Image Compression Using Lifting Based Haar Wavelet Transform Coupled With Spiht Algorithm

M.Gnanesh Goud¹, S.Balakrishna², N.Naresh³, B.Purender Reddy⁴
Electronics and Communication Engineering Dept.¹, ², ³, ⁴, TKR College of Engineering and Technology, Affiliated to JNTU
Hyderabad, Telangana, India

Abstract: In the coming of era the digitized image is an important challenge to deal with the storage and transmission requirements of enormous data, including medical images. Compression is one of the indispensable techniques to solve this problem. In this paper, we propose an algorithm for medical image compression based on lifting base wavelet transform coupled with SPIHT (Set Partition in Hierarchical Trees) coding algorithm, of which we applied the lifting structure to improve the drawbacks of conventional wavelet transform. We compared the results with various wavelet based compression algorithm. Experimental results show that the proposed algorithm is superior to traditional methods for all tested images at low bit rate. Our algorithm provides better PSNR and MSSIM values for medical images only at low bit rate.

Keywords- Compression, Haar wavelet, Lifting scheme, SPIHT, Entropy coding.

INTRODUCTION

Uncompressed digital image requires considerable storage capacity and transmission bandwidth. Despite of rapid progress in mass-storage density, processor speeds, and digital-communication system, it is need to improve the data storage capacity and data-transmission technologies. Medical records are an important dataset of patients’ medical history, and are needed by physicians to engage in an accurate diagnostic process. Thus, the repository for medical images in the medical institution will require huge storage capacities. This kind of system is very expensive. It also takes time to transfer the images. Over the past ten years, the wavelets (DWT) have had a huge success in the field of image processing, and have been used to solve many problems such as image compression and restoration [1]. However, despite of the success of wavelets in various fields of image processing such as encoding, weaknesses detection and representation of the objects’ contours. The wavelets transform and other classical multi resolutions decompositions seem to form a restricted and limited class of opportunities for multi-scale representations of multi dimensional signals. To overcome this problem, new multi resolution decompositions better adapted to the representation the image. This is the case of decomposition by lifting scheme. In this work we proposed the modified lifting structure algorithm for image compression. In order to improve image compression, we compare the PSNR and MSSIM results with the existing techniques.

II. WAVELET TRANSFORMS

The wavelet transform (WT), in general, produces floating point coefficients. Although these coefficients are used to reconstruction original image perfectly in theory, the use of finite precision arithmetic and quantization results in a lossy scheme. The lifting scheme based wavelet Transform can be implemented as shown in fig.1 for reducing computational complexity. Only the decomposition part of WT is depicted in Fig.1 because the reconstruction process is just the reverse version of Fig.1. The lifting-based WT consists of splitting, lifting, and scaling modules and the WT is treated as prediction-error decomposition. It provides a complete spatial interpretation of WT.

A. Spliting

In this stage the input signal is divided in to two disjoint sets, the odd (X[2n+1]) and the even samples (X[2n]).

B. Lifting

Figure 1. The Lifting-based WT.
In this module, the prediction operation P is used to estimate \( X_0(n) \) from \( X_e(n) \) and results in an error signal \( d(n) \). Then we update \( d(n) \) by applying it to the update operation U, and the resulting signal is combined with \( X_e(n) \) to \( S(n) \) estimate, which represents the smooth part of the original signal.

C. Scaling

A normalization factor is applied to \( d(n) \) and \( s(n) \), respectively. In the even-indexed part \( S(n) \) is multiplied by a normalization factor \( K_e \) to produce the wavelet subband \( XL_1 \). Similarly in the odd-indexed part the error signal \( d(n) \) is multiplied by \( K_0 \) to obtain the wavelet sub band \( XH_1 \). The output result is \( XL_1 \) and \( XH_1 \) by using the lifting-based WT are the same as those of using the convolution approach for the same input. For lifting implementation, the CDF 9/7 wavelet filter pair can be factorized into a sequence of primal and dual lifting. The most efficient factorization of the poly phase matrix for the 9/7 filter is as follows [2]:

\[
P(Z) = \begin{bmatrix}
0 & a(1+Z^{-1}) \\
1 & 0
\end{bmatrix} \begin{bmatrix} 1 & 0 \\
0 & 1/K
\end{bmatrix} \begin{bmatrix} 1 & c(1+Z^{-1}) \\
0 & 1
\end{bmatrix}
\]

