

# Support Anti Intuitionistic Fuzzy A-Ideal

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**Abstract:** In this paper, we combine a intuitionistic fuzzy set with a fuzzy set. This raise a new concept called support-intuitionistic fuzzy (SIF) set. In which there are three membership function of an element in a given set. Here I conclude some operations of support anti intuitionistic fuzzy a-ideal theorems and examples.

**Index terms:** Fuzzy set , support-intuitionist fuzzy, a-ideal, BCI-Algebras.

**Introduction :** Fuzzy set theory was introduced by L-Zadeh since 1965. Immediately it became a useful method to study in the problems of imprecision and uncertainty. Since, a lot of new theories treating imprecision and uncertainty have been introduced. For instance intuitionistic fuzzy sets were introduced in 1986 by K. Atanassov which is a generalization of the notion of a fuzzy set. When fuzzy set give the degree of membership of an element in a given set, intuitionistic fuzzy set give a degree of membership and a degree of non-membership of an element in a given set. Then, the concept of fuzzy relations and intuitionistic fuzzy relations introduced. They together with their logic operators and are applied in many different fields.

In this paper, we combine a intuitionistic fuzzy set with a fuzzy set. This raise a new concept called support-intuitionistic fuzzy (SIF) set. In which there are three membership function of an element in a given set.

To develop the theory of BCI-algebras, the ideal theory plays an important role. Liu and meng introduced the notation of q-ideals and a-ideals in BCI-algebras.

## Preliminaries

**Definition 1:** A fuzzy set  $A$  on the universe  $U$  is an object of the form

$$A = \{x, \mu_A(x) : x \in U\}$$

Where  $\mu_A(x)$  ( $X \in [0,1]$ ) is called the degree of membership of  $x$  in  $A$ .

**Definition 2:** An Intuitionistic fuzzy (IF) set  $A$  on the universe  $U$  is an object of the form  $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in U\}$  where  $\mu_A(x)$  ( $\in [0,1]$ ) is called the “degree of membership of  $x$  in  $A$ ”,  $\lambda_A(x)$  ( $X \in [0,1]$ ) is called the “degree of non-membership of  $x$  in  $A$ ” and where  $\mu_A$  and  $\lambda_A$  satisfy the following condition  $\mu_A(x) + \lambda_A(x) \leq 1, (\forall x \in U)$ .

**Definition 3:** A support-intuitionist fuzzy (SIF) set  $A$  on the universe  $U$  is an object of the form  $A = \{(x, \mu_A(x), \lambda_A(x), \gamma_A(x)) : \forall x \in U\}$  where  $\mu_A(x)$  ( $\in [0,1]$ ) is called the “degree of membership of  $x$  in  $A$ ”,  $\lambda_A(x)$  ( $\in [0,1]$ ) is called the “degree of non-membership of  $x$  in  $A$ ”, and  $\gamma_A(x)$  is called the “degree of support membership of  $x$  in  $A$ ”, and where  $\mu_A, \lambda_A$  and  $\gamma_A$  satisfy the following condition.  $\mu_A(x) + \lambda_A(x) \leq 1, 0 \leq \gamma_A(x) \leq 1, (\forall x \in U)$ .

The family of all support intuitionistic fuzzy set in  $U$  is denoted by SIFS ( $U$ )

**Definition 4:** Algebra  $(X; *, 0)$  of type  $(2,0)$  is called a **BCI-algebra** if it satisfies the following conditions:

- (I)  $\forall x, y, z \in X, ((x*y) *(x*z)) *(z*y) = 0,$
- (II)  $\forall x, y \in X, (x*(x*y)) *y = 0,$
- (III)  $\forall x \in X, x*x = 0,$
- (IV)  $\forall x, y \in X, x*y = 0, y*x = 0 \implies x = y$

**Definition 5:** A nonempty subset  $A$  of a BCI-algebra  $X$  is called an **ideal** of  $X$  if it satisfies: (I)  $0 \in A$ , (II)  $\forall x, y \in X, \forall y \in A, x-y \in A \implies x \in A$ .

**Definition 6:** A nonempty subset  $A$  of a BCI-algebra  $X$  is called **a-ideal** of  $X$  if it satisfies: (I)  $0 \in A$ , (II)  $x, y \in X, \forall z \in A ((x-z) - (0-y) \in A \implies y - x \in A)$

**Definition 7:** An intuitionist fuzzy set  $A$  in a nonempty set  $X$  is an object having the form  $A = \{x, \mu_A(x), \lambda_A(x) : \forall x \in X\}$ , Where the function  $\mu_A: X \rightarrow [0,1]$  and  $\lambda_A: X \rightarrow [0,1]$  denoted the **degree of membership** (namely  $\mu_A(x)$ ) and the **degree of non membership** (namely  $\lambda_A(x)$ ) of each element  $x \in X$  to the set  $A$  respectively, and  $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$  for all  $x \in X$ .

