# Chemical reaction and Viscous Dissipation effects on MHD free convective Rivlin-Ericksen fluid flow past a semi-infinite stationary vertical porous plate with radiation absorption

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Abstract: In this paper we examine the Chemical reaction and Viscous Dissipation effects on MHD free convective Rivlin-Ericksen fluid flow past a semi-infinite stationary vertical porous plate with radiation absorption. The flow was unsteady and restricted to the laminar domain. The dimensionless governing equations for this investigation are solved analytically by using multiple regular perturbation law. The effects of various physical parameters on velocity, temperature and concentration fields are presented graphically. With the aid of these, the expression for the skin friction, Nusselt number and Sherwood number profiles was done with the help of tables. It was found that an increases in chemical reaction, Schmidt number, Prandtl number, thermal radiation, leads to decrease in both velocity and temperature. However an increase in both chemical reactions, Schmidt number, leads to decreases in concentration. And also an increase in porous medium, solutal Grashof number and Radiation parameter leads to increases in both velocity and temperature.

**Keywords:** Radiation absorption, viscous dissipation, MHD, chemical reaction, multiple regular Perturbation law.

	Nomenclature
<i>B</i> <sub>0</sub>	Magnetic induction (A.m <sup>-1</sup> )
С	Non dimensional Concentration
$C_p$	Specific heat for constant pressure(J. $kg^{-1}$ .K)
$C^*_w$	Concentration of the plate $(kg m^{-3})$
$C_{f}$	The local skin-friction $(N.m^{-1})$
${C_{\scriptscriptstyle \infty}}^*$	Concentration in the fluid far away from the plate( $kg.m^{-3}$ )
D	Chemical molecular diffusivity $(m^2.S^{-1})$
е	electron charge( $C$ )
E	Electric field intensity $(vm^{-1})$
Ec	Eckert number
<i>g</i> *	Acceleration due to gravity $(m.S^{-2})$
Gm	Solutal Grashof number
Gr	local temperature Grashof number
J	Current density $(Am^{-2})$

Κ	Permeability of porous medium
Kr	Chemical reaction parameter $(m.S^{-1})$
$k_{e}^{*}$	Mean absorption coefficient
k	Thermal conductivity( $W.m^{-1}.K^{-1}$ )
М	Magnetic parameter
N	Dimensionless material parameter
N <sub>w</sub>	Nusselt number
n	Mining
$P_y$	Yield stress of fluid
Pr	Prandtl number
$Q_0$	Heat generation/absorption
$q_r^*$	Radiation heat flux density $(W.m^{-2})$
R <sub>a</sub>	Thermal radiation parameter
$R^*$	Radiation parameter
S <sub>h</sub>	Sherwood number
Sc	Schmidt number
$T_{\infty}$	Dimensional free stream temperature (K)
$T^*$	Dimensional Temperature ( <i>K</i> )
<i>t</i> *	Dimensional time (S)
Τ	Temperature of the fluid (K)
$T_w$	Fluid temperature at walls (K)
$T_w^*$	Fluid temperature at the wall (K)
$T_{\infty}$	Dimensional free stream temperature (K)
$U^*_{\infty}$	Free stream velocity (K)
u	Components of velocity vector in x direction(m.S <sup>-1</sup> )
v	Velocity component in y direction $(m.S^{-1})$
U <sub>0</sub>	Reference velocity at the plate( $m S^{-1}$ )
<i>x</i> *, <i>y</i> *	Coordinate axis along the $plate(m)$ ,
	Co-ordinate axis normal to the plate( $m$ )
F —	Complex velocity
F	Complex conjugate of F
	Greek Symbols
$\beta_e$	Hall current parameter
$\alpha^*$	Shear rate
$\mathcal{O}_{e}$	Cyclotron frequency( $Hz$ )

