Adaptive Beamforming in Planar Array Using Leaky LMS and Variable Step Size Leaky LMS Algorithms

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Abstract: One of the key technologies that are being used in present day cellular network is adaptive smart antenna which, with the help of cell site is able to direct its main beam towards the desired direction only and minimises the undesired interferences by producing nulls in those directions. This paper presents a modified least mean square algorithm known as leaky least mean square (LLMS) algorithm and variable step size leaky least mean square (VSLMS) for beamforming in smart antenna. Analysis of beamforming using leaky LMS and variable step size leaky LMS for uniform rectangular planar array (URPA) is done based on main beam direction, null direction, maximum side lobe level ($SLL_{\max}$) and convergence by varying the direction of arrival of the desired signal.

Index Terms - smart antenna; beam-forming; leaky least mean square algorithm; variable step size leaky least mean square algorithm.

I. INTRODUCTION

With the rapid growth in the usage of wireless technology, we need greater bandwidth to accommodate more and more number of users, this can be done with the help of smart antenna (SA) [1]. Smart antenna uses multiple antennas in array form, which can be arranged in different manner such as in circular or rectangular manner, to achieve different kind of results. The antenna array helps in achieving high directivity of the beam in desired direction, placing nulls towards the interferer and also helps in reducing the side lobe level, as higher value of side lobe level in not desirable. With the help of smart signal processing algorithms, digital signal processing modules and by using multiple antennas the above mentioned functions can be performed by smart antenna.

By the smart signal processing algorithm, it tries to estimate the spatial signal signature of the desired signal like the direction of arrival of the signal, and uses the same information to update its beam forming vectors, which will be then used to track and locate the antenna main radiation beam toward the mobile target [2]. Performance analysis of various adaptive beamforming algorithms are available in literature, having their own advantages and disadvantages. With many schemes of direction of arrival estimations, various methods of adaptive beamforming are described in [3]. For multilobe pattern and adaptive nulling a sequential quadratic programming based algorithm is used in [5]. Hybridization of soft-computing methods used for beamforming is reported in [6]. Beamforming of polarization-sensitive electromagnetic vector-sensor is done using a complex quaternion LMS algorithm [7]. Performance comparison of least mean square (LMS) algorithm and recursive least square algorithm (RLS) based on beamforming is reported in [8]. A constrained LMS (CSLMS) algorithm is used for adaptive beamforming using perturbation sequences [9]. An adaptive beamforming technique, MRVSS-LMS for the uplink of LTE system was presented by using channel estimation in the receiver to remove the effect of multipath in the channel. A significant improvement in performance of VSS-LMS as compared to LMS algorithm was seen over AWGN channel with good ability to generate multiple nulls [14]. Report on adaptive beamforming in SA using Leaky LMS algorithm and variable step-size Leaky LMS algorithm is relatively less.

This paper presents beamforming for smart antenna using Leaky LMS (LLMS) algorithm and variable step-size Leaky LMS (VSLMS) algorithm. Analysis of beamforming using LLMS and VSLMS algorithm is done based on main beam direction, null direction, $SLL_{\max}$, first null beamwidth (FNBW) and convergence for different angle of arrival of user and interferer.

II. LEAKY LEAST MEAN SQUARE ALGORITHM AND VARIABLE STEP SIZE LEAKY LMS ALGORITHM

Leaky least mean square (LLMS) algorithm is one of the most commonly used variant of LMS algorithm. Leaky LMS was introduced to overcome the slow convergence speed of LMS in case of high value eigen spread [11]. In LLMS algorithm, a leakage factor ($\phi$) is introduced in the weight update equation, which solves the drifting problem that occurs in LMS algorithm by bounding the parameter estimate [11]. Also, leak factor helps in improving capability, stability and convergence of the LMS algorithm [12]. In variable step size leaky LMS algorithm, the step size is varied adaptively along with the weights of the filter. An adaptive beamforming system is shown in Fig. 1. Planar array of MxN antenna elements is shown in Fig. 2 [10].
In Fig. 1, LLMS algorithm is used to minimize the error $e(n)$ between the desired signal $d(n)$ and the array output $y(n)$:[11]

$$
    e(n) = d(n) - y(n)
$$

(1)

The weights update equation using LLMS algorithm at ‘n’th iteration is[11]

$$
    w(n+1) = (1 - 2\mu\psi) * w(n) + \mu * e(n) * x(n)
$$

(2)

Where ‘$\mu$’ is the step size parameter, $\psi$ is the leak factor, $w(n)$ is the filter coefficients vector, $x(n)$ is received signal.

When $\psi = 0$, the leaky LMS algorithm will be equal to standard LMS algorithm.