**Definition 8:** An intuitionist fuzzy set  $A = \langle X, \mu_A, \lambda_A \rangle$  in  $X$  is called an **intuitionist fuzzy ideal** of  $X$ , if it satisfies the following axioms:

- (I)  $\mu_A(0) \geq \mu_A(x)$  and  $\lambda_A(0) \leq \lambda_A(x),$
- (II)  $\mu_A(x) \geq \min \{\mu_A(x-y), \mu_A(y)\}$
- (III)  $\lambda_A(x) \leq \max \{\lambda_A(x-y), \lambda_A(y)\}, \forall x, y \in X.$

**Definition 9:** An intuitionist fuzzy set  $A = \langle X, \mu_A, \gamma_A \rangle$  in  $X$  is called an **intuitionist fuzzy ideal** of  $X$ , if it satisfies the following axioms

- (I)  $\lambda_A(x) \leq \max \{\lambda_A(x-y), \lambda_A(y)\}, \forall x, y \in X.$

And  $(\forall x, y, z \in X) (\mu_A(y-x) \geq \min \{\mu_A((x-z) - (0-y)), \mu_A(z)\}),$

$$(\forall x, y, z \in X) (\lambda_A(y-x) \leq \max\{\lambda_A((x-z) - (0-y)), \lambda_A(z)\}).$$

**Definition 10:** An intuitionist fuzzy set  $A = \langle X, \mu_A, \lambda_A, \gamma_A \rangle$  in  $X$  is called an **Support Intuitionistic Fuzzy set**  $X$ , if it satisfies the following axioms For all  $A, B \in X$

- i)  $A \subseteq B$  iff  $\mu_A(x) \leq \mu_B(x)$ ,  $\lambda_A(x) \geq \lambda_B(x)$  and  $\gamma_A(x) \leq \gamma_B(x)$ ,  $\forall x \in X$
- ii)  $A \supseteq B$  iff  $\mu_A(x) \geq \mu_B(x)$ ,  $\lambda_A(x) \leq \lambda_B(x)$  and  $\gamma_A(x) \geq \gamma_B(x)$
- iii)  $A = B$  iff  $A \subseteq B$  and  $B \subseteq A$

**Definition 11:** Let  $A$  and  $B$  be any two Support Intuitionistic Fuzzy set  $X$  then  $A \cup B$  is also Support Intuitionistic Fuzzy set  $X$ , if it satisfies the following axioms

$$A \cup B = \{(x, \mu_{A \cup B}(x), \lambda_{A \cup B}(x), \gamma_{A \cup B}(x)) / x \in X\}$$

$$\text{Where } \mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\},$$

$$\lambda_{A \cup B}(x) = \min\{\lambda_A(x), \lambda_B(x)\},$$

$$\gamma_{A \cup B}(x) = \max\{\gamma_A(x), \gamma_B(x)\}, \forall x \in X.$$

**Definition 12:** Let  $A$  and  $B$  be any two Support Intuitionistic Fuzzy set  $X$  then  $A \cap B$  is also Support Intuitionistic Fuzzy set  $X$ , if it satisfies the following axioms

$$A \cap B = \{(x, \mu_{A \cap B}(x), \lambda_{A \cap B}(x), \gamma_{A \cap B}(x)) / x \in X\}$$

$$\text{Where } \mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\},$$

$$\lambda_{A \cap B}(x) = \max\{\lambda_A(x), \lambda_B(x)\},$$

$$\gamma_{A \cap B}(x) = \min\{\gamma_A(x), \gamma_B(x)\}, \forall x \in X.$$

**Definition 13:** Let  $A$  be any Support Intuitionistic Fuzzy set  $X$  then  $\sim A$  is also Support intuitionistic Fuzzy set  $X$ , if it satisfies the following axioms

The complement of  $A$

$$\sim A = \{(x, \mu_A(x), \lambda_A(x), 1 - \gamma_A(x)) / \forall x \in X\}.$$

**Definition 14:** Let  $U$  and  $V$  be two universes and let  $A = \{(x, \mu_A(x), \lambda_A(x), \gamma_A(x)) / \forall x \in U\}$  and  $B = \{(y, \mu_B(y), \lambda_B(y), \gamma_B(y)) / \forall y \in U\}$  be two Support Intuitionistic Fuzzy sets on  $U$  and  $V$  respectively. We define the Cartesian product of these two Support Intuitionistic Fuzzy sets

$$A \times B = \{((x,y), \mu_{A \times B}(x,y), \lambda_{A \times B}(x,y), \gamma_{A \times B}(x,y)) / \forall x \in U, \forall y \in U\}$$

$$\text{Where } \mu_{A \times B}(x,y) = \mu_A(x) \mu_B(y)$$

$$\lambda_{A \times B}(x,y) = \lambda_A(x) \lambda_B(y)$$

$$\gamma_{A \times B}(x,y) = \gamma_A(x) \gamma_B(y)$$

**Definition 15:** Let  $U$  and  $V$  be two universes and let  $A = \{(x, \mu_A(x), \lambda_A(x), \gamma_A(x)) / \forall x \in U\}$  and  $B = \{(y, \mu_B(y), \lambda_B(y), \gamma_B(y)) / \forall y \in U\}$  be two Support Intuitionistic Fuzzy sets on  $U$  and  $V$  respectively. We define the Cartesian product of these two Support Intuitionistic Fuzzy sets

$$A \otimes B = \{((x,y), \mu_{A \otimes B}(x,y), \lambda_{A \otimes B}(x,y), \gamma_{A \otimes B}(x,y)) / \forall x \in U, \forall y \in V\} \text{ Where } \mu_{A \otimes B}(x) = \min\{\mu_A(x), \mu_B(x)\},$$

$$\lambda_{A \otimes B}(x) = \max\{\lambda_A(x), \lambda_B(x)\},$$

$$\gamma_{A \otimes B}(x) = \min\{\gamma_A(x), \gamma_B(x)\}, \forall x \in U, y \in V.$$

