Stress Analysis of Rotating Multilayer Cylinder Subjected to Internal Pressure and Radial Temperature Gradient

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Abstract: Rotating multilayer cylinder subjected to internal pressure and/or radial temperature gradient finds several applications, e.g. mold of centrifugal casting machine. The cylinder wall may undergo elastic deformation during the operation. The present study deals with thermo-elastic stress analysis of rotating multilayer cylinder subjected to internal pressure and radial thermal gradient. Multilayer compound cylinder is usually tailor made in order to satisfy the requirement of working in extreme operating conditions characterized by high temperature, high pressure and/or corrosive environment. This paper presents a basic model that can be used to study the effects of rotation, internal pressure and temperature on the stress distributions and displacement fields in multilayer compound cylinders. The results of the developed analytical approach are compared with published research paper and validated.

Index Terms: Multilayer cylinder, rotation, internal pressure, temperature gradient.

I. INTRODUCTION

In many industrial applications, the cylinders are often subjected to extreme operating conditions characterized by high temperature, high pressure and/or corrosive environment. Conventional cylinder made of single material can barely satisfy the requirements for service in extreme operating condition. Therefore, a multilayered composite cylinder is usually tailor-made to satisfy the particular requirements, using different layer. Most cylinders used in industry, are subjected to thermal loads produced by temperature variation in addition to mechanical loads. Thermal load has a significant influence on the stress distribution of a multilayered composite pressure vessel because it is made of materials with different coefficients of thermal expansion and constructed by shrink fitting. So, it is important to derive solution for multilayer cylinder subjected to thermo-mechanical load. In case of multilayer cylinder, a larger number of design variables are available to the designer.

A cylinder made of two or more layer is known as multilayer cylinder. In a multilayer compound cylinder the outer cylinder is shrunk fit over the inner cylinder by heating and cooling. On cooling, the contact pressure is developed at the junction of the two cylinders, induces compressive tangential stress in the material of the inner cylinder and tensile tangential stress in the material of the outer cylinder. When the cylinder is loaded, the compressive stresses are first relieved and then tensile stresses are induced. Hence, a multilayer compound cylinder is effective in resisting higher internal pressure than single cylinder with the same overall dimension.

A deformation of cylinder subjected to various thermo-mechanical loads has been investigated by many researchers. Initially the French mathematician Gabriel Lame presented an analytical solution of cylinder made of homogeneous and isotropic material exposed to external and internal pressure. This problem has been extended to consider the thermal stresses in number of work. Whalley has presented a series of papers on design of shell subjected to thermal stresses in addition to pressure with different authors. In part IV of that series, Whalley and Morris [1] have discussed the design of multilayer vessel subjected to thermal stress. It was shown that thermal and pressure stresses may add or subtract at any point within the vessel. In part IV, Whalley and Mackinnon [2] applied this theory considering transient thermal stress. Bahoum and Diany [3] presented a basic model that can be used to study the effects of temperature and internal pressure on the stress distributions and displacement fields in compound cylinders. In this model, two layer compound cylinder was considered with logarithmic radial temperature distribution. Stress distribution is obtained for two cylinder layer made of same material steel ASTM A564 H1150 and cylinder layer made of different material steel ASTM A564 H1150 and aluminum 1050A-H9. It was shown that to ensure permanent contact between two cylinders the minimum temperature $T_1$is around 100 ˚C when T_3 is the ambient temperature. The results of the analytical solution were compared and validated to finite element axis symmetric model. Anani and Rahimi [4] analyzed rotating thick-walled hollow cylindrical shell composed of functionally graded material by using the theory of hyper elasticity. Radial stress, circumferential stress and longitudinal stress as a function of radial direction are plotted for different values of n. The obtained results show that the material inhomogeneity parameter (n) and structure parameter (β: ratio of outer radius to inner radius) have a significant influence on the mechanical behavior of rotating thick-walled hollow cylindrical shell made of functionally graded materials with power law varying properties.

A cylinder subjected pressure and thermal load made of a homogenous material or functionally graded material (FGM) has been investigated by number of researches till now. Research work related to cylinder subjected to pressure, rotation and radial temperature gradient is missing in this literature survey. Hence, aim of the present investigation is to develop analytical solution of multilayer cylinder subjected to internal pressure, rotation and radial temperature gradient.

II. ANALYTICAL FORMULATION OF MULTILAYER CYLINDER

A Two layer cylinder with internal radius a, mid radius b and external radius c is considered for the mathematical modeling. A general equation for the cylinder is derived first and then this equation is applied to each layer of the multilayer cylinder and these equations are solved using the boundary condition for the multilayer cylinder.

