FUZZY ADOMIAN DECOMPOSITION METHOD FOR VISCOUS FLOW AND HEAT TRANSFER OVER A NONLINEARLY STRETCHING SHEET

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Abstract: Unsteady boundary layer flow of an incompressible fluid over a stretching surface in the presence of a heat source or sink is studied. The unsteadiness in the flow and temperature fields is caused by the time dependence of the stretching velocity and the surface heat flux. The nonlinear boundary layer equations are transformed to nonlinear ordinary differential equations containing the Prandt l number, heat source or sink parameter and unsteadiness parameter. These equations are solved by applying Fuzzy Adomian technique and compared with the existing numerical results obtained by using Shooting with Runge Kutta method. This focuses on solving the nonlinear ordinary and partial differential equations using Fuzzy Adomian decomposition method**.**

IndexTerms - Fuzzy Adomian Decomposition Method ,non-linear ordinary differential equation, ,non-linear partial differential equation , Runge Kutta Method.

I. INTRODUCTION

The study of two-dimensional boundary layer flow due to a stretching surface is important in variety of engineering applications such as cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of paper and plastic sheets. In all these cases, a study of flow field and heat transfer can be of significant importance since the quality of the final product depends on skin friction coefficient and surface heat transfer rate.

The problem of heat transfer from boundary layer flow driven by a continuous moving surface is of importance in a number of industrial manufacturing processes. Several authors have been analysed in various aspects of the pioneering work of Sakiadis (1961). Crane (1970) have investigated the steady boundary layer flow due to stretching with linear velocity.Vleggaar et al. (1977) have analysed the stretching problem with constant surface temperature and Soundalgekar et al. (1980) have analysed the constant surface velocity.

Perturbation techniques are based on the existence of small or large parameters, the so-called perturbation quantity. Unfortunately, many nonlinear problems in science and engineering do not contain those kinds of perturbation quantities. Therefore, many different methods have recently introduced some ways to eliminate the small parameter. One of the semi exact methods which do not need small parameters is the Adomian decomposition method.

Also this method is employed for many researches in engineering sciences. He's Homotopy perturbation method is applied to obtain approximate analytical solutions for the motion of a spherical particle in a plane couette flow Jalaal et al. (1977) Then Jalaal et al. (1986) showed the effectiveness of HPM for unsteady motion of a spherical particle falling in a Newtonian fluid. Ghotbi et al. (1997) used HPM to approximate the solution of the ratio-dependent predatorprey system with constant effort prey harvesting. Also homotopy perturbation method was used for solving nonlinear MHD Jeffery Hamel problem by Moghimi et al. (2000) Recently, Ganji et al.(2007) studied the steady-state flow of a Hagen-Poiseuille model in a circular pipe and entropy generation due to fluid friction and heat transfer using HPM.

II. FORMULATION OF THE PROBLEM

We consider the flow of an incompressible viscous fluid past a flat sheet coinciding with the plane y = 0, the flow being confined to y > 0. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The basic boundary layer equations that govern momentum and energy respectively are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0 \qquad (2.1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2} \qquad (2.2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\propto\frac{\partial^2 T}{\partial y^2}\frac{v}{C_P}\left[\frac{\partial u}{\partial y}\right]^2 \qquad (2.3)$$

Subject to the boundary conditions are

$$u_\omega(x)=Cx^{n,},v=0 \qquad (2.4)$$

$$u \to 0, y \to \infty$$
$$T = T_\omega \; at \; y = 0; T \to T_\infty \; as \; y \to \infty$$

where *(x,y)* denotes the Cartesian coordinates along the sheet and normal to it, u and v are the velocity components of the fluid in the x and y directions, respectively, and v is the kinematic viscosity. C and n are parameters related to the surface stretching speed. c_p and α are the specific heat of the fluid at constant pressure and the thermal diffusivity respectively.
The equation of continuity is satisfied if we choose a stream function ψ(x, y) such that

$$u = \frac{\partial\psi}{\partial y}, v = \frac{-\partial\psi}{\partial x}$$

The mathematical analysis of the problem is simplified by introducing the following dimensionless similarity variables:

$$\eta = y\sqrt{\frac{C(n+1)}{2v}}x^{\frac{n-1}{2}}$$
$$u = Cx^n f'(\eta), \tag{2.5}$$
$$v = -\sqrt{\frac{Cv(n+1)}{2}}x^{\frac{n-1}{2}}\left[f + \frac{n-1}{n+1}\eta f'\right]$$