β	Spin gradient viscosity (K <sup>-1</sup> )											
$\beta^*$	Coefficient of volumetric expansion (m <sup>3</sup> .kg <sup>-1</sup> )											
μ	Fluid dynamic viscosity											
ρ	Density of the fluid $(kg.m^{-3})$											
9	Fluid kinematic viscosity( $m^2.S^{-1}$ )											
θ	Non dimensional temperature ( <i>K</i> )											
ε	Scalar constant(<<1)											
$\beta_i$	Ion-Slip parameter											
σ	Electrical conductivity ( $\Omega^{-1}m^{-1}$ )											
$\sigma_{s}^{*}$	Stefan-Boltzmann constant( $Wm^{-2}K^{-4}$ )											
τ <sub>w</sub>	Skin-friction coefficient( $m^2/s$ )											
η	Heat generation parameter											
$ au_0$	Casson yield stress											
$\tau_e$	Electron collision time(S)											
	<b>Subscripts</b>											
Р	Plate											
x	Free stream condition											
W	Wall condition											
*	Dimensionless properties											

The investigation of viscoelastic fluid has turned out to be vital over the most recent couple of years. Subjective examination of these investigations has huge bearing on a few mechanical applications, for example, polymer sheet expulsion from a drawing of plastic firms. When fabricating forms at high temperature require cooling, the fluid may require viscoelastic fluid to create great impact or decrease the temperature. Then again, the flow and heat transfer of a viscoelastic fluid between parallel plates have noteworthy part in numerous designing fields, for example, petroleum production, chemical catalytic reactors and solar power collectors. The consciousness of Visco-elastic fluids past a porous media plays very important role in a lot of scientific and engineering applications. Most commonly this flow was mainly utilized in the fields of petroleum engineering concerned with the oil, gas and water through reservoir to the hydrologist in the analysis of the migration of underground water. This progression in chemical engineering is used for mutually decontamination and filtration. Its application is also found in the process of drug permeation through human skin. To recover the water for drinking and irrigation purposes the principles of this flow are followed. Free convection flow over a vertical surface embedded in porous media also comes across in many engineering areas like the design of nuclear reactors, catalytic reactors, and condensed heat exchangers, geothermal energy renovation, the use of fibrous materials, thermal insulation of buildings, heat transfer from storage of agricultural products which generate heat as a result of metabolism, petroleum reservoirs, nuclear wastes, etc.

Magneto Hydrodynamic free convection flow has critical, mechanical and logical applications in the field of stellar and planetary magnetospheres, aeronautical plasma streams, compound designing and hardware. The fluid flow and heat transfer through a permeable medium have been widely contemplated in the past due to its significance to atomic waste transfer, strong lattice heat transfer, heat protection and other down to earth application. Common convective streams are regularly experienced in physical and designing issues, for

example, concoction synergist reactors, atomic waste materials, and geothermal framework. The idea of concurrent heat and mass move is utilized as a part of different science and designing issues. It is utilized as a part of sustenance handling, wet-globule thermometer and polymer arrangement and furthermore in different liquids stream related designing issues. In our day by day life, the joined heat and mass exchange wonder is seen in the arrangement and scattering of mist, conveyance of temperature and dampness over agrarian fields and forests of natural product trees, trim harm because of solidifying, and ecological contamination.