The equation of equilibrium in radial direction and strain-displacement relationship for the rotating cylinder under axisymmetric loading condition is written as [5],

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_t}{r} + \gamma\omega^2 r = 0 \tag{1}$$

$$\varepsilon_r = \frac{du}{dr} \qquad \& \qquad \varepsilon_t = \frac{u}{r} \tag{2}$$

Hooke's constitutive laws for the tri-axial stress state in the linear elastic field with thermal loading are as follows

$$\varepsilon_r = \frac{1}{E}\left(\sigma_r - v(\sigma_t + \sigma_z)\right) + \alpha T \tag{3}$$

$$\varepsilon_t = \frac{1}{E}\left(\sigma_t - v(\sigma_r + \sigma_z)\right) + \alpha T \tag{4}$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - v(\sigma_r + \sigma_t)\right) + \alpha T \tag{5}$$

From above Eq. 3 - 5 values of σ_r, σ_t and σ_z is given by

$$\sigma_r = \frac{E}{(1+v)}\left(\frac{v \cdot e}{1-2v} + v\varepsilon_r\right) - \frac{E}{(1-2v)} \cdot \alpha T \tag{6}$$

$$\sigma_t = \frac{E}{(1+v)}\left(\frac{v \cdot e}{1-2v} + v\varepsilon_t\right) - \frac{E}{(1-2v)} \cdot \alpha T \tag{7}$$

$$\sigma_z = \frac{E}{(1+v)}\left(\frac{v \cdot e}{1-2v} + v\varepsilon_z\right) - \frac{E}{(1-2v)} \cdot \alpha T \tag{8}$$

Obtaining the value of $\frac{d\sigma_r}{dr}$ and $\frac{\sigma_r - \sigma_t}{r}$ using the Eq. 6 and 7.

$$\frac{d\sigma_r}{dr} = \frac{E}{1+v}\left(\frac{v}{1-2v}\left(\frac{d^2u}{dr^2} + \frac{1}{r} \cdot \frac{du}{dr} - \frac{u}{r^2}\right) + \frac{d^2u}{dr^2}\right) - \frac{E}{1-2v} \cdot \alpha \frac{dT}{dr} \tag{9}$$

$$\frac{\sigma_r - \sigma_t}{r} = \frac{E}{1+v}\left(\frac{1}{r} \cdot \frac{du}{dr} - \frac{u}{r^2}\right) \tag{10}$$

Substituting the value of Eq. 9 and 10 in Eq. 1, we get the following governing differential equation for the rotating cylinder subjected to radial thermal gradient.

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} - \left(\frac{1+v}{1-v}\right)\alpha \frac{dT}{dr} + \frac{(1+v)(1-2v)}{E(1-v)}\gamma\omega^2 r = 0 \tag{11}$$

$$\frac{d}{dr}\left(\frac{1}{r} \cdot \frac{d}{dr}(u \cdot r)\right) - \left(\frac{1+v}{1-v}\right)\alpha \frac{dT}{dr} + \frac{(1+v)(1-2v)}{E(1-v)}\gamma\omega^2 r = 0 \tag{12}$$

Integrating the above equation to obtain the displacement u which involves two integration constants C_1 and C_2,

$$u = \alpha \cdot \frac{1+v}{1-v} \cdot \frac{1}{r} \cdot \int_{r_i}^{r} T \cdot r \cdot dr - \frac{(1+v)(1-2v)}{E(1-v)} \cdot \frac{\gamma\omega^2 r^3}{8} + \frac{C_1}{2} \cdot r + \frac{C_2}{r} \tag{13}$$

Differentiating the above equation, we get

$$\frac{du}{dr} = \frac{1+v}{1-v} \cdot \alpha \cdot T - \alpha \cdot \frac{1+v}{1-v} \cdot \frac{1}{r^2}\int_{r_i}^{r} T \cdot r \cdot dr = -\frac{(1+v)(1-2v)}{E(1-v)} \cdot \frac{3\gamma\omega^2 r^2}{8} + \frac{C_1}{2} - \frac{C_2}{r^2} \tag{14}$$

