacent to v_{p+1} and v_{p-2} is adjacent to v_p, then $G \cong C_p$. If $v_i = v_j$ for $j = 2,3, \dots \dots . . p - 3$, then $S_1 = S - \{v_1, v_{p-2}\} \cup \{ v_{p-1}\}$ is a triple connected dominating set of cardinality $p - 3$ and hence, $\gamma_{tct}(G) < p - 3$, which is a contradiction.

Case ii $(V - S) = \overline{K_2}$

Since G is connected and S is a γ_{tct} set of G, then there exists a vertex, say, v_i in P_{p-2} which is adjacent to both vertices, say, v_{p-1} and v_p in $\overline{K_2}$. If $v_i = v_1 \, or \, (v_{p-2})$, then by taking the vertex $v_1 \, or \, (v_{p-2})$, we can construct a triple connected dominating set which contains fewer elements than $p - 2$, which is a contradiction. Hence no graph exists. If $v_i = v_j$ for $j = 2,3, \dots \dots . . n - 3$, then by taking the vertex v_j , we can construct a triple connected dominating set which contains fewer elements than $p - 2$, which is contradiction. Hence no graph exists. Suppose there exits a vertex say v_i in P_{p-2} which is adjacent to v_{p-1} in $\overline{K_2}$ and a vertex v_j $(i \neq j)$ in P_{p-2} which is adjacent to v_p in $\overline{K_2}$. If $v_i = v_1$ and $v_j = v_{p-2}$, then $S = \{v_1, v_2, \dots \dots . . v_{p-2}\}$ is a γ_{tct} set of G and hence $G \cong P_p$. If $v_i = v_1$ and $v_j = v_k$ for $k = 2,3, \dots \dots . . n - 3$, then by taking the vertex v_1 and v_k we can construct a triple connected dominating set which contains fewer elements than $p - 2$, which is a contradiction.

Hence no graph exists. If $v_i = v_k$ and $v_j = v_1$ for $k = 2,3, \dots \dots . . n - 3$, then by taking the vertex v_1 and v_k , we can can construct a triple connected dominating set which contains fewer elents than $p - 2$, which is a contradiction.

Theorem 2.10

Let T be a tree. Then $\bar{T}$ is triple connected if and only if $T \ncong K_{1,r}$.

Proof

Let T be a tree. Assume that $T \ncong K_{1,r}$. Let u, v, w be any three vertices in $V(\overline{T})$ and set $S = \{u, v, w\}$

Case i $\langle S \rangle = \overline{K_3}$ in T.

Then $\langle S \rangle = K_3$ in $\bar{T}$ and uvw is a path in $\bar{T}$.

Case ii $\langle S \rangle = K_2 \cup K_1$ in T.

Without loss of generality,. let u and v be adjacent in T. Thus uw and $vw \in E(\bar{T})$ and hence uvw is a path in $\bar{T}$.

Case iii $\langle S \rangle = P_3$ in T.

Without loss of generality, let u be a adjacent to both v and w in T. Thus $\in E(\bar{T})$. Since $T \ncong K_{1,r}$, there exists another vertex x which is not adjacent to u in T. Thus $\in E(\bar{T})$. Since T is a tree, x can not be a adjacent to both v and w in T. Without loss of generality , assume that x is not adjacent w in T. Then $xw \in E(\bar{T})$ and $uxwv$ is a path in $\bar{T}$.Then any three vertices lie on a path in $\bar{T}$. Hence $\bar{T}$ is triple connected.

Conversely, assume that $\bar{T}$ is triple connected. Assume that $\bar{T} \ncong K_{1,r}$,This implies that $\bar{T} \ncong K_1 \cup K_r . r \geq 2$ which is disconnected. Thus $\bar{T}$ is not triple connected, which is a contradiction. Thus $\ncong K_{1,r}$.

III. Triple connected complementary tree domination number for some special graphs are given below

1) The ***Franklin graph*** 3-regular graph with 12 vertices and 18 edges as shown below in Figure 3.1

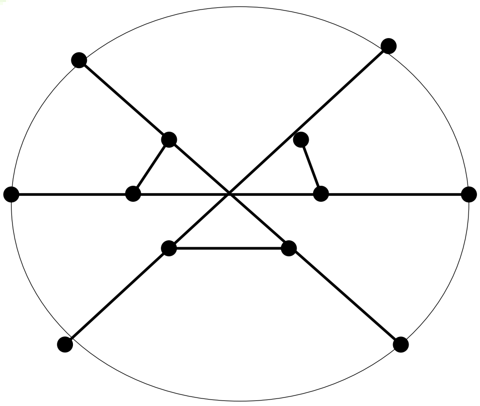

Fig 3.1

For the Franklin graph G, $\gamma_{tct}(G) = 6$.