**Definition 16:** Let  $U$  and  $V$  be two universes and let  $A = \{(x, \mu_A(x), \lambda_A(x), \gamma_A(x)) / \forall x \in U\}$  and  $B = \{(y, \mu_B(y), \lambda_B(y), \gamma_B(y)) / \forall y \in U\}$  be two Support Intuitionistic Fuzzy sets on  $U$  and  $V$  respectively. We define the Cartesian product of these two Support Intuitionistic Fuzzy sets

$$A \boxtimes B = \{((x,y), \mu_{A \boxtimes B}(x,y), \lambda_{A \boxtimes B}(x,y), \gamma_{A \boxtimes B}(x,y)) / \forall x \in U, \forall y \in V\}$$

$$\text{Where } \mu_{A \boxtimes B}(x) = \max\{\mu_A(x), \mu_B(x)\},$$

$$\lambda_{A \boxtimes B}(x) = \min\{\lambda_A(x), \lambda_B(x)\},$$

$$\gamma_{A \boxtimes B}(x) = \max\{\gamma_A(x), \gamma_B(x)\}, \forall x \in U, y \in V.$$

### **Support Anti Intuitionistic Fuzzy A-Ideal in Subtraction BCI-Algebras Theorems**

**Theorem 1:** If  $A$  and  $B$  is a support intuitionistic fuzzy a-ideal in BCI-algebra  $X$ , then  $A \cup B$  is also support intuitionistic fuzzy a-ideal in BCI-algebra  $X$ .

**Proof:** An AIFS  $A = \langle X, \mu_A, \lambda_A, \gamma_A \rangle$  in  $X$  is called an Anti intuitionistic fuzzy a-ideal of  $X$  if  $\mu_A(0) \leq \mu_A(x)$ ,  $\mu_B(0) \leq \mu_B(x)$  and

$$(i) \max\{\mu_A(0), \mu_B(0)\} \leq \max\{\mu_A(x), \mu_B(x)\} \Rightarrow \mu_{A \cup B}(0) \leq \mu_{A \cup B}(x), \forall x \in X.$$

$$\lambda_A(0) \geq \lambda_A(x), \lambda_B(0) \geq \lambda_B(x) \text{ and}$$

$$\min\{\lambda_A(0), \lambda_B(0)\} \geq \min\{\lambda_A(x), \lambda_B(x)\} \Rightarrow \lambda_{A \cup B}(0) \geq \lambda_{A \cup B}(x), \forall x \in X.$$

$$\gamma_A(0) \leq \gamma_A(x), \gamma_B(0) \leq \gamma_B(x) \text{ and}$$

$$\max\{\gamma_A(0), \gamma_B(0)\} \leq \max\{\gamma_A(x), \gamma_B(x)\} \Rightarrow \gamma_{A \cup B}(0) \leq \gamma_{A \cup B}(x), \forall x \in X$$

$$(ii) \mu_A(y-x) \leq \max\{\mu_A((x-z) - (0-y)), \mu_A(z)\} \text{ and}$$

$$\mu_B(y-x) \leq \max\{\mu_B((x-z) - (0-y)), \mu_B(z)\}$$

$$\max\{\mu_A(y-x), \mu_B(y-x)\} \leq \max\{\max\{\mu_A((x-z) - (0-y)), \mu_A(z)\}, \max\{\mu_B((x-z) - (0-y)), \mu_B(z)\}\}$$

$$\mu_{A \cup B}(y-x) \leq \max\{\mu_{A \cup B}((x-z) - (0-y)), \mu_{A \cup B}(z)\}$$

$$\text{If } \lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0-y)), \lambda_A(z)\} \text{ and}$$

$$\lambda_B(y-x) \geq \min\{\lambda_B((x-z) - (0 - y)), \lambda_B(z)\}$$

$$\min\{\lambda_A(y-x), \lambda_B(y-x)\} \geq \min\{\min\{\lambda_A((x-z) - (0 - y)), \lambda_A(z)\}, \min\{\lambda_B((x-z) - (0 - y)), \lambda_B(z)\}\}$$

$$\lambda_{A \cup B}(y-x) \geq \min\{\lambda_{A \cup B}((x-z) - (0 - y)), \lambda_{A \cup B}(z)\}.$$