From this, value of radial and tangential strain is given by following equation

$$\varepsilon_r = \frac{du}{dr} = \frac{1+v}{1-v} \cdot \alpha \cdot T - \alpha \cdot \frac{1+v}{1-v} \cdot \frac{1}{r^2}\int_{r_i}^{r} T \cdot r \cdot dr = -\frac{(1+v)(1-2v)}{E(1-v)} \cdot \frac{3\gamma\omega^2 r^2}{8} + \frac{C_1}{2} - \frac{C_2}{r^2} \tag{15}$$

$$\varepsilon_t = \frac{u}{r} = \alpha \cdot \frac{1+v}{1-v} \cdot \frac{1}{r^2} \cdot \int_{r_i}^{r} T \cdot r \cdot dr - \frac{(1+v)(1-2v)}{E(1-v)} \cdot \frac{\gamma\omega^2 r^2}{8} + \frac{C_1}{2} + \frac{C_2}{r^2} \tag{16}$$

As per the generalized plane strain assumption, axial strain for multilayer cylinder is given by Eq. 17.

$$\varepsilon_z = \text{constant} = \varepsilon_0 \tag{17}$$

Substituting the values of ε_r, ε_t and ε_z from Eq. 15 to 17 in Eq. 6 to 8 and simplifying, we get

$$\sigma_r = \frac{E \cdot C_1}{2(1+v)(1-2v)} - \frac{E \cdot C_2}{(1+v)} \cdot \frac{1}{r^2} - \frac{(3-2v)}{8(1-v)} \cdot \gamma\omega^2 r^2 - \frac{\alpha E}{(1-v)} \frac{1}{r^2} \int_{r_i}^{r} T \cdot r \cdot dr + \frac{Ev}{(1+v)(1-2v)} \cdot \varepsilon_0 \tag{18}$$

$$\sigma_t = \frac{E \cdot C_1}{2(1+v)(1-2v)} + \frac{E \cdot C_2}{(1+v)} \frac{1}{r^2} - \frac{(1+2v)}{8(1-v)} \cdot \gamma\omega^2 r^2 + \frac{\alpha E}{(1-v)} \frac{1}{r^2} \int_{r_i}^{r} T \cdot r \cdot dr + \frac{Ev}{(1+v)(1-2v)} \cdot \varepsilon_0 - \frac{\alpha TE}{(1-v)} \tag{19}$$

$$\sigma_z = \frac{E \cdot C_1 \cdot v}{(1+v)(1-2v)} - \frac{v}{(1-v)} \cdot \frac{\gamma\omega^2 r^2}{2} + \frac{E(1-v)}{(1+v)(1-2v)} \cdot \varepsilon_0 - \frac{\alpha TE}{(1-v)} \tag{20}$$

Using equations 18 to 20 of a single layer cylinder, the equation of a multilayer cylinder is given by Eq. 21 to 23. Where i is the number of layers (i=2 for this case). Equation involves 2i+1 unknown (5 unknown for this case).

$$\sigma_r^i(r) = \frac{E_i \cdot C_{i1}}{2(1+v_i)(1-2v_i)} - \frac{E_i \cdot C_{i2}}{(1+v_i)} \cdot \frac{1}{r^2} - \frac{(3-2v_i)}{8(1-v_i)} \cdot \gamma_i \omega^2 \cdot r^2 - \frac{\alpha_i E_i}{(1-v)} \cdot \frac{1}{r^2} \cdot \int_{r_i}^{r} T_{cyi} \cdot rdr + \frac{E_i v_i}{(1+v_i)(1-2v_i)} \cdot \varepsilon_0 \tag{21}$$

$$\sigma_t^i(r) = \frac{E_i \cdot C_{i1}}{2(1+v_i)(1-2v_i)} + \frac{E_i \cdot C_{i2}}{(1+v_i)} \cdot \frac{1}{r^2} - \frac{(1+2v_i)}{8(1-v_i)} \cdot \gamma_i \omega^2 \cdot r^2 + \frac{\alpha_i E_i}{(1-v_i)} \cdot \frac{1}{r^2} \cdot \int_{r_i}^{r} T_{cyi} \cdot r \cdot dr - \frac{\alpha_i E_i T_{cyi}}{(1-v_i)} + \frac{E_i v_i}{(1+v_i)(1-2v_i)} \cdot \varepsilon_0 \tag{22}$$

$$\sigma_z^i(r) = \frac{E_i \cdot C_{i1} \cdot v_i}{(1+v_i)(1-2v_i)} - \frac{v_i}{(1-v_i)} \cdot \frac{\gamma_i \omega^2 r^2}{2} - \frac{\alpha_i E_i T_{cyi}}{(1-v_i)} + \frac{E_i(1-v_i)}{(1+v_i)(1-2v_i)} \cdot \varepsilon_0 \tag{23}$$

$$u^i(r) = \alpha_i \cdot \frac{1+v_i}{1-v_i} \cdot \frac{1}{r} \cdot \int_{r_i}^{r} T_{cyi} \cdot r \cdot dr - \frac{(1+v_i)(1-2v_i)}{E(1-v_i)} \cdot \frac{\gamma_i \omega^2 r^3}{8} + \frac{C_{i1}}{2} \cdot r + \frac{C_{i2}}{r} \tag{24}$$