Viscous dissipation plays an important role in changing the temperature distribution, just like an energy source, which affects the heat transfer rates considerably. In fact, the shear stresses can induce a significant amount of heat generation. In the flow of fluids, mechanical energy is degraded into heat and this process is called viscous dissipation. The viscous dissipation effects are important in geophysical flows and also in certain industrial operations are usually characterized by the Eckert number. In the literature, extensive research work is available to examine the effect of natural convection on flow past a plate. Umamaheswar et al [1] Viscous dissipation and Ohmic heating effects on unsteady magneto hydrodynamic free convective Visco-elastic fluid flow embedded in porous medium bounded by an infinite inclined porous plate in the presence of heat source. In this investigation the Non-Linear coupled partial differential equations are solved by using multiple regular perturbation method. From this paper I observed that for different values of heat source and Grashof number increases then it leads to increase in Velocity, Temperature, Skin friction and Nusselt number but it shows the reverse effects in case of magnetic field and Prandtl number S. Mohammed Ibrahim [2] considered Viscous dissipation and chemical reaction effects on unsteady MHD flow of a viscoelastic fluid past spontaneously taking place infinite vertical plate in existence of heat source/sink and Soret. Here in this investigation the Non-Linear coupled partial differential equations are solved by using multiple regular perturbation method. From this paper I observed that for various values of Eckert number increase then it leads to increase in both velocity and temperature but it shows the reverse effects in case of Prandtl number and heat source parameter. Soundalgekar [3] excellent paper was explained this paper viscous dissipation effect on the unsteady free convective flow of an elastic-viscous fluid past an infinite vertical plate with constant suction was investigated. P. Chandra Reddy [4] Analyzed viscous dissipation effect on MHD free convection flow of Rivlin-Ericksen fluid past a semi infinite vertical plate with constant mass flux numerically in presence of diffusion thermo and thermal diffusion effects. Here in this investigation the Non-Linear coupled partial differential equations are solved by finite difference method. In this paper I observed that the skin friction coefficient increases with increasing values of magnetic parameter, but a reverse effect is found in the case of visco-elastic parameter. Sherwood number increases with increase of Schmidt number but a reverse effect is noticed in the case of Soret and Dufour numbers. The Nusselt number decreases with an increase in Eckert number. Rita Choudhury et al [5] analyzed significance of viscous dissipation effects on MHD free convection flow of an incompressible and electrically conducting Visco-elastic fluid with heat and mass transfer past a vertical porous plate. Here in this investigation the Non-Linear coupled partial differential equations are solved by using multiple regular perturbation method. Satya Sagar Saxena et al [6] analyzed Viscous dissipation and chemical reaction effects on unsteady two dimensional MHD free convective oscillatory Rivlin-Ericksen fluid flow of a visco-elastic fluid past an infinite vertical porous plate in presence of heat source. In this consideration the Non-Linear coupled partial differential equations are solved by using multiple regular perturbation method. In this research I observed that for different values of Visco-elastic parameter increases then it leads to skin friction increases. V. Suresh babu et al [7] studied the effect of viscous dissipation and chemical reaction on unsteady hydro-magnetic free convective flow of a Walter's memory fluid past a vertical plate in presence of thermal radiation. In this consideration the Non-Linear coupled partial differential equations are solved by using multiple regular perturbation method.

A number of investigators extended their research the effects of chemical reaction on fluid flow in different physical situations. Due to its huge practical applications in several branches of science and engineering, a lot of diverse areas together with combustion, polymer melt flows, geothermic geophysics and solar collectors,

manufacturing of ceramics, food processing. The order of chemical reaction depends on many factors, for instance foreign mass, active fluid and stretching of sheet and so on. Chemical reactions can be classified as homogeneous, heterogeneous, exothermic, or endothermic reactions. Gireesha et al [8] analyzed chemical reaction and thermal radiation effects on MHD free convective flow of time dependent an electrically conducting viscoelastic fluid in non-uniform vertical channel by means of convective boundary condition in the pressure of heat absorption and Hall current. In this consideration the Non-Linear coupled partial differential equations are solved by using multiple regular perturbation method. From this paper I observed for different values of Prandtl number and heat absorption parameter increases then it leads to temperature profile decreases. Venkateswarlu et al [9] Considered chemical reaction and heat generation effects on MHD free convective Visco-elastic fluid flow over a continuously moving vertical surface with uniform suction. In this consideration the Non-Linear coupled partial differential equations are solved by using multiple regular perturbation method. In this paper I observed that for different values of Visco-elastic parameter and magnetic field increases then it leads to decrease velocity field in the boundary layer. A similar effect for magnetic field is seen in the velocity profiles. Dada, M. S. et al [10] studied chemical reaction and radiation effects on unsteady 2-D MHD free convective Rivlin-Ericksen flow of fluids past a porous vertical plate. In this consideration the Non-Linear coupled partial differential equations are solved by using multiple regular perturbation method. From this investigation I observed that velocity increases with an increase in permeability of the porous medium, Grashof number for heat transfer and Grashof number for mass transfer, Prandtl number, Schmidt number, magnetic parameter, dimensionless Visco-elasticity parameter of, magnetic parameter, the Rivlin-Ericksen fluid and dimensionless heat absorption coefficient. Hridi Ranjan Deb [11] considered chemically reaction and heat source effect on 2D MHD free convective and mass transfer boundary layer flow of a Visco-elastic fluid flowing along a semi-infinite vertical porous plate with time dependent suction velocity in presence thermal effects. The transformed governing equations are solved by multiple regular perturbation method. In this paper I observed that the temperature and concentration fields were not significantly affected by the Visco-elastic parameters. The velocity field is significantly affected with the variation of Visco-elastic parameter. H. M. El-Hawary et al [12] examined heat and mass transfer of an MHD viscoelastic fluid moving over a stretching surface near a stagnation point in the existence of slip velocity, heat source/sink, and the concentration dependent thermal diffusivity. In this research article I observed that the governing non-dimensional coupled partial differentials are solved by using Lie group method. In this investigation I found that different values of viscoelastic parameter increases then it refers that velocity decreases near the surface. Excellent research work was done by Pooja Sharma et al [13] from this paper I observed that The chemical reaction effect on MHD free convective flow of Rivlin-Ericksen fluid past moving semi-infinite vertical permeable plate in the existence of pressure gradient and crosswise magnetic field. Ramesh Babu et al [14] analyzed the effects of Chemical reaction and heat source effects on unsteady MHD free convective flow of a viscoelastic fluid past a moving vertical porous plate with time dependent oscillatory permeability in existence of a uniform transverse magnetic field. In this consideration the Non-Linear coupled partial differential equations are solved by using multiple regular perturbation method.