Substituting (2.5) into (2.2) and (2.3), we obtain the following set of ordinary differential equations:

$$f''' + ff'' - (f')^2\left[\frac{2n}{n+1}\right] = 0 \tag{2.6}$$
$$\theta'' + Prf\theta' + PrEc(f'')^2 = 0 \tag{2.7}$$

The boundary conditions (2.4) now become

$$\eta = 0: f = 0, f' = 1, \theta = 1 \tag{2.8}$$
$$\eta \to \infty: f' = 0, \theta = 0$$

Where the primes denote differentiation with respect to η

$E_c = \frac{u_\omega{}^2}{C_p(T_\omega - T_\infty)}$ is the Eckert number , $Pr\left[= \frac{v}{\alpha}\right]$ is the Prandt1 number Further , the constants T_ω, T_∞ denote the temperature at the wall and at large distance from the wall,respectively.

III. ADOMIAN DECOMPOSITION METHOD

To solve the system of coupled ODEs using Adomian decomposition method,
rearranging (2.6) and (2.7) as follows

$$f''' = -ff'' - f'^2\frac{2n}{n+1} \tag{3.1}$$
$$\theta'' = -Pr[f\theta' + Ec(f'')^2] \tag{3.2}$$

While applying the standard procedure of Adomian decomposition method
Eqs (3.1) and (3.2) becomes

$$L_1 f = \left[-ff'' - f'^2\frac{2n}{n+1}\right] \tag{3.3}$$

$$L_2\theta = -Pr\left[[f\theta' + Ec(f'')^2]\right] \tag{3.4}$$

Where
$L_1 = \frac{d^3}{d\eta^3}$ and inverse operator $L^{-1}{}_1(.) = \int_0^\eta\int_0^\eta\int_0^\eta(.)d\eta d\eta d\eta$ and
$L_2 = \frac{d^2}{d\eta^2}$ and inverse operator $L^{-1}{}_2(.) = \int_0^\eta\int_0^\eta(.)d\eta d\eta$
Applying the inverse operator on both sides of (3.3) and (3.4)

$$L^{-1}{}_1 L_1 f = L^{-1}{}_1\left[-ff'' - f'^2\frac{2n}{n+1}\right] \tag{3.5}$$

$$L^{-1}{}_2 L_2\theta = -PrL^{-1}{}_2\left[[f\theta' + Ec(f'')^2]\right] \tag{3.6}$$

Simplify eqs (3.5) and (3.6) we get

$$f(\eta) = \eta + \frac{\alpha_1\eta^2}{2} + \int_0^\eta\int_0^\eta\int_0^\eta\left[-N_1(f) - N_2(f)\frac{2n}{n+1}\right]d\eta d\eta d\eta \tag{3.7}$$

And

$$\theta(\eta) = \alpha_2 - \eta - Pr\int_0^\eta\int_0^\eta[N_3(f,\theta) + EcN_4(f)]\,d\eta d\eta \tag{3.8}$$