Temperature distribution in each layer is given by the following equation.

$$T_{cyi} = T_{ou} + (T_{in} - T_{ou}) \frac{\ln\left(\frac{r}{r_{ou}}\right)}{\ln\left(\frac{r_{in}}{r_{ou}}\right)} \tag{25}$$

Equation for two layer cylinder involves 5 unknowns to be determined. These unknowns are determined using the following five conditions [3, 5].

1. $\sigma_r^1(a) = -P_i$
2. $\sigma_r^2(c) = 0$
3. $\sigma_r^1(b) = \sigma_r^2(b)$
4. $u^1(b) = u^2(b)$
5. $F_z = \sum_{i=1}^{2} \int \sigma_z^i \cdot dA = P_i \cdot \pi r_i^2$

I. $\sigma_r^1(a) = -P_i$

$$\frac{E_1}{2(1+v_1)(1-2v_1)} C_{11} - \frac{E_1}{(1+v_1)a^2} C_{12} + \frac{E_1 v_1}{(1+v_1)(1-2v_1)} \varepsilon_0 = \frac{3-2v_1}{8(1-v_1)} \gamma_1 \omega^2 a^2 - P_i$$

$$k_1 C_{11} - k_2 C_{12} + k_3 \varepsilon_0 = M_1 \tag{26}$$

II. $\sigma_r^2(c) = 0$

$$\frac{E_2}{2(1+v_2)(1-2v_2)} C_{21} - \frac{E_2}{(1+v_2)c^2} C_{22} + \frac{E_2 v_2}{(1+v_2)(1-2v_2)} \varepsilon_0 = \frac{3-2v_2}{8(1-v_2)} \gamma_2 \omega^2 c^2 + \frac{\alpha_2 E_2}{(1-v_2)c^2} \int_b^c T \cdot r dr$$

$$k_4 C_{21} - k_5 C_{22} + k_6 \varepsilon_0 = M_2 \tag{27}$$

III. $\sigma_r^1(b) = \sigma_r^2(b)$

$$\frac{E_1}{2(1+v_1)(1-2v_1)} C_{11} - \frac{E_1}{(1+v_1)b^2} C_{12} - \frac{E_2}{2(1+v_2)(1-2v_2)} C_{21} + \frac{E_2}{(1+v_2)b^2} C_{22} + \varepsilon_0 \left(\frac{E_1 v_1}{(1+v_1)(1-2v_1)} - \frac{E_2 v_2}{(1+v_2)(1-2v_2)} \right) =$$
$$\frac{3-2v_1}{8(1-v_1)} \gamma_1 \omega^2 b^2 - \frac{3-2v_2}{8(1-v_2)} \gamma_2 \omega^2 b^2 + \frac{\alpha_1 E_1}{(1-v_1)b^2} \int_a^b T \cdot r \cdot dr$$

$$k_1 C_{11} - k_7 C_{12} - k_4 C_{21} + k_8 C_{22} + k_9 \varepsilon_0 = M_3 \tag{28}$$

IV. $u^1(b) = u^2(b)$

$$\frac{C_{11}}{2} + \frac{C_{12}}{b^2} - \frac{C_{21}}{2} - \frac{C_{22}}{b^2} = \frac{(1+v_1)(1-2v_1)}{1-v_1} \cdot \frac{\gamma_1 \omega^2 b^2}{8E_1} - \frac{(1+v_2)(1-2v_2)}{1-v_2} \cdot \frac{\gamma_2 \omega^2 b^2}{8E_2} - \frac{1+v_1}{1-v_1} \cdot \frac{\alpha_1}{b^2} \int_a^b T \cdot r \cdot dr$$

$$\frac{1}{2} C_{11} + k_{10} C_{12} - \frac{1}{2} C_{21} - k_{10} C_{22} = M_4 \tag{29}$$

V. $\sum_{i=1}^{2} \int \sigma_z^i \cdot dA = P_i \cdot \pi r_i^2$

$\therefore \int_a^b \sigma_z^1 \cdot 2\pi r \cdot dr + \int_b^c \sigma_z^2 \cdot 2\pi r \cdot dr = P_i \cdot \pi r_i^2$