If  $\gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\}$  and  $\gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0 - y)), \gamma_B(z)\}$

$$\text{Max}\{\gamma_A(y-x), \gamma_B(y-x)\} \leq \max\{\max\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\}, \max\{\gamma_B((x-z) - (0 - y)), \gamma_B(z)\}\}$$

$$\gamma_{A \cup B}(y-x) \leq \max\{\gamma_{A \cup B}((x-z) - (0 - y)), \gamma_{A \cup B}(z)\}.$$

Hence AUB is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

**Example 1:** Consider a BCI-algebra X = {0, a, b} with the following Cayley table:

X	0	a	B
$\mu_A$	0.2	0.7	0.2
$\lambda_A$	0.5	0.2	0.5
$\gamma_A$	0.4	0.6	0.4

X	0	a	B
$\mu_B$	0.3	0.8	0.3
$\lambda_B$	0.7	0.4	0.7
$\gamma_B$	0.6	0.7	0.6

- I) Let x=a,  $\max\{\mu_A(0), \mu_B(0)\} \leq \max\{\mu_A(x), \mu_B(x)\} \Rightarrow 0.3 \leq 0.8$   
 Let x=b,  $\min\{\lambda_A(0), \lambda_B(0)\} \geq \min\{\lambda_A(x), \lambda_B(x)\} \Rightarrow 0.5 \geq 0.5$   
 And let x=a,  $\max\{\gamma_A(0), \gamma_B(0)\} \leq \max\{\gamma_A(x), \gamma_B(x)\} \Rightarrow 0.6 \leq 0.7$  is satisfied.
- II) If x=b, y=a, z=0  $\mu_A(y-x) \leq \max\{\mu_A((x-z) - (0 - y)), \mu_A(z)\}$  and  $\mu_B(y-x) \leq \max\{\mu_B((x-z) - (0 - y)), \mu_B(z)\}$   
 $0.3 \leq 0.3$  is satisfied.
- If x=b, y=a, z=0  $\lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0 - y)), \lambda_A(z)\}$  and  $\lambda_B(y-x) \geq \min\{\lambda_B((x-z) - (0 - y)), \lambda_B(z)\}$   
 $0.7 \geq 0.7$  is satisfied.
- If x=b, y=a, z=0  $\gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\}$  and  $\gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0 - y)), \gamma_B(z)\}$   
 $0.6 \leq 0.6$  is satisfied.

This completes the proof.

**Theorem 2:** If A and B is a support intuitionistic fuzzy a-ideal in BCI-algebra X, then  $A \cap B$  is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

**Proof:**

An AIFS  $A = \langle X, \mu_A, \lambda_A, \gamma_A \rangle$  in X is called an Anti intuitionistic fuzzy a-ideal of X if  $\mu_A(0) \leq \mu_A(x)$ ,  $\mu_B(0) \leq \mu_B(x)$  and

- I)  $\min\{\mu_A(0), \mu_B(0)\} \leq \min\{\mu_A(x), \mu_B(x)\} \Rightarrow \mu_{A \cap B}(0) \leq \mu_{A \cap B}(x), \forall x \in X.$   
 $\lambda_A(0) \geq \lambda_A(x), \lambda_B(0) \geq \lambda_B(x)$  and  
 $\max\{\lambda_A(0), \lambda_B(0)\} \geq \max\{\lambda_A(x), \lambda_B(x)\} \Rightarrow \lambda_{A \cap B}(0) \geq \lambda_{A \cap B}(x), \forall x \in X.$   
 $\gamma_A(0) \leq \gamma_A(x), \gamma_B(0) \leq \gamma_B(x)$  and  
 $\min\{\gamma_A(0), \gamma_B(0)\} \leq \min\{\gamma_A(x), \gamma_B(x)\} \Rightarrow \gamma_{A \cap B}(0) \leq \gamma_{A \cap B}(x), \forall x \in X.$
- II)  $\mu_A(y-x) \leq \max\{\mu_A((x-z) - (0 - y)), \mu_A(z)\}$  and  $\mu_B(y-x) \leq \max\{\mu_B((x-z) - (0 - y)), \mu_B(z)\}$   
 $\mu_{A \cap B}(y-x) \leq \max\{\mu_{A \cap B}((x-z) - (0 - y)), \mu_{A \cap B}(z)\}.$   
 $\lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0 - y)), \lambda_A(z)\}$  and  $\lambda_B(y-x) \geq \min\{\lambda_B((x-z) - (0 - y)), \lambda_B(z)\}$   
 $\lambda_{A \cap B}(y-x) \geq \max\{\lambda_{A \cap B}((x-z) - (0 - y)), \lambda_{A \cap B}(z)\}.$   
 $\gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\}$  and  $\gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0 - y)), \gamma_B(z)\}$   
 $\gamma_{A \cap B}(y-x) \leq \max\{\min\{\gamma_A((x-z) - (0 - y)), \gamma_B((x-z) - (0 - y))\}, \min\{\gamma_A(z), \gamma_B(z)\}\}$   
 $\gamma_{A \cap B}(y-x) \leq \max\{\gamma_{A \cap B}((x-z) - (0 - y)), \gamma_{A \cap B}(z)\}.$