The effect of radiation on MHD flow and heat transfer problem has turned out to be more critical modernly. At high working temperature, radiation impact can be very noteworthy. Many procedures in building territories happen at high temperature and learning of radiation heat transfer turns out to be vital for the plan of the related gear. Atomic power plants, gas turbines and the different drive gadgets for air ship, rockets, satellites and space vehicles are cases of such building regions. Vijayakumar et al [15] analyzed thermal radiation and diffusion effects on MHD flow elastic-viscous fluid of second order fluid past a vertical porous plate bounded by a porous medium. Here in this investigation the Non-Linear coupled partial differential equations are solved by finite difference method. In this paper I observed that for various values of Prandtl number and magnetic parameter increases then it leads to Velocity of the fluid decreases. For different values of Eckert number increases then it leads to Temperature decrease but a reverse effect is noticed in the case of absorption parameter and Prandtl number. Different values of Sc increases then it leads to concentration decreases but a

reverse effect is noticed in the case of Sherwood number. Nagalakshmi et al [16] analyzed thermal radiation effects on unsteady MHD free convective Visco-Elastic fluid flow over a stretching sheet in the presence of suction/injection. In this consideration the Non-Linear coupled partial differential equations are solved by using R-K fourth order method with shooting technique. Hridi Ranjan Deb [17] analyzed radiation effect on the unsteady MHD convective flow of a Visco-elastic fluid over an inclined plate implanted in a porous medium. Ramaiah et al [18] radiation absorption effects on MHD convective flow of a viscoelastic fluid past an oscillating porous plate in presence of chemical reaction and heat absorption was analyzed. The primary velocity decreases with an increase in radiation absorption parameter, where as secondary velocity increases with an increase in radiation absorption parameter. Ravikumar et al [18] analyzed heat absorption and thermal buoyancy effects on MHD on free convective Rivlin-Ericksen flow past a semi-infinite vertical porous plate. Here in this investigation the Non-Linear coupled partial differential equations are solved by using multiple regular perturbation method. From this paper I observed that time-dependent wall suction and uniform transverse magnetic field was considered. Here different values on heat absorption and visco-elasticity parameter of the Rivlin– Ericksen fluid increases then it leads to velocity decreases. Chandra et al [19] investigated thermal diffusion effects on fully developed MHD free convective flow of a visco-elastic fluid past a vertical porous plate enclosed by a porous medium in the influence of time dependent variable suction, heat source, variable suction and variable permeability. S B Kulkarni [20] study a class of exact solutions for the flow of incompressible electrically conducting elastic-viscous fluid of second order fluid by taking into account the magnetic field, angle of inclination and porosity factor of the bounding surfaces. B. V Padmavathi et al [21] Considered the hall effect on unsteady MHD flow of a liquid of the Walter B' model with simultaneous heat and mass transfer near an oscillating porous plate in slip flow regime. Here in this investigation the Non-Linear coupled partial differential equations are solved by using multiple regular perturbation method. In the above research Radiation absorption and viscous dissipations were not consider. Santhosha et al [22] employs perturbation method for solving governing equation. In this investigation radiation absorption and chemical reaction effects on MHD free convective flow of viscoelastic fluid through porous medium finite porous plate were consider. Noran Nur Wahida Khalili, et al [23] considered radiation effects on MHD flow past an exponentially extending sheet with chemical reaction and heat sink was examined. B. M. Jewel Rana et al [24] studied effect of radiation absorption and chemical reaction on high-speed Magneto Hydrodynamics free convective flow past an exponential accelerated inclined plate. Here viscous dissipation effect and viscoelastic fluid was not considered.