$\therefore \int_a^b \sigma_z^1 \cdot r \cdot dr + \int_b^c \sigma_z^2 \cdot r \cdot dr = \frac{1}{2} P_i \cdot r_i^2$

$$\frac{E_1 v_1 (b^2-a^2)}{2(1+v_1)(1-2v_1)} C_{11} + \frac{E_2 v_2 (c^2-b^2)}{2(1+v_2)(1-2v_2)} C_{21} + \varepsilon_0 \left(\frac{E_1(1-v_1)(b^2-a^2)}{2(1+v_1)(1-2v_1)} - \frac{E_2(1-v_2)(c^2-b^2)}{2(1+v_2)(1-2v_2)} \right) = \frac{v_1}{8(1-v_1)} \gamma_1 \omega^2 (b^4 - a^4) + \frac{v_2}{8(1-v_2)} \gamma_2 \omega^2 \left(c^4 - b^4\right) + \frac{\alpha_1 E_1}{(1-v_1)} \int_a^b T \cdot r \cdot dr + \frac{\alpha_2 E_2}{(1-v_2)} \int_b^c T \cdot r \cdot dr + \frac{1}{2} P_i \cdot r_i^2$$

$$k_{11} C_{11} + k_{12} C_{21} + k_{13} \varepsilon_0 = M_5 \qquad (30)$$

Where,

$$k_1 = \frac{E_1}{2(1+v_1)(1-2v_1)} \qquad k_2 = \frac{E_1}{(1+v_1)a^2}$$

$$k_3 = \frac{E_1 v_1}{(1+v_1)(1-2v_1)} = 2k_1 v_1 \qquad k_4 = \frac{E_2}{2(1+v_2)(1-2v_2)}$$

$$k_5 = \frac{E_2}{(1+v_2)c^2} \qquad k_6 = \frac{E_2 v_2}{(1+v_2)(1-2v_2)} = 2k_4 v_2$$

$$k_7 = \frac{E_1}{(1+v_1)b^2} \qquad k_8 = \frac{E_2}{(1+v_2)b^2}$$

$$k_9 = \frac{E_1 v_1}{(1+v_1)(1-2v_1)} - \frac{E_2 v_2}{(1+v_2)(1-2v_2)} = k_3 - k_6$$

$$k_{10} = \frac{1}{b^2}$$

$$k_{11} = \frac{E_1 v_1 (b^2 - a^2)}{2(1+v_1)(1-2v_1)} \qquad k_{12} = \frac{E_2 v_2 (c^2 - b^2)}{2(1+v_2)(1-2v_2)}$$

$$k_{13} = \frac{E_1(1-v_1)(b^2-a^2)}{2(1+v_1)(1-2v_1)} + \frac{E_2(1-v_2)(c^2-b^2)}{2(1+v_2)(1-2v_2)}$$

$$M_1 = \frac{3-2v_1}{8(1-v_1)} \gamma_1 \omega^2 a^2 - P_i$$

$$M_2 = \frac{3-2v_2}{8(1-v_2)} \gamma_2 \omega^2 c^2 + \frac{\alpha_2 E_2}{(1-v_2)c^2} \int_b^c T \cdot rdr$$

$$M_3=\frac{3-2v_1}{8(1-v_1)}\gamma_1\omega^2b^2-\frac{3-2v_2}{8(1-v_2)}\gamma_2\omega^2b^2+\frac{\alpha_1E_1}{(1-v_1)b^2}\int_a^b T\cdot r\cdot dr$$

$$M_4=\frac{(1+v_1)(1-2v_1)}{1-v_1}\cdot\frac{\gamma_1\omega^2b^2}{8E_1}-\frac{(1+v_2)(1-2v_2)}{1-v_2}\cdot\frac{\gamma_2\omega^2b^2}{8E_2}-\frac{1+v_1}{1-v_1}\cdot\frac{\alpha_1}{b^2}\int_a^b Trdr$$

$$M_5=\frac{v_1\gamma_1\omega^2}{8(1-v_1)}(b^4-a^4)+\frac{v_2\gamma_2\omega^2}{8(1-v_2)}\left(c^4-b^4\right)+\frac{\alpha_1E_1}{(1-v_1)}\int_a^b T\cdot rdr+\frac{\alpha_2E_2}{(1-v_2)}\int_b^c T\cdot rdr+\frac{1}{2}P_i\cdot r_i^2$$