Where $\alpha_1 = f''(0) and\; \alpha_2 = \theta(0)$ are to be determined from the boundary conditions at infinity in (2.8). The non linear terms $ff'', f'^2, f\theta' and\; f'\theta$ can be decomposed as Adomian Polynomials $\sum_{n=0}^\infty B_n, \sum_{n=0}^\infty C_n, \sum_{n=0}^\infty D_n and\; \sum_{n=0}^\infty E_n$ as follows

$$N_1(f) = \sum_{n=0}^\infty B_n = ff'' \tag{3.9}$$
$$N_2(f) = \sum_{n=0}^\infty C_n = (f')^2 \tag{3.10}$$

$$N_3(f,\theta) = \sum_{n=0}^{\infty} D_n = f\theta' \qquad (3.11)$$

$$N_4(f,\theta) = \sum_{n=0}^{\infty} E_n = (f'')^2 \qquad (3.12)$$

Where $B_n(f_0, f_1, \dots\dots, f_n), C_n(f_0, f_1, \dots\dots, f_n)$ and $D_n(f_0, f_1, \dots\dots, f_n, \theta_0, \theta_1, \dots., \theta_n)$, $E_n(f_0, f_1, \dots\dots, f_n)$ are the so called Adomian polunomials. In the Adomian decomposition method (1994) f and θ can be expanded as the infinite series

$$f(n) = \sum_{n=0}^{\infty} f_n = f_0 + f_1 + \cdots + f_m + \cdots$$

$$\theta(\eta) = \sum_{n=0}^{\infty} \theta_n = \theta_0 + \theta_1 + \cdots + \theta_m + \cdots \qquad (3.13)$$

Substituting (3.9),(3.10),(3.11) and (3.12) into (3.7) and (3.8) gives

$$\sum_{n=0}^{\infty} f_n(\eta) = \eta + \frac{\alpha_1\eta^2}{2} + \int_0^{\eta}\int_0^{\eta}\int_0^{\eta}\left[-\sum_{n=0}^{\infty} B_n - \frac{2n}{n+1}\sum_{n=0}^{\infty} C_n\right] d\eta d\eta d\eta \qquad (3.14)$$

$$\text{and} \sum_{n=0}^{\infty} \theta_n(\eta) = \alpha_{2\_}\eta - Pr\int_0^{\eta}\int_0^{\eta}\left[\sum_{n=0}^{\infty} D_n + Ec\sum_{n=0}^{\infty} C_n\right] d\eta d\eta \qquad (3.15)$$

Hence, the individual terms of the Adomian series solution of the equation (2.6)-(2.8) are provided below by the simple recursive algorithm

$$f_0(\eta) = \eta + \frac{\alpha_1\eta^2}{2} \qquad (3.16)$$

$$\theta_0(\eta) = 1 + \alpha_2\eta \qquad (3.17)$$

$$f_{n+1}(\eta) = \int_0^{\eta}\int_0^{\eta}\int_0^{\eta}[-B_n - C_n]\, d\eta d\eta d\eta \qquad (3.18)$$

$$\theta_{n+1}(\eta) = -Pr\int_0^{\eta}\int_0^{\eta}[D_n + EcE_n]\, d\eta d\eta \qquad (3.19)$$

For practical numerical computation , we take the m-term approximation of $f(\eta) and\ \theta(n)$ as

$$\phi_m(\eta) = \sum_{n=0}^{m-1} f_n(\eta) \text{and}$$

$$\Omega_m(\eta) = \sum_{n=0}^{m-1} \theta_n(\eta)$$

The recursive algorithms (3.16)-(3.19) are programmed in MATLAB. We have obtained upto 15th term of approximations to both $f(\eta) and\ \theta(n)$. We provided below only first few terms due to lack of space.

$$f_0(\eta) = \eta + \frac{\alpha_1\eta^2}{2}$$

$$f_1 = \left[\frac{1}{6} - \frac{1}{3n+3}\right]\eta^3 + \left[\frac{\alpha_1}{8} - \frac{\alpha_1}{6n+6}\right]\eta^4 + \left[\frac{{\alpha_1}^2}{40} - \frac{{\alpha_1}^2}{30n+30}\right]\eta^5$$