Hence  $A \cap B$  is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

**Example 2:** Consider a BCI-algebra X = {0, a, b} with the following Cayley table:

X	0	a	b
$\mu_A$	0.2	0.7	0.2
$\lambda_A$	0.5	0.2	0.5
$\gamma_A$	0.4	0.6	0.4

X	0	a	B
$\mu_A$	0.3	0.8	0.3
$\lambda_A$	0.7	0.4	0.7
$\gamma_A$	0.6	0.7	0.6

- I) Let  $x=a, \min\{\mu_A(0), \mu_B(0)\} \leq \min\{\mu_A(x), \mu_B(x)\} \Rightarrow 0.2 \leq 0.7$   
 Let  $x=b, \max\{\lambda_A(0), \lambda_B(0)\} \geq \max\{\lambda_A(x), \lambda_B(x)\} \Rightarrow 0.7 \geq 0.7$   
 And let  $x=a, \min\{\gamma_A(0), \gamma_B(0)\} \leq \min\{\gamma_A(x), \gamma_B(x)\} \Rightarrow 0.4 \leq 0.6$  is satisfied.
- II) If  $x=b, y=a, z=0$   $\mu_A(y-x) \leq \min\{\mu_A((x-z) - (0 - y)), \mu_A(z)\}$  and  
 $\mu_B(y-x) \leq \min\{\mu_B((x-z) - (0 - y)), \mu_B(z)\}$   
 $\min\{\mu_A(y-x), \mu_B(y-x)\} \leq \min\{\max\{\mu_A((x-z) - (0 - y)), \mu_A(z)\}, \max\{\mu_B((x-z) - (0 - y)), \mu_B(z)\}\}$   
 $0.2 \leq 0.3$  is satisfied.
- If  $x=b, y=a, z=0$   $\lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0 - y)), \lambda_A(z)\}$   
 and  $\lambda_B(y-x) \geq \min\{\lambda_B((x-z) - (0 - y)), \lambda_B(z)\}$   
 $\max\{\lambda_A(y-x), \lambda_B(y-x)\} \geq \max\{\min\{\lambda_A((x-z) - (0 - y)), \lambda_A(z)\}, \min\{\lambda_B((x-z) - (0 - y)), \lambda_B(z)\}\}$   
 $0.7 \geq 0.5$  is satisfied.
- If  $x=b, y=a, z=0$   $\gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\}$  and  
 $\gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0 - y)), \gamma_B(z)\}$   
 $\min\{\gamma_A(y-x), \gamma_B(y-x)\} \leq \min\{\max\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\}, \max\{\gamma_B((x-z) - (0 - y)), \gamma_B(z)\}\}$   
 $0.4 \leq 0.6$  is satisfied.
- This completes the proof.

**Theorem 3:** If A and B is a support intuitionistic fuzzy a-ideal in BCI-algebra X, then  $\sim A$  is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

**Proof:** An AIFS  $A = \langle X, \mu_A, \lambda_A, \gamma_A \rangle$  in X is called an Anti intuitionistic fuzzy a-ideal of X if I)  $\mu_A(0) \leq \mu_A(x), \lambda_A(0) \geq \lambda_A(x), \gamma_A(0) \leq \gamma_A(x)$ .

- II)  $\mu_A(y-x) \leq \max\{\mu_A((x-z) - (0 - y)), \mu_A(z)\}, (\forall x, y, z \in X)$   
 $\lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0 - y)), \lambda_A(z)\} (\forall x, y, z \in X)$  and  
 $1-\gamma_A(y-x) \leq 1 - \max\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\} (\forall x, y, z \in X)$   
 $1-\gamma_A(y-x) \leq \min\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\} (\forall x, y, z \in X)$ .

**Example: 3**

Consider a BCI-algebra  $X = \{0, a, b\}$  with the following Cayley table:

-	0	a	B
0	0	b	A
A	A	0	B
B	B	a	0

X	0	a	b
$\mu_A$	0.3	0.8	0.3
$\lambda_A$	0.6	0.3	0.6
$\gamma_A$	0.6	0.7	0.6

Let  $x=b, y=a, z=0. \mu_A(0) \leq \mu_A(x) \Rightarrow \mu_A(0.3) \leq \mu_A(0.3),$   
 $\lambda_A(0) \geq \lambda_A(x) \Rightarrow \lambda_A(0.6) \geq \lambda_A(0.6)$  and  
 $\gamma_A(0) \leq \gamma_A(x) \Rightarrow \gamma_A(0.6) \leq \gamma_A(0.6)$

Then,  $\mu_A(y-x) \leq \max\{\mu_A((x-z) - (0 - y)), \mu_A(z)\} \Rightarrow 0.3 \leq 0.3$   
 $\lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0 - y)), \lambda_A(z)\} (\forall x, y, z \in X) \Rightarrow 0.6 \geq 0.6.$   
 And  $1-\gamma_A(y-x) \leq 1 - \max\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\} (\forall x, y, z \in X) \Rightarrow 0.5 \leq 0.5$   
 This completes the proof.