Arranging the equation in the matrix form

$$\begin{bmatrix} k_1 & -k_2 & 0 & 0 & k_3 \\ 0 & 0 & k_4 & -k_5 & k_6 \\ k_1 & -k_7 & -k_4 & k_8 & k_9 \\ 1/2 & k_{10} & -1/2 & -k_{10} & 0 \\ k_{11} & 0 & k_{12} & 0 & k_{13} \end{bmatrix}\begin{bmatrix} C_{11} \\ C_{12} \\ C_{21} \\ C_{22} \\ \varepsilon_0 \end{bmatrix}=\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{bmatrix}$$

Solution above 5x5 system of equation gives the value of C_{11}, C_{12}, C_{21}, C_{22} and ε_o. Hence, stress distribution and displacement field for the two layer cylinder is given by the equation 31 to 34, where i =1, 2.

$$\sigma_r^i(r)=\frac{E_i\cdot C_{i1}}{2(1+v_i)(1-2v_i)}-\frac{E_i\cdot C_{i2}}{(1+v_i)}\cdot\frac{1}{r^2}-\frac{(3-2v_i)}{8(1-v_i)}\cdot\gamma_i\omega^2\cdot r^2-\frac{\alpha_iE_i}{(1-v)}\cdot\frac{1}{r^2}\cdot\int_{r_i}^r T_{cyi}\cdot rdr+\frac{E_iv_i}{(1+v_i)(1-2v_i)}\cdot\varepsilon_0 \quad (31)$$

$$\sigma_t^i(r)=\frac{E_i\cdot C_{i1}}{2(1+v_i)(1-2v_i)}+\frac{E_i\cdot C_{i2}}{(1+v_i)}\cdot\frac{1}{r^2}-\frac{(1+2v_i)}{8(1-v_i)}\cdot\gamma_i\omega^2\cdot r^2+\frac{\alpha_iE_i}{(1-v_i)}\cdot\frac{1}{r^2}\cdot\int_{r_i}^r T_{cyi}\cdot r\cdot dr-\frac{\alpha_iE_iT_{cyi}}{(1-v_i)}+\frac{E_iv_i}{(1+v_i)(1-2v_i)}\cdot\varepsilon_0 \quad (32)$$

$$\sigma_z^i(r)=\frac{E_i\cdot C_{i1}\cdot v_i}{(1+v_i)(1-2v_i)}-\frac{v_i}{(1-v_i)}\cdot\frac{\gamma_i\omega^2r^2}{2}-\frac{\alpha_iE_iT_{cyi}}{(1-v_i)}+\frac{E_i(1-v_i)}{(1+v_i)(1-2v_i)}\cdot\varepsilon_0 \quad (33)$$

$$u^i(r)=\alpha_i\cdot\frac{1+v_i}{1-v_i}\cdot\frac{1}{r}\cdot\int_{r_i}^r T_{cyi}\cdot r\cdot dr-\frac{(1+v_i)(1-2v_i)}{E(1-v_i)}\cdot\frac{\gamma_i\omega^2r^3}{8}+\frac{C_{i1}}{2}\cdot r+\frac{C_{i2}}{r} \quad (34)$$

III. RESULTS AND DISCUSSION

Solution of the multilayer cylinder is used to study the stress distribution of the mold. Table 1 presents material properties, dimensions and loading conditions of the mold used for the investigation of the mold. Mold made of same material for both layer and both layer of equal thickness is considered to study the results of multilayer cylinder.

Table 1 Mold dimensions, loading conditions and material properties

Mold material	AISI-4340
Density (ϒ) (Kg/m3)	7800
Poisson's ratio	0.29
Young's Modulus (GPa)	210
Yield Strength (MPa)	910
Thermal expansion (/°C)	1.21E-05
Internal Radius (a) (mm)	1000
Mid Radius (b)(mm)	1110
External Radius (c) (mm)	1220
Length (mm)	1000
Internal Pressure (P_i) (MPa)	5
Angular velocity (ω)(rpm)	600
Temperature at Internal radius (T_1) (°C)	700
Temperature at Mid radius (T_2) (°C)	370
Temperature at Internal radius (T_2) (°C)	100

For obtaining stress distribution for multilayer cylinder, we require the value of temperature at mid radius T_2. This value of temperature T_2 can be found using following relation for heat transfer. Once the value of T_2 is known stress distribution is obtained using the equations of two layer cylinder. Stress distribution for interference pressure equals to 5 MPa is obtained and by using the principle of super position the final stress distribution for multilayer cylinder is shown in fig. 1.

$$Q = \frac{2\pi k_1 L(T_1 - T_2)}{\ln(\frac{b}{a})} = \frac{2\pi k_2 L(T_2 - T_3)}{\ln(\frac{c}{b})}$$