Etc.,

And

$$\theta_0(\eta) = 1 + \alpha_2\eta$$

$$\theta_1 = -\mathrm{Pr}\left[\left[\frac{Ec{\alpha_1}^2}{2}\right]\eta^2 + \left[\frac{\alpha_2}{6}\right]\eta^3 - \left[\frac{\alpha_1\alpha_2}{24}\right]\eta^4\right]$$

Etc.,

The undetermined values of $\alpha_1 and\ \alpha_2$ are computed using the boundary conditions at infinity in (3.8). The difficulty at infinity is tackled by applying the diagonal Padé approximants Boyd (1997). that approximate $f'(\eta) and\ \theta(\eta) using\ \phi_{15}{}'(\eta) and\ \Omega_{15}(\eta) respectively$. The numerical results of $\alpha_1 and\ \alpha_2$ from $\lim_{\eta\to\infty} \Phi'_{15}(\eta) = 0$ and $\lim_{\eta\to\infty} \Omega_{15}(\eta) = 0$ for selected m in the range from 4 to 8 are shown below tables .

IV. RESULT ANALYSIS

Table 4.1

The velocity gradient ($\alpha_1 = f''(0)$) for various values of n using ADM and FUZZY ADM.

	Present Result		**R Cortell (1970)**
	ADM	Fuzzy ADM	

A	□ *f* ''(0)		□ *f* ''(0)		□ *f* ''(0)
	[7/7]	[8/8]	[7/7]	[8/8]	
0	0.62821	0.62802	0.62725	0.62754	0.62754
0.2	0.76775	0.76597	0.76675	0.76664	0.76675
0.5	0.88999	0.88974	0.88889	0.88947	0.88947
0.75	0.95861	0.95378	0.95382	0.95012	0.95378
1	1.05	1	1.066	1.023	1

Table 4.2

The velocity gradient ($\alpha_2 = \theta'(0)$) for various values of *n* with *Ec=0, Pr=1* using ADM and Fuzzy ADM techniques

n	$-\theta'(0)$	$-\theta'(0)$		
	Exact	Fuzzy ADM	ADM	R Cortell (1970)
0.5	0.52665	0.56199	0.56269	0.52674
1	0.57104	0.54021	0.54693	0.57123
3	0.52568	0.52549	0.52793	0.52569
10	0.47792	0.47576	0.47895	0.47785

Table 4.3

The velocity gradient ($\alpha_2 = \theta'(0)$) for various values of *Ec* at *n=1* and *Pr=1* using ADM and Fuzzy ADM techniques.

Ec	$-\theta'(0)$	$-\theta'(0)$		
	Exact	Fuzzy ADM	ADM	R Cortell (1970)
0	0.57104	0.54021	0.54693	0.54693
0.2	0.19981	0.19979	0.19927	0.19929
0.5	0.49988	0.49957	0.4998	0.4998
1	1	0.99998	0.99995	0.99995

Table 4.4

The velocity gradient ($\alpha_2 = \theta'(0)$) for various values of *Pr* at *n=3* and *Ec=1* using ADM and Fuzzy ADM techniques.

Ec	$-\theta'(0)$	$-\theta'(0)$		R Cortell (1970)
	Exact	Fuzzy ADM	ADM	
0.5	0.26181	0.28264	0.26181	0.28894
0.72	0.42176	0.46637	0.42176	0.46964
1	0.76478	0.75348	0.76478	0.75567
2	1.5194	1.4984	1.5194	1.4995

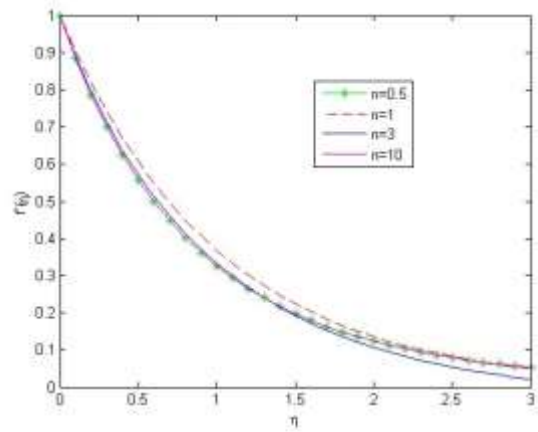

Fig. 4.1 Velocity profiles $f'(\eta)$ for various values of *n* when *Pr*= 1 and *Ec*=1 Using $\Phi'_{15[7/7]}$.