**Theorem 4:** If A and B is a support intuitionistic fuzzy a-ideal in BCI-algebra X, then  $A \times B$  is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

**Proof:**

An AIFS  $A = \langle X, \mu_A, \lambda_A, \gamma_A \rangle$  in X is called an Anti intuitionistic fuzzy a-ideal of X if  $\mu_B(0) \leq \mu_B(x)$  and  $\mu_A(0) \leq \mu_A(x)$ ,

- I)  $\max\{\mu_A(0), \mu_B(0)\} \leq \max\{\mu_A(x), \mu_B(x)\} \Rightarrow \mu_{A \times B}(0) \leq \mu_{A \times B}(x), \forall x \in X.$   
 $\lambda_A(0) \geq \lambda_A(x), \lambda_B(0) \geq \lambda_B(x)$  and  
 $\min\{\lambda_A(0), \lambda_B(0)\} \geq \min\{\lambda_A(x), \lambda_B(x)\} \Rightarrow \lambda_{A \times B}(0) \geq \lambda_{A \times B}(x), \forall x \in X.$   
 $\gamma_A(0) \leq \gamma_A(x), \gamma_B(0) \leq \gamma_B(x)$  and  
 $\max\{\gamma_A(0), \gamma_B(0)\} \leq \max\{\gamma_A(x), \gamma_B(x)\} \Rightarrow \gamma_{A \times B}(0) \leq \gamma_{A \times B}(x), \forall x \in X.$
- II)  $\mu_A(y-x) \leq \max\{\mu_A((x-z) - (0-y)), \mu_A(z)\}$  and  
 $\mu_B(y-x) \leq \max\{\mu_B((x-z) - (0-y)), \mu_B(z)\}$   
 $\mu_{A \times B}(y-x) \leq \max\{\mu_{A \times B}((x-z) - (0-y)), \mu_{A \times B}(z)\}.$   
 $\lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0-y)), \lambda_A(z)\}$  and  
 $\lambda_B(y-x) \geq \min\{\lambda_B((x-z) - (0-y)), \lambda_B(z)\}$   
 $\lambda_{A \times B}(y-x) \geq \min\{\lambda_{A \times B}((x-z) - (0-y)), \lambda_{A \times B}(z)\}.$   
 $\gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\}$  and  
 $\gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}$   
 $\gamma_{A \times B}(y-x) \leq \max\{\gamma_{A \times B}((x-z) - (0-y)), \gamma_{A \times B}(z)\}.$

Hence  $A \times B$  is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

**Example: 4**

Consider a BCI-algebra  $X = \{0, a, b\}$  with the following Cayley table

X	0	a	b
$\mu_A$	0.2	0.7	0.2
$\lambda_A$	0.5	0.2	0.5
$\gamma_A$	0.4	0.6	0.4

X	0	a	b
$\mu_B$	0.3	0.8	0.3
$\lambda_B$	0.7	0.4	0.7
$\gamma_B$	0.6	0.7	0.6

- Let  $x=a,$   $\max\{\mu_A(0), \mu_B(0)\} \leq \max\{\mu_A(x), \mu_B(x)\}, 0.3 \leq 0.8$
- Let  $x=b,$   $\min\{\lambda_A(0), \lambda_B(0)\} \geq \min\{\lambda_A(x), \lambda_B(x)\}$  gives  $0.5 \geq 0.5$
- And let  $x=a,$   $\max\{\gamma_A(0), \gamma_B(0)\} \leq \max\{\gamma_A(x), \gamma_B(x)\}$  gives  $0.6 \leq 0.7$  is satisfied.
- II) If  $x=b, y=a, z=0$   $\mu_A(y-x) \leq \max\{\mu_A((x-z) - (0-y)), \mu_A(z)\}$  and  
 $\mu_B(y-x) \leq \max\{\mu_B((x-z) - (0-y)), \mu_B(z)\}$   
 $0.1 \leq 0.1$

Therefore  $\mu_{A \times B}(y-x)$  is satisfied.

- If  $x=b, y=a, z=0$   $\lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0-y)), \lambda_A(z)\}$  and  
 $\lambda_B(y-x) \geq \min\{\lambda_B((x-z) - (0-y)), \lambda_B(z)\}$   
 $0.4 \geq 0.4$

Therefore  $\lambda_{A \times B}(y-x)$  is satisfied.

- If  $x=b, y=a, z=0$   $\gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\}$  and  
 $\gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}$   
 $0.3 \leq 0.3$

Therefore  $\gamma_{A \times B}(y-x)$  is satisfied.

This completes the proof.