Equations for variable step size leaky LMS[14]:

$$
    p = \beta * p + (1 - \beta) * e * e_1
$$

(3)

$$
    \mu = \alpha * \mu + \gamma * (p)^2
$$

(4)

The weight for variable step size Leaky LMS algorithm at ‘n’th iteration is

$$
    w(n+1) = (1 - 2\mu\psi) * w(n) + \mu * e(n) * x(n)
$$

(5)

where step size becomes a function of $p$ and $p$ is the error signal correlation at iteration time $n$ and $n+1$, $\mu$ is the step size, $\alpha$ and $\gamma$ are constants with values lying between $0<\alpha, \gamma>0$ and $\beta$ is the time average of square error signal, which is used to control the sensitivity of $p$ to the instantaneous error correlation.
Figure 2 shows a uniform rectangular planar array (URPA) configuration with M number of antenna elements arranged linearly in x-direction and N number of antenna elements arranged linearly in y-direction. Each element in x-axis and y-axis has an inter-element spacing of $d_x$ and $d_y$ respectively.

The array factor (AF) of M x N planar array can be expressed as [14]

$$AF_R(\varphi, \theta) = \sum_{m=1}^{M-1} \sum_{n=1}^{N-1} w_{mn} e^{j[(m-1)kd_x \sin \varphi + (n-1)kd_y \sin \theta]}$$  

$$AF = AF_x \cdot AF_y$$  

(6)  

(7)

where $w_{mn}$ represents the weight vector, which is used to steer the main beam of the planar array towards the desired direction and $k$ is the wave number ($k = \frac{2\pi}{\lambda}$), $\lambda$ is the carrier wavelength.

Where to generate the main beam at wavelength $\lambda$ toward the desired beam direction from the broadside direction, the progressive phase shift is,

$$\alpha = \frac{2\pi d}{\lambda} \sin \theta_0$$  

(8)

Normalized Array factor is,

$$AF_{norm} = \frac{AF}{AF_{max}}$$  

(9)

III. SIMULATION RESULTS

Simulations are done using MATLAB for URPA with inter-element spacing of $d_x = d_y = 0.5\lambda$ for 20 antenna elements and SNR of 20dB. Programs are run for 100 iterations, angle of desired user (AOA) is varies as 0° and 20°, angle of interferer are varies from -20° to +10°.

Figure 3-Figure 6 shows normalized array factor plot for 20 element URPA using Leaky LMS algorithm and variable step size Leaky LMS algorithm respectively with $d_x = d_y = 0.5\lambda$, SNR=20dB, $\psi=0.001$ and step size is 0.02. The main beam is taken at 0° and nulls are placed at +5°, -5°, +10°, and -10°.

Figure 3: Normalized AF using LLMS (AOA=0°, AOI=+5°, +10° and -10°)
Figure 4: Normalized AF using VSLLMS (AOA=0°, AOI=+5°, +10° and -10°)

Figure 5: Normalized AF using LLMS (AOA=0°, AOI=+5°, -5° and -10°)

Figure 6: Normalized AF using VSLLMS (AOA=0°, AOI=+5°, -5° and -10°)
Figure 7-Figure 8 shows normalized array factor plot for 20 element URPA using Leaky LMS algorithm and variable step size Leaky LMS algorithm respectively with $d_x=d_y=0.5\lambda$, SNR=20dB, $\psi=0.001$ and step size is 0.02. The main beam is taken at $20^0$ and nulls are placed at $-5^0$, $-15^0$ and $-20^0$.

Fig. 9-Fig. 10 shows the convergence performance of Leaky LMS algorithm and variable step size Leaky LMS algorithm for 20 number of antenna elements with $d_x=d_y=0.5\lambda$, SNR=20dB, $\psi=0.001$ and step size is 0.02.
Performance analysis of LLMS algorithm is summarized in Table 1.

Table 1. Performance analysis of LLMS and VSLLMS algorithm with $\mu=0.02$, $d_x=d_y=0.5\lambda$, $\psi=0.001$, SNR=20 dB

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Desired Main Beam (in degree)</th>
<th>Desired Null (in degree)</th>
<th>Achieved Main Beam (in degree)</th>
<th>Achieved Null (in degree)</th>
<th>SLL$_{\text{max}}$ (in dB)</th>
<th>FNBW</th>
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<tr>
<td>VSLLMS</td>
<td>0°</td>
<td>+5°</td>
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<td>+5.4°</td>
<td>-24.94</td>
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<td>LLMS</td>
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<td>+5°</td>
<td>-0.2</td>
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IV. CONCLUSION

A new method of adaptive beam formation for smart antenna using VSLLMS algorithm is presented here. Performance analysis is done for LLMS and VSLLMS by varying the direction of main beam and interferer. Multiple nulls are achieved with maximum 16% deviation from desired direction. VSLLMS converges with 10 number of iterations. But LLMS takes more number of iterations to converge.

REFERENCES