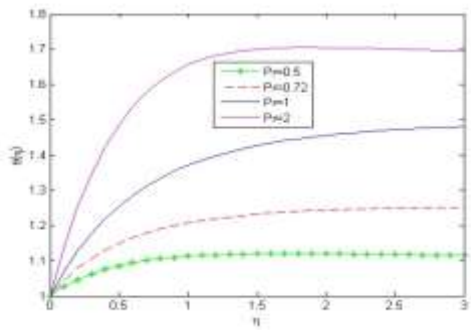

Fig. 4.2 Temperature profiles $\theta(\eta)$ for various values of Pr at n = 3 and Ec= 1 Using $\Omega_{15[7/7]}$.

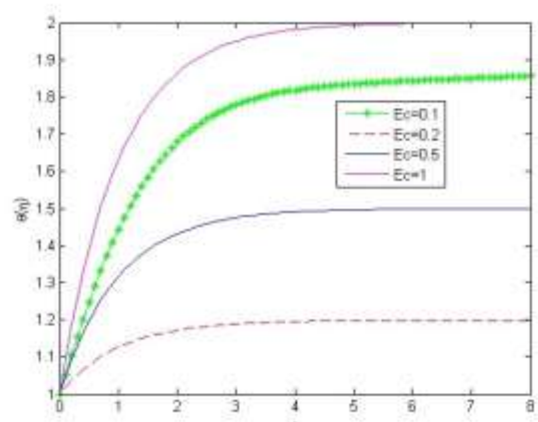

Fig. 4.3 Temperature profiles $\theta(\eta)$ for various values of Ec at n = 1 and Pr= 1 Using $\Omega_{15[7/7]}$.

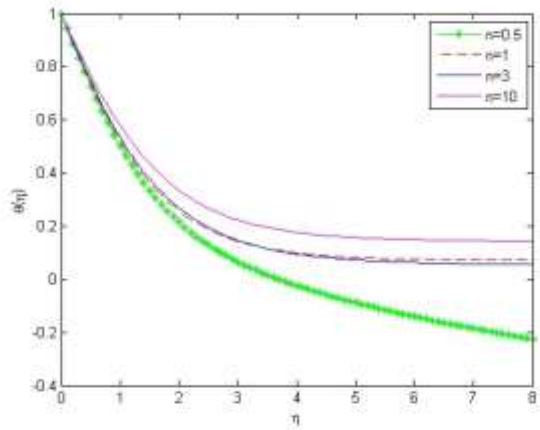

Fig. 4.4 Temperature profiles $\theta(\eta)$ for various values of n at Ec = 0 and Pr= 1 Using $\Omega_{15[8/8}$

From Fig.4.1 we note that when unsteadiness parameter *n* increases, the velocity profiles decreases. In Figs. 4.2 and 4.3 we note that when Prandtl Number (Pr) increases that implies the temperature decreases within the boundary layer for all values of the Prandtl number. This is consistent with the well-known fact that the thermal boundary layer thickness decreases with increasing Prandtl number. In Fig 4.4 we note that when unsteadiness parameter *n* increases the temperature Profiles is decreases.

V. CONCLIUSION

The Fuzzy Adomian decomposition method is applied to solve a system of two nonlinear ordinary differential equations with the specified boundary conditions that describes viscous flow and heat transfer over a nonlinearly stretching sheet. The obtained solutions have matched with the existing numerical result. The Fuzzy Adomian decomposition method techniques are very efficient alternative tools to solve nonlinear models with infinite boundary conditions.

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