Therefore  $A \times B$  is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

**Theorem 5:** If A and B is a support intuitionistic fuzzy a-ideal in BCI-algebra X, then  $A \otimes B$  is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

**Proof:**

An AIFS  $A = \langle X, \mu_A, \lambda_A, \gamma_A \rangle$  in X is called an Anti intuitionistic fuzzy a-ideal of X if

- I)  $\mu_A(0) \leq \mu_A(x), \mu_B(0) \leq \mu_B(x)$  and  
 $\min\{\mu_A(0), \mu_B(0)\} \leq \min\{\mu_A(x), \mu_B(x)\} \Rightarrow \mu_{A \otimes B}(0) \leq \mu_{A \otimes B}(x), \forall x \in X.$   
 $\lambda_A(0) \geq \lambda_A(x), \lambda_B(0) \geq \lambda_B(x)$  and  
 $\max\{\lambda_A(0), \lambda_B(0)\} \geq \max\{\lambda_A(x), \lambda_B(x)\} \Rightarrow \lambda_{A \otimes B}(0) \geq \lambda_{A \otimes B}(x), \forall x \in X.$   
 $\gamma_A(0) \leq \gamma_A(x), \gamma_B(0) \leq \gamma_B(x)$  and

$$\begin{aligned}
 & \min \{ \gamma_A(0), \gamma_B(0) \leq \min \{ \gamma_A(x), \gamma_B(x) \} \Rightarrow \gamma_{A \otimes B}(0) \leq \gamma_{A \otimes B}(x), \forall x \in X. \\
 \text{II)} & \quad \mu_A(y-x) \leq \max \{ \mu_A((x-z) - (0-y)), \mu_A(z) \} \text{ and} \\
 & \quad \mu_B(y-x) \leq \max \{ \mu_B((x-z) - (0-y)), \mu_B(z) \} \\
 & \quad \mu_{A \otimes B}(y-x) \leq \max \{ \mu_{A \otimes B}((x-z) - (0-y)), \mu_{A \otimes B}(z) \}. \\
 & \quad \lambda_A(y-x) \geq \min \{ \lambda_A((x-z) - (0-y)), \lambda_A(z) \} \text{ and} \\
 & \quad \lambda_B(y-x) \geq \min \{ \lambda_B((x-z) - (0-y)), \lambda_B(z) \} \\
 & \quad \lambda_{A \otimes B}(y-x) \geq \max \{ \lambda_{A \otimes B}((x-z) - (0-y)), \lambda_{A \otimes B}(z) \}. \\
 & \quad \gamma_A(y-x) \leq \max \{ \gamma_A((x-z) - (0-y)), \gamma_A(z) \} \text{ and} \\
 & \quad \gamma_B(y-x) \leq \max \{ \gamma_B((x-z) - (0-y)), \gamma_B(z) \} \\
 & \quad \gamma_{A \otimes B}(y-x) \leq \max \{ \gamma_{A \otimes B}((x-z) - (0-y)), \gamma_{A \otimes B}(z) \}.
 \end{aligned}$$

Hence  $A \otimes B$  is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

**Example 5:** Consider a BCI-algebra  $X = \{0, a, b\}$  with the following Cayley table

X	0	a	b
$\mu_A$	0.2	0.7	0.2
$\lambda_A$	0.5	0.2	0.5
$\gamma_A$	0.4	0.6	0.4

X	0	a	b
$\mu_B$	0.3	0.8	0.3
$\lambda_B$	0.7	0.4	0.7
$\gamma_B$	0.6	0.7	0.6

$$\begin{aligned}
 \text{I)} \quad & \text{Let } x=a, \quad \min \{ \mu_A(0), \mu_B(0) \} \leq \min \{ \mu_A(x), \mu_B(x) \} \Rightarrow 0.2 \leq 0.7 \\
 & \text{If } x=b, \quad \min \{ \mu_A(0), \mu_B(0) \} \leq \min \{ \mu_A(x), \mu_B(x) \} \Rightarrow 0.2 \leq 0.2 \\
 & \text{Let } x=b, \quad \max \{ \lambda_A(0), \lambda_B(0) \} \geq \max \{ \lambda_A(x), \lambda_B(x) \} \Rightarrow 0.7 \geq 0.7 \\
 & \text{If } x=a, \quad \max \{ \lambda_A(0), \lambda_B(0) \} \geq \max \{ \lambda_A(x), \lambda_B(x) \} \Rightarrow 0.7 \geq 0.4 \\
 & \text{And let } x=a \quad \min \{ \gamma_A(0), \gamma_B(0) \} \leq \min \{ \gamma_A(x), \gamma_B(x) \} \quad 0.4 \leq 0.6 \\
 & \text{Let } x=b, \quad \min \{ \gamma_A(0), \gamma_B(0) \} \leq \min \{ \gamma_A(x), \gamma_B(x) \} \Rightarrow 0.4 \leq 0.4 \text{ is satisfied.}
 \end{aligned}$$

$$\begin{aligned}
 \text{II)} \quad & \text{If } x=b, y=a, z=0 \quad \mu_A(y-x) \leq \min \{ \mu_A((x-z) - (0-y)), \mu_A(z) \} \text{ and} \\
 & \quad \mu_B(y-x) \leq \min \{ \mu_B((x-z) - (0-y)), \mu_B(z) \} \\
 & \min \{ \mu_A(y-x), \mu_B(y-x) \} \leq \min \{ \max \{ \mu_A((x-z) - (0-y)), \mu_A(z) \}, \\
 & \quad \max \{ \mu_B((x-z) - (0-y)), \mu_B(z) \} \} \\
 & \quad 0.2 \leq 0.3 \text{ is satisfied.}
 \end{aligned}$$