2) The ***Moser spindle*** graph (also called the Moser's spindle or Moser graph) is a undirected graph, named after mathematics Leo Moser and his brother William , with 7 vertices and 7 edges as shown in figure 3.2

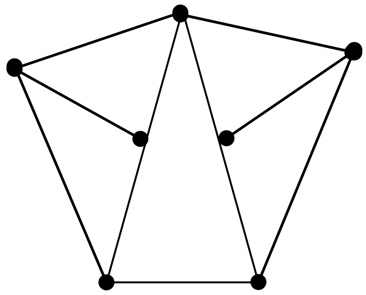

Fig 3.2

For the Moser spindle graph G, $\gamma_{tct}(G) = 3$.

3) The ***Herschel*** graph is a bibartite undirected graph with 11 vertices and 18 edges as shown in figure 3.3, the smallest non Hamiltonian polyhedral graph.

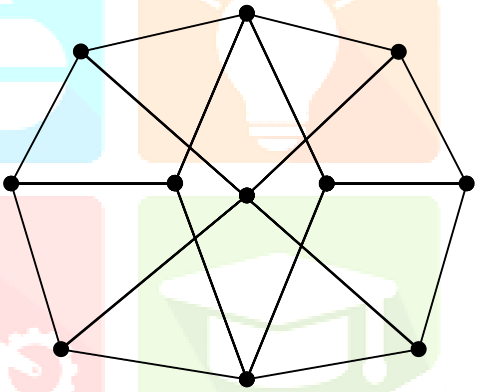

Fig 3.3

For the Herschel graph G, $\gamma_{tct}(G) = 5$.

4) The ***Bidiakis cube*** is a 3-regular graph with 12 vertices and 18 edges as shown in figure 3.4..

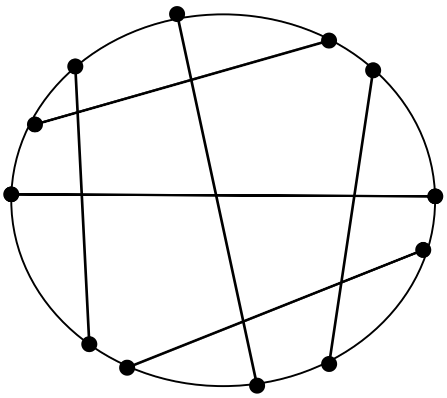

Fig 3.4

For the Bidiakis cube graph G, $\gamma_{tct}(G) = 8$.

5) The ***Bull graph*** is planer graph 5 vertices and 5 edges , in the form of a triangle with two disjoint pendent edges as shown in figure 3.5

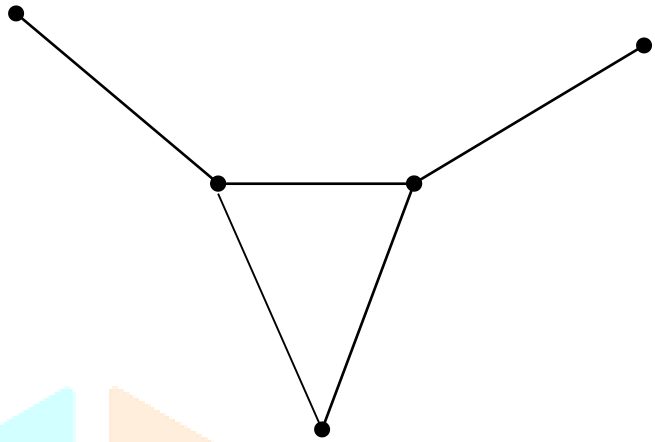

Fig 3.5

For the Bull graph, tct does not exist..

References

[1] Arumugam.S and Ramachandran.S, Invitation to graph Theory, SCITECH Publications (India) Pvt.Ltd, Chennai-600 017.

[2] Caro.Y, Yuster.R and West D.B, Connected domination and spanning trees, SIAM.J.Discrete Math.13 (2000), 202-211.

[3] Hedetnieml.S.T and Laskar.R.C, Connected domination in graphs, In Bollobas.B, editor, Graph Theory and combinatories. Acadamic press, London,1984.

[4] John Clark and Derek Allan Holton, First look at Graph Theory,Allied Publishers Ltd, 1991.

[5] Mahadevan.G, Selvam Avadayappan,Ramesh.N and Subramanian.T, International Mathematical Forum,vol.8, 2013,no.14, 659-670 HIKARI Ltd.

[6] J. Paulraj Joseph and S. Arumugam, Domination and connectivity in graphs, International Journal of Management Systems, 8(3)(1992),233-236.

[7] J.Paulraj Joseph and S. Arumugam, Domination and coloring in graphs, International Journal of Management Systems, 8(1)(1997),37-44.