(1)

The following equation describes the four “lifting” steps and the two “Scaling” steps. Where the values of the parameters:

\[
a = -1.586134342,
b = -0.0529801185,
c = 0.8829110762,
d = -0.4435068522,
K = 1.149604398.
\]

In stands of using equation 3 we update the four lifting steps by using a second order filter with same parameter. Which handles both FIR and IIR filters [3].

\[
\begin{align*}
Y(2n+1) & = Y(2n) + Y_e(z) \\
Y(2n) & = Y(2n-1) + Y_e(z)
\end{align*}
\]

(2)

Where \( Y_e(z) = \frac{b(1)+b(2)z^{-1}+...+b(nb+1)z^{-nb}}{1+a(2)z^{-1}+...+a(na+1)z^{-na}} \) \( Y(2n) \)

And \( Y_e(z) = \frac{b(1)+b(2)z^{-1}+...+b(nb+1)z^{-nb}}{1+a(2)z^{-1}+...+a(na+1)z^{-na}} \) \( Y(2n+1) \)

\[
\begin{align*}
Y(2n+1) & = -K \times Y(2n+1), \\
Y(2n) & = \left(\frac{1}{K}\right) \times Y(2n),
\end{align*}
\]

(3)

The filter function is implemented as a direct form II transposed structure,

\[
y(n) = b(1)x(n) + b(2)x(n-1) + ... + b(nb+1)x(n-b) - a(2)y(n-1) - ... - a(na+1)y(n-na)
\]

Where \( n-1 \) is the filter order, which handles both FIR and IIR filters [4], \( na \) is the feedback filter order, and \( nb \) is the feed forward filter order. The operation of filter at sample is given:

\[
y(m) = b(1)x(m) + z_1(m-1) \\
z_1(m) = b(2)x(m) + z_2(m-1) - a(2)y(m)
\]

The input-output description of this filtering operation in the -transform domain is a rational transfer function,

\[
Y(z) = \frac{b(1)+b(2)z^{-1}+...+b(nb+1)z^{-nb}}{1+a(2)z^{-1}+...+a(na+1)z^{-na}} X(z)
\]

(4)

III. HAAR WAVELET

In mathematics, the Haar wavelet is a sequence of rescaled "square-shaped" functions which together form a wavelet family or basis. Wavelet analysis is similar to Fourier analysis in that it allows a target function over an interval to be represented in terms of an orthonormal basis. The Haar sequence is now recognised as the first known wavelet basis and
extensively used as a teaching example. The Haar sequence was proposed in 1909 by Alfréd Haar. Haar used these functions to give an example of an orthonormal system for the space of square-integrable functions on the unit interval $[0, 1]$. The study of wavelets, and even the term "wavelet", did not come until much later. As a special case of the Daubechies wavelet, the Haar wavelet is also known as Db1.

The Haar wavelet is also the simplest possible wavelet. The technical disadvantage of the Haar wavelet is that it is not continuous, and therefore not differentiable. This property can, however, be an advantage for the analysis of signals with sudden transitions, such as monitoring of tool failure in machines.

Figure 3: Haar Wavelet

The Haar wavelet's mother wavelet function $\psi(t)$ can be described as

$$\psi(t) = \begin{cases} 1 & 0 \leq t < 1/2 \\ -1 & 1/2 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

Its scaling function $\phi(t)$ can be defined as

$$\phi(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

IV. SPIHT CODING SCHEME

SPIHT (Set Partition in Hierarchical Trees) [5] is one of the most advanced schemes, even outperforming the state-of-the-art JPEG 2000 in some situations. The basic principle is the same; a progressive coding is applied, processing the image respectively to a lowering threshold. The difference is in the concept of zero trees (spatial orientation trees in SPIHT). There is a coefficient at the highest level of the transform in a particular subband which considered insignificant against a particular threshold; it is very probable that its descendants in lower levels will be insignificant too. Therefore we can code quite a large group of coefficients with one symbol. A spatial orientation tree is defined in a pyramid constructed with recursive four subbands splitting. According to this relationship, the SPIHT algorithm saves many bits that specify insignificant coefficients [6].

Figure 4: Flow chart of SPIHT.