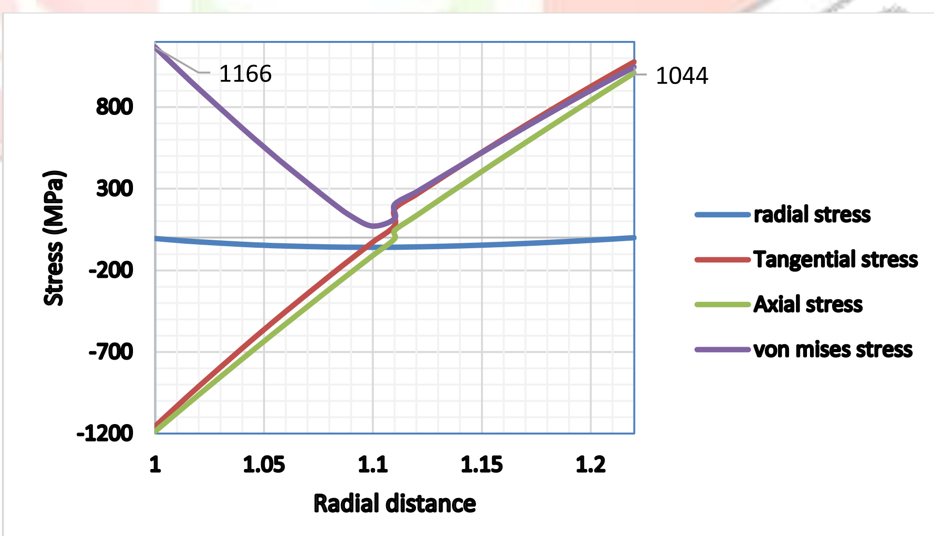

Fig. 1 Stress distribution for multilayer cylinder with interference pressure = 5 MPa

Figure. 1 presents the stress distribution for multilayer cylinder subjected to internal pressure of 5 MPa, Angular velocity of 600 rpm, internal wall temperature of 700 °C and external wall temperature of 400 °C. Radial stress is zero at internal and external radius and compressive in-between. Tangential and axial stress varies from compressive at internal radius to tensile at external radius. Von mises stress is also obtained in order study the combined effect of radial, tangential and axial stress.

Results of this case are compared with K. Bahoum paper to validate results of the developed analytical approach. Using the material properties and other conditions from the paper and considering angular velocity as 0 rad/sec in the obtained analytical solution, results are obtained and it shows good agreement with the paper result.