Motivated by the above studies, the main objective of this paper is to study Chemical reaction and Viscous Dissipation effects on MHD free convective Rivlin-Ericksen fluid flow past a semi-infinite stationary vertical porous plate with radiation absorption. The flow was unsteady and restricted to the laminar domain. The dimensionless governing equations for this investigation are solved analytically by using multiple regular perturbation law.

#### 2. Mathematical formulation:

Walter's B Liquid Model: The constitutive equations for the rheological equation of the state for the Viscoelastic fluid (Walters liquid B) are (B. J. Gireesha et al )

$$p_{ik} = -\rho g_{ik} + p_{ik}^{*}$$
(1)  
$$p_{ik}^{*} = 2 \int_{-\infty}^{t} \xi \left(t - t^{*}\right) e_{ik}^{(1)} \left(t^{*}\right) dt^{*}$$
(2)

Where,

$$\xi(t-t^*) = \int_0^\infty \frac{N(\tau)}{\tau} \exp\left(-\left(\frac{t-t^*}{\tau}\right)\right) d\tau$$
(3)

Here the stress tensor is  $p_{ik}$ , p is an arbitrary isotropic pressure,  $g_{ik}$  is the metric tensor of a fixed coordinate system  $x^{i}$ .  $e_{ik}^{(1)}$  is the rate of strain tensor.  $N(\tau)$  is the distribution function of relaxation time( $\tau$ ). Equation (2) can be put in the following generalized form which is valid for all types of motion and stress; it was shown by Walters [28]:

$$p_{ik}^{*} = 2 \int_{-\infty}^{t} \xi \left( t - t^{*} \right) \frac{\partial x^{i}}{\partial x_{m}^{*}} \cdot \frac{\partial x^{k}}{\partial x_{r}^{*}} e_{mr}^{(1)} \left( x^{*} t^{*} \right) dt^{*}$$

$$\tag{4}$$

Where  $x^{*i}$  is the position at the time  $t^*$  of the element that is instantaneously at the print  $x^i$  at time't'. The fluid with equation of state (1) and (4) has been designated as liquid  $B^*$ . In the case of liquids with short memories, that is short relaxation time, the above equation of state can be written in the flowing simplified form.

$$p_{ik}^{*}(x,t) = 2 \eta_{0} e_{ik}^{(1)} - 2 k_{0} \frac{\partial e_{ik}^{(1)}}{\partial t}$$
(5)

Where  $\eta_0 = \int_0^\infty N(\tau) d\tau$  is the limiting viscosity at small rates of shear.  $k_0 = \int_0^\infty \tau N(\tau) d\tau$ 

is the Walters-B Visco-elasticity parameter, and  $\frac{\partial}{\partial t}$  denotes the convicted time derivative. This rheological

model is very versatile and robust and provides a relatively simple mathematical formulation which is easily incorporated into boundary layer theory for engineering applications.

The momentum equation is based on the principle of conservation of momentum, i.e., that the time rate of change of momentum in a material region is equal to the sum of the forces on that region. The equations governing the flow are in vector form.