$$\begin{aligned}
 & \text{If } x=0, y=a, z=b, \quad \mu_A(y-x) \leq \min \{ \mu_A((x-z) - (0-y)), \mu_A(z) \} \text{ and} \\
 & \quad \mu_B(y-x) \leq \min \{ \mu_B((x-z) - (0-y)), \mu_B(z) \} \\
 & \min \{ \mu_A(y-x), \mu_B(y-x) \} \leq \min \{ \max \{ \mu_A((x-z) - (0-y)), \mu_A(z) \}, \\
 & \quad \max \{ \mu_B((x-z) - (0-y)), \mu_B(z) \} \} \\
 & \quad 0.2 \leq 0.3 \text{ is satisfied.}
 \end{aligned}$$

$$\begin{aligned}
 & \text{If } x=a, y=0, z=b, \quad \mu_A(y-x) \leq \min \{ \mu_A((x-z) - (0-y)), \mu_A(z) \} \text{ and} \\
 & \quad \mu_B(y-x) \leq \min \{ \mu_B((x-z) - (0-y)), \mu_B(z) \} \\
 & \min \{ \mu_A(y-x), \mu_B(y-x) \} \leq \min \{ \max \{ \mu_A((x-z) - (0-y)), \mu_A(z) \}, \\
 & \quad \max \{ \mu_B((x-z) - (0-y)), \mu_B(z) \} \} \\
 & \quad 0.2 \leq 0.3 \text{ is satisfied.}
 \end{aligned}$$

$$\begin{aligned}
 & \text{If } x=b, y=a, z=0, \quad \lambda_A(y-x) \geq \min \{ \lambda_A((x-z) - (0-y)), \lambda_A(z) \} \text{ and} \\
 & \quad \lambda_B(y-x) \geq \min \{ \lambda_B((x-z) - (0-y)), \lambda_B(z) \} \\
 & \max \{ \lambda_A(y-x), \lambda_B(y-x) \} \geq \max \{ \min \{ \lambda_A((x-z) - (0-y)), \lambda_A(z) \}, \\
 & \quad \min \{ \lambda_B((x-z) - (0-y)), \lambda_B(z) \} \} \\
 & \quad 0.7 \geq 0.5 \text{ is satisfied.}
 \end{aligned}$$

$$\begin{aligned}
 & \text{If } x=a, y=0, z=b, \quad \lambda_A(y-x) \geq \min \{ \lambda_A((x-z) - (0-y)), \lambda_A(z) \} \text{ and} \\
 & \quad \lambda_B(y-x) \geq \min \{ \lambda_B((x-z) - (0-y)), \lambda_B(z) \} \\
 & \max \{ \lambda_A(y-x), \lambda_B(y-x) \} \geq \max \{ \min \{ \lambda_A((x-z) - (0-y)), \lambda_A(z) \}, \\
 & \quad \min \{ \lambda_B((x-z) - (0-y)), \lambda_B(z) \} \} \\
 & \quad 0.7 \geq 0.5 \text{ is satisfied.}
 \end{aligned}$$

$$\begin{aligned}
 & \text{If } x=0, y=b, z=a, \quad \lambda_A(y-x) \geq \min \{ \lambda_A((x-z) - (0-y)), \lambda_A(z) \} \text{ and} \\
 & \quad \lambda_B(y-x) \geq \min \{ \lambda_B((x-z) - (0-y)), \lambda_B(z) \} \\
 & \max \{ \lambda_A(y-x), \lambda_B(y-x) \} \geq \max \{ \min \{ \lambda_A((x-z) - (0-y)), \lambda_A(z) \}, \\
 & \quad \min \{ \lambda_B((x-z) - (0-y)), \lambda_B(z) \} \} \\
 & \quad 0.7 \geq 0.2 \text{ is satisfied.}
 \end{aligned}$$

$$\begin{aligned} \text{If } x=b, y=a, z=0, \quad & \gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\} \text{ and} \\ & \gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\} \\ \min\{\gamma_A(y-x), \gamma_B(y-x)\} & \leq \min\{\max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\}, \\ & \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}\} \end{aligned}$$

0.4 ≤ 0.6 is satisfied.

$$\begin{aligned} \text{If } x=0, y=a, z=b, \quad & \gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\} \text{ and} \\ & \gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\} \\ \min\{\gamma_A(y-x), \gamma_B(y-x)\} & \leq \min\{\max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\}, \\ & \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}\} \end{aligned}$$

0.6 ≤ 0.6 is satisfied.

$$\begin{aligned} \text{If } x=b, y=0, z=a, \quad & \gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\} \text{ and} \\ & \gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\} \\ \min\{\gamma_A(y-x), \gamma_B(y-x)\} & \leq \min\{\max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\}, \\ & \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}\} \end{aligned}$$

0.6 ≤ 0.7 is satisfied.

This completes the proof.

### Conclusion

In this project, we derived Support Anti Intuitionistic Fuzzy A-Ideal subtraction BCI-Algebras Theorems and Some Operations on Support Anti Intuitionistic Fuzzy A-ideal theorems and problems, also New results of anti intuitionistic subtraction Fuzzy Soft A-Ideal theorems. All the theorems are newly derived and suitable examples are provided to and valuate the findings verify the results.

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