The flowchart of SPIHT is presented in Fig.3 as a First step; the original image is decomposed into subbands. Then the method finds the maximum iteration number. Second, the method puts the DWT coefficients into a sorting pass that finds the significance coefficients in all coefficients and encodes the sign of these significance coefficients. Third, the significance coefficients that can be found in the sorting pass are put into the refinement pass that uses two bits to exact the reconstruct value for approaching to real value.

The first, second and third steps are iterative, and then iteration decreases the threshold (Tn) and the reconstructive value (Rn – Rn-1/2). As a fourth step, the encoding bits access entropy coding and then transmit or store the bit. The result is in the form of a bit stream. All of the wavelet-based-image encoding algorithms improve the compression rate and the visual quality, but the wavelet-transform computation is a serious disadvantage of those algorithms.

V. HUFFMANN CODING

The technique works by creating a binary tree of nodes. These can be stored in a regular array, the size of which depends on the number of symbols. A node can be either a leaf node or an internal node. Initially, all nodes are leaf nodes, which contain the symbol itself, the weight (frequency of appearance) of the symbol and optionally, a link to a parent node which makes it easy to read the code (in reverse) starting from a leaf node. Internal nodes contain a weight, links to two child nodes and an optional link to a parent node. As a
common convention, bit '0' represents following the left child and bit '1' represents following the right child. A finished tree has up to leaf nodes and internal nodes. A Huffman tree that omits unused symbols produces the most optimal code lengths.

The process begins with the leaf nodes containing the probabilities of the symbol they represent. Then, the process takes the two nodes with smallest probability, and creates a new internal node having these two nodes as children. The weight of the new node is set to the sum of the weight of the children. We then apply the process again, on the new internal node and on the remaining nodes (i.e., we exclude the two leaf nodes), we repeat this process until only one node remains, which is the root of the Huffman tree.

The simplest construction algorithm uses a priority queue where the node with lowest probability is given highest priority:

- Create a leaf node for each symbol and add it to the priority queue.
- While there is more than one node in the queue:
  - Remove the two nodes of highest priority (lowest probability) from the queue
  - Create a new internal node with these two nodes as children and with probability equal to the sum of the two nodes' probabilities.
  - Add the new node to the queue.
- The remaining node is the root node and the tree is complete.

Since efficient priority queue data structures require $O(\log n)$ time per insertion, and a tree with $n$ leaves has $2n-1$ nodes, this algorithm operates in $O(n \log n)$ time, where $n$ is the number of symbols.

If the symbols are sorted by probability, there is a linear-time ($O(n)$) method to create a Huffman tree using two queues, the first one containing the initial weights, and combined weights being put in the back of the second queue. This assures that the lowest weight is always kept at the front of one of the two queues:

- Start with as many leaves as there are symbols.
- Enqueue all leaf nodes into the first queue (by probability in increasing order so that the least likely item is in the head of the queue).
- While there is more than one node in the queues:
  - Dequeue the two nodes with the lowest weight by examining the fronts of both queues.
  - Create a new internal node, with the two just-removed nodes as children (either node can be either child) and the sum of their weights as the new weight.
  - Enqueue the new node into the rear of the second queue.
  - The remaining node is the root node; the tree has now been generated.

In many cases, time complexity is not very important in the choice of algorithm here, since $n$ here is the number of symbols in the alphabet, which is typically a very small number (compared to the length of the message to be encoded); whereas complexity analysis concerns the behavior when $n$ grows to be very large.

It is generally beneficial to minimize the variance of codeword length. For example, a communication buffer receiving Huffman-encoded data may need to be larger to deal with especially long symbols if the tree is especially unbalanced. To minimize variance, simply break ties between queues by choosing the item in the first queue. This modification will retain the mathematical optimality of the Huffman coding while both minimizing variance and minimizing the length of the longest character code.

### VI. PERFORMANCE CRITERION:

The Peak Signal to Noise Ratio (PSNR) is the most commonly used as a measure of quality of reconstruction in image compression. The PSNR are formulating as:

$$PSNR = 10 \log_{10} \left( \frac{(\text{Dynamics of image})^2}{\text{MSE}} \right)$$  \hspace{1cm} (5)

Mean Square Error (MSE) which requires two MN grayscale images $I$ and $I_f$ where one of the images is considered as a compression of the other is defined:

$$\text{MSE} = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} (I(i,j) - I_f(i,j))^2$$  \hspace{1cm} (6)

Usually an image is encoded on 8 bits. It is represented by 256 gray levels.