Equation of Momentum is defined as:

$$\begin{bmatrix} \frac{\partial \vec{q}}{\partial t^*} + (\vec{q} \cdot \nabla) \vec{q} \end{bmatrix} = -\frac{1}{\rho} [grad \ p] + \vartheta \left[ \nabla^2 \vec{q} \right] + \overline{F_B} + \left[ \frac{k_0}{\rho} \right] [grad \ p_{ik} ] \\ - \left[ \frac{\vartheta}{K^*} \right] \vec{q} - \frac{\sigma}{\rho} B_0^2 \vec{q}$$
(6)

The first term on the left hand side is the local acceleration and the second term is known as the convective acceleration term. The second term is the term which makes the Navie-Stokes equation nonlinear, which is the source of the great complexity of the mathematics and physics of fluid motion. On the right hand side, the first term is the pressure gradient force, the second term is known as the viscous force, the third term  $\overline{F_B}$  is the buoyancy force and it is defined as  $\overline{F_B} = g \beta (T^* - T^*_{\infty}) + g \beta^* (C^* - C^*_{\infty}) \overline{i}$ , forth tern grad  $p_{ik}$  is the stress

gradient tensor. Fifth term in the square brackets  $\left[\frac{9}{\kappa^*}\right]$  denotes the bulk matrix resistance, i.e., Darcy term.

The energy equation decouples from the rest of the Navier-Stokes equations for incompressible flow. This can be seen from a non-dimensionalisation of the energy equation by using the definition of the enthalpy.

Equation of Energy is defined as:

$$\frac{\partial T^*}{\partial t^*} + \left(\vec{q}.\nabla\right)T^* = \frac{K}{\rho C_p}\nabla^2 T^* - \frac{1}{k_\rho C_p} \operatorname{grad} q_r + \frac{9}{C_p}\left(\nabla \vec{q}\right)^2 + \frac{Q_0}{\rho C_p}(T^* - T^*_{\infty}) + R^*\left(C^* - C^*_{\infty}\right) - \frac{k_0}{\rho C_p}\left(\nabla \vec{q}\frac{\partial}{\partial t^*}\left(\nabla \vec{q}\right)\right)$$

$$(7)$$

The first term on the left hand side is the temporal thermal gradient and the second term describes convection. On the right hand side, first term K is the thermal diffusivity,  $\nabla^2 T$  is the thermal diffusion term., second term  $grad q_r$  is the gradient radiative heat flux, fourth term  $Q_0 \left(T^* - T_{\infty}^*\right)$  is the amount of heat generated per unit volume.  $Q_0$  is the constant where either  $Q_0 < 0$  or  $Q_0 > 0$ ,  $C_p$  is the specific heat of the fluid at constant pressure.

A Concentration equation is an equation that describes the transport of some quantity. It is particularly simple and particularly powerful when applied to a conserved quantity, but it can be generalised to apply to any extensive quantity.

Concentration species diffusion equation is defined as:

$$\frac{\partial C^*}{\partial t^*} + \left(\vec{q} \cdot \nabla\right) C^* = D \nabla^2 C - K_r (C^* - C^*_{\infty}) \tag{8}$$

Where the opening term on the left hand side signifies the temporal concentration gradient and the second term describes the convection term. On the right hand side, the first term represents species diffusion and the last term is the chemical reaction term.

Consider the two dimensional unsteady laminar flow of a viscous incompressible and heat generation/absorption electrically conducting fluid past a semi infinite vertical stationary porous plate in a uniform of a pressure grading has been considered with double-diffusive free convection, chemical reaction and thermal radiation effects. The following assumptions are listed below.

• Let the  $x^*$ -axis be taken along the porous plate in the upward direction and  $y^*$ -axis in the direction perpendicular to the flow and t\* is the dimensional time. u\* and v\* are the components of dimensional velocities along x\*and y\* directions. It is assume that the porous plate moves with a constant velocity in the direction of fluid flow.

• Assumed transverse magnetic field of the uniform strength  $B_0$  is to be applied normal to the plate.

• The radiate heat flux in the  $x^*$ - direction is considered negligible in comparison with that in  $y^*$  - direction.

• Viscous dissipation, Radiation absorption are taken into account the constant permeability porous medium. Since the plate is semi-infinite in length the entire flow variable are functions of y and t only.

• The fluid has constant thermal conductivity and kinematic viscosity, the Boussinesqu's approximation has been taken for the flow.

• The homogeneous chemical reaction is of first order with rate constant  $K_r$  between the diffusing species and the fluid is considered.