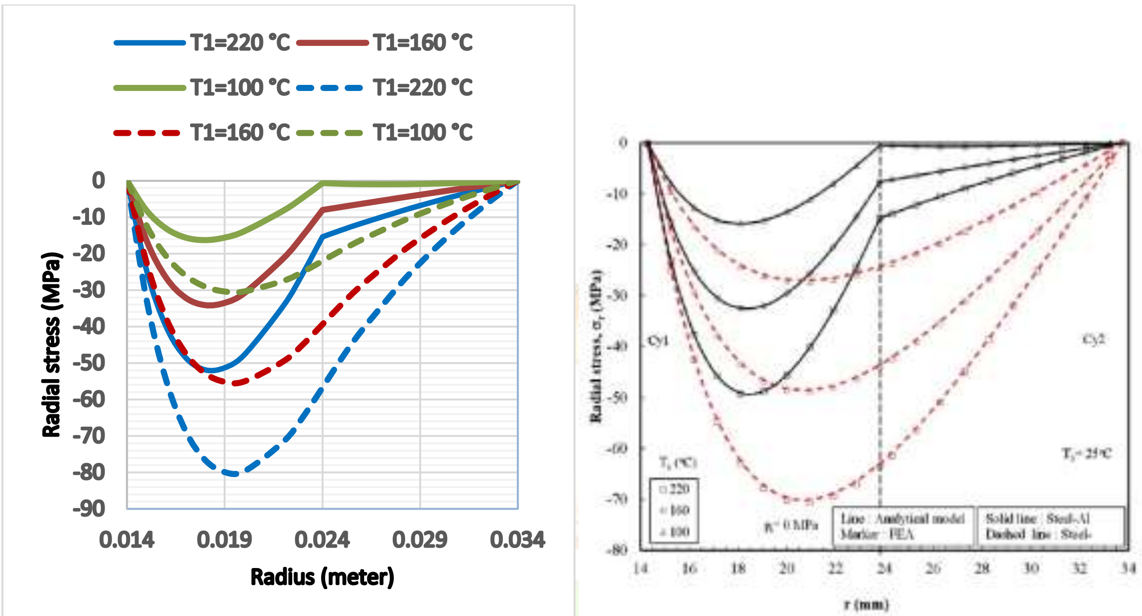

Fig. 2 Comparison of radial stress with paper

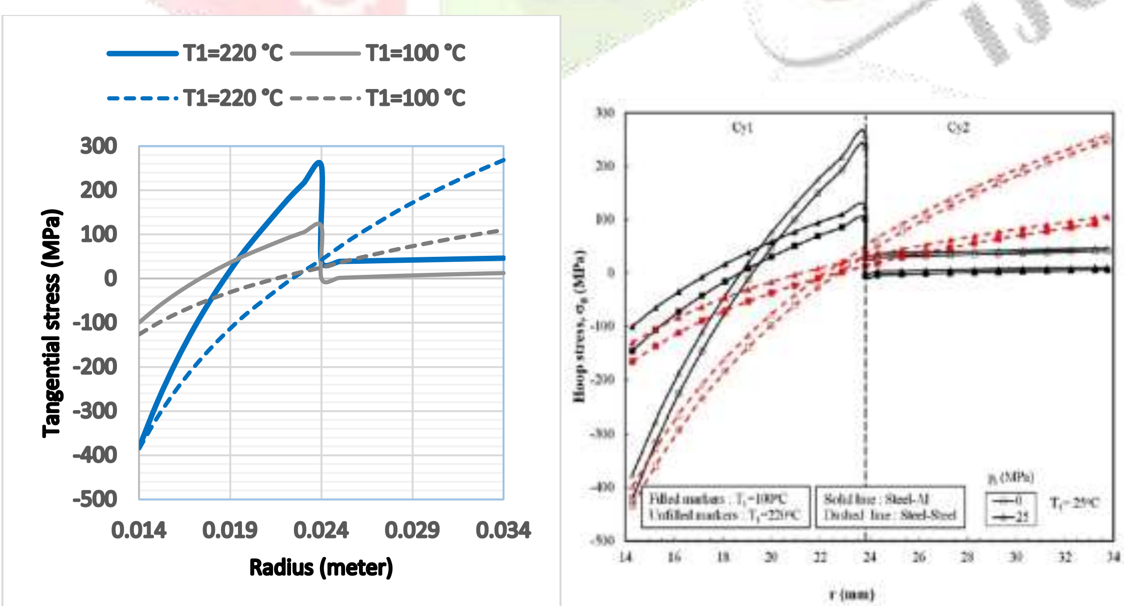

Fig. 3 Comparison of tangential stress with paper

The results presented in this study are just for a selected loading combinations and for single material selected. However, analytical solution (stress distribution) for any combination of loading and material properties can be obtained.

IV. CONCLUSION

The main objective of the current investigation is to derive analytical solution for rotating multilayer cylinder subjected to internal pressure and radial thermal gradient. The expressions of radial displacement and different stresses in the two cylinders, inner and outer, were deducted for the cases where the cylinders are made from the same materials. The results obtained in the current investigation lead to the following conclusions.

- The combined effects of rotation, internal pressure and temperature must be taken into account when designing compounds cylinders, to ensure their maximum efficiency and maximum availability.
- Effect of thermal load is dominant over the load due to internal pressure and rotation. When multilayer cylinder is subjected to combined load. The hoop stress is more affected by the variation of the temperature then the internal pressure value.

REFERENCES:

[1] Whalley E, Morris S. The Design of Pressure Vessels Subjected to Thermal Stresses-IV Multilayer Vessels. International Journal of Mechanical Sciences 1960; 1:369-378.

[2] Whalley E, Mackinnon RF. The Design of Pressure Vessels Subjectded to Thermal Stresses- III Thermal Shock. International Journal of Mechanical Sciences 1960;1: 301-308.

[3] K. Bahoum and M. Diany, “Stress analysis of compound cylinders subjected to thermo-mechanical loads”, Journal of Mechanical Science and Technology 31 (4), November 2016, 1805-1811

[4] Y. Anani and G. H. Rahimi, “Stress analysis of rotating cylindrical shell composed of functionally graded incompressible hyperelastic materials”, International Journal of Mechanical Sciences 108-109, February 2016, 122–128

[5] Vullo V, Vivio F. Rotors: Stress Analysis and Design, Mechanical Engineering Series, DOI 10.1007/978-88-470-2562-2_1, Springer-Verlag, Italia 2013.