### A. The structural similarity index (SSIM)

The PSNR measurement gives a numerical value on the damage, but it does [7] not describe its type. For medical imaging applications, where images are degraded must eventually be examined by experts, traditional evaluation remains insufficient. For this reason, objective approaches are needed to assess the medical imaging quality. We then evaluate a new
paradigm based on the assumption of human visual system (HVS). The similarity index compares the brightness, contrast and structure between each pair of vectors, where the structural similarity index (SSIM) between two signals x and y is given by the following expression [8]:  

$$\text{SSIM}(x,y) = l(x,y) c(x,y) s(x,y)$$  \hspace{1cm} (7)

However, the comparison of brightness is determined by the following expression:

$$l(x,y) = \frac{2\mu_x\mu_y + C_1}{\mu_x^2 + \mu_y^2 + C_1}$$  \hspace{1cm} (8)

Where the average intensity of signal x is given by mean intensity $\mu_x$, $(C_1=K_1L)^2$ the constant $K_1<<1$, and L is the dynamic range of the pixel values. The function of contrast comparison takes the following form:

$$c(x,y) = \frac{2\sigma_x\sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2}$$  \hspace{1cm} (9)

Where $\sigma_x$ is the standard deviation of the original signal $x$, $(C_1=K_2L)$ and the constant $K_1<<1$. The function of structure comparison is defined as follows:

$$s(x,y) = \frac{\text{cov}(x,y) + C_3}{\sigma_x\sigma_y + C_3}$$  \hspace{1cm} (10)

SSIM Becomes: $(C_3=C_2/2)$

$$\text{SSIM}(x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_x\sigma_y + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}$$  \hspace{1cm} (11)

Finally the quality measurement can provide a spatial map of the local image quality, which provides more information on the image quality degradation. For application, we require a single overall measurement formula as:

$$MSSIM(I,\hat{I}) = \frac{1}{M} \sum_{i=1}^{M} \text{SSIM}(I_i,\hat{I}_i)$$  \hspace{1cm} (12)

Where I and $\hat{I}$ are respectively the reference and degraded images, $I_i$ and $\hat{I}_i$ are the contents of images at the ith local window. M is the total number of local windows in image. The MSSIM values exhibit greater consistency with the visual quality.

VII. RESULTS AND DISCUSSION

We are interested in lossy compression methods based on 2D wavelet transforms because their properties are interesting. Indeed, the 2D wavelets transform combines good spatial and frequency locations. As work on any image, the spatial location and frequency are important [9].
CONCLUSION

The objective of this paper is undoubtedly the improvement of medical images quality after the compression step at low bit rate. The latter is regarded as an essential tool to aid diagnosis (storage or transmission) in medical imaging. We used the CDF Lifting based wavelet transform, coupled with the SPIHT coding. After several applications, we found that this algorithm gives better results than the other compression techniques only at low bit rate. To develop our algorithm, we have applied this technique on different types of medical images. We have noticed that for 0.75 bpp bit-rate, the algorithm provides better PSNR and MSSIM values for various medical images. In future this algorithm can use for all bit rates by modifying it. Thus, we conclude that the results obtained are very satisfactory in terms of compression ratio and compressed image quality only at low bit rate.

REFERENCE


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S.Bala Krishna received his B.Tech degree in Electronics and Communication Engineering from Jawaharlal Nehru technological University in 2010, and he did his M.Tech in Embedded Systems from Jawaharlal Nehru technological University in 2014, currently he is working as Assistant professor in TKR College of engineering and technology, Hyd.

N.Naresh received his B.Tech degree in Electronics and Communication Engineering from Jawaharlal Nehru technological University in 2011, and he did his M.Tech in Communication Systems from Jawaharlal Nehru technological University in 2014, currently he is working as Assistant professor in TKR College of engineering and technology, Hyd.

B.Purender Reddy received his B.Tech degree in Electronics and Communication Engineering from Jawaharlal Nehru technological University in 2010, and he did his M.Tech in Digital Electronics Communication Systems from PES University in 2014, currently he is working as Assistant professor in TKR College of engineering and technology, Hyd.

AUTHORS BIODATA

M.Gnanesh goud, received his B.Tech degree in Electronics and Communication Engineering from Jawaharlal Nehru technological University in 2012, and he completed his M.Tech from Jawaharlal Nehru technological University in 2014. He worked as Assistant professor in TKR College of Engineering and technology, Hyd.