Under the above stated assumptions and usual Boussinesq approximation, the governing equations in their component form are reduced to:

$$\frac{\partial u^{*}}{\partial t^{*}} + \left(u^{*}\frac{\partial u^{*}}{\partial x^{*}} + v^{*}\frac{\partial u^{*}}{\partial y^{*}}\right) = \frac{-1}{\rho}\frac{\partial p^{*}}{\partial x^{*}} + \vartheta\left(\frac{\partial^{2}u^{*}}{\partial x^{*2}} + \frac{\partial^{2}u^{*}}{\partial y^{*2}}\right) + g\beta(T^{*} - T_{\infty}^{*}) + g\beta^{*}\left(C^{*} - C_{\infty}^{*}\right) - \frac{k_{0}}{\rho}\frac{\partial^{3}u^{*}}{\partial y^{*2}\partial t^{*}} - \frac{\vartheta}{K^{*}}u^{*} - \frac{\vartheta}{\rho}B_{0}^{2}u^{*}$$

$$(9)$$

$$\frac{\partial T^*}{\partial t^*} + \left(u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*}\right) = \frac{K}{\rho C_p} \left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right) + \frac{Q_0}{\rho C_p} (T^* - T^*_{\infty})$$
(10)

$$-\frac{1}{K\rho C_p}\frac{\partial q_r^*}{\partial y^*} - \frac{k_0}{\rho C_p}\frac{\partial u^*}{\partial y^*} \left(\frac{\partial^2 u^*}{\partial t^* \partial y^*}\right) + \frac{g}{C_p} \left(\frac{\partial u^*}{\partial y^*}\right)^2 + R^* \left(C^* - C^*_{\infty}\right)$$

$$\frac{\partial C^*}{\partial t^*} + \left(u^* \frac{\partial C^*}{\partial y^*} + v^* \frac{\partial C^*}{\partial y^*}\right) = D\left(\frac{\partial^2 C^*}{\partial x^{*2}} + \frac{\partial^2 C^*}{\partial y^{*2}}\right) - K_r(C^* - C_\infty^*)$$
(11)

The boundary conditions for velocity, Temperature and Concentration field are given by

$$at \ y^{*} = 0 \ u^{*} = 0 \ T^{*} = T_{W}^{*} + \varepsilon \left( T_{W}^{*} - T_{\infty}^{*} \right) e^{n^{*}t^{*}}, C^{*} = C_{W}^{*} + \varepsilon \left( C_{W}^{*} - C_{\infty}^{*} \right) e^{n^{*}t^{*}}$$

$$As \ y^{*} \to \infty \ u^{*} = U_{0}(1 + \varepsilon e^{n^{*}t^{*}}), \ T^{*} \to T_{\infty}^{*}, \ C^{*} \to C_{\infty}^{*}$$

$$(12)$$

From the equation of momentum equation gives

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} = \frac{dU_{\infty}^*}{dt^*} + \frac{9}{k}U_{\infty}^* + \frac{\sigma}{\rho}B_0^2 U_{\infty}^*$$
(13)

Since the motion is two dimensional and length of the plate is large enough so all the physical variables are independent of x - axis.

Therefore 
$$\frac{\partial u^*}{\partial x^*} = \frac{\partial T^*}{\partial x^*} = \frac{\partial C^*}{\partial x^*} = 0$$
 (14)

From the equation of the continuity, it is clear that the suction velocity is to be assumed as,

$$v^* = 0 \tag{15}$$

The radiative heat flux term by using the Rosseland approximation is given by

$$q_r^* = -\frac{4\sigma_s^*}{3k_e^*} \frac{\partial T^{*4}}{\partial y^*} \tag{16}$$

We assume that the temperature difference within the flow is sufficiently small such that  $T^{*^{*}}$  can be expressed as a linear function of the temperature; this is expanding in Taylor series about  $T^{*}_{\infty}$  and neglecting higher order terms to give:

$$T^* \cong 4T^* T_{\infty}^{*^3} - 3T_{\infty}^*$$
(17)

By using equations (13), (14), (16), and (17) then the governing equations (9), (10) and (11) can be write as follows:

$$\frac{\partial u^{*}}{\partial t^{*}} = \frac{dU_{\infty}^{*}}{dt^{*}} + g \frac{\partial^{2} u^{*}}{\partial y^{*2}} + g \beta (T^{*} - T_{\infty}^{*}) + g \beta^{*} \left(C^{*} - C_{\infty}^{*}\right) + \frac{g}{K^{*}} \left(U_{\infty}^{*} - u^{*}\right) - \frac{k_{0}}{\rho} \frac{\partial^{3} u^{*}}{\partial y^{*2} \partial t^{*}} + \frac{\sigma}{\rho} B_{0}^{2} \left(U_{\infty}^{*} - u^{*}\right) - \frac{k_{0}}{\rho} \frac{\partial^{2} T^{*}}{\partial y^{*2} \partial t^{*}} + \frac{\sigma}{\rho} B_{0}^{2} \left(U_{\infty}^{*} - u^{*}\right) - \frac{k_{0}}{\rho} \frac{\partial^{2} T^{*}}{\partial y^{*2} \partial t^{*}} + \frac{\sigma}{\rho} B_{0}^{2} \left(U_{\infty}^{*} - u^{*}\right) - \frac{k_{0}}{\rho} \frac{\partial^{2} T^{*}}{\partial y^{*2} \partial t^{*}} + \frac{\sigma}{\rho} B_{0}^{2} \left(U_{\infty}^{*} - u^{*}\right) - \frac{k_{0}}{\rho} \frac{\partial^{2} T^{*}}{\partial y^{*2} \partial t^{*}} + \frac{g}{\rho} \left(\frac{\partial^{2} T^{*}}{\partial y^{*}}\right) - \frac{k_{0}}{\rho} \frac{\partial u^{*}}{\partial y^{*}} \left(\frac{\partial^{2} u^{*}}{\partial t^{*} \partial y^{*}}\right) + R^{*} \left(C^{*} - C_{\infty}^{*}\right)$$

$$(18)$$

$$(18)$$

$$(18)$$

$$(18)$$

$$(18)$$

$$(19)$$

$$(19)$$

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - K_r (C^* - C_\infty^*)$$
(20)

The suitable initial and boundary conditions for the present problem are

$$at \quad y^{*} = 0 \quad u^{*} = 0 \quad T^{*} = T_{W}^{*} + \varepsilon \left( T_{W}^{*} - T_{\infty}^{*} \right) e^{n^{*}t^{*}}, C^{*} = C_{W}^{*} + \varepsilon \left( C_{W}^{*} - C_{\infty}^{*} \right) e^{n^{*}t^{*}} \\As \quad y^{*} \to \infty \quad u^{*} \to U_{\infty}^{*} = U_{0} \left( 1 + \varepsilon e^{n^{*}t^{*}} \right), T^{*} \to T_{\infty}^{*}, \quad C^{*} \to C_{\infty}^{*}$$

$$(21)$$

Now Non-dimensional quantities are defined as

$$u = \frac{u^{*}}{U_{0}}, U_{\infty} = \frac{U_{\infty}^{*}}{U_{0}}, y = \frac{V_{0}y^{*}}{g}, t = \frac{V_{0}^{2}t^{*}}{g}, k = \frac{k_{0}V_{0}^{2}}{g}$$

$$K^{*} = \frac{Kg^{2}}{V_{0}^{2}}, n^{*} = \frac{V_{0}^{2}n}{g}, T^{*} = T_{\infty}^{*} + \theta(T_{w}^{*} - T_{\infty}^{*}), C^{*} = C_{\infty}^{*} + C(C_{w}^{*} - C_{\infty}^{*})$$
(22)

After substituting the boundary conditions and non-dimensional variables in the governing equations (18), (19) and (20) and taking into account equation (22) then we get

$$\frac{\partial u}{\partial t} = \frac{dU_{\infty}}{dt} + \frac{\partial^2 u}{\partial y^2} - \gamma \frac{\partial^3 u}{\partial y^2 \partial t} + G_m C + N(U_{\infty} - u)$$
(23)

$$\frac{\partial\theta}{\partial t} = (\Pr)^{-1} \left( 1 + \frac{4R}{3} \right) \frac{\partial^2 \theta}{\partial y^2} + \eta \theta + Ec \left( \left( \frac{\partial u}{\partial y} \right)^2 - \gamma \left( \frac{\partial u}{\partial t} \right) \left( \frac{\partial^2 u}{\partial t \partial y} \right) \right) + R_a C$$
(24)

$$\frac{\partial C}{\partial t} = (Sc)^{-1} \frac{\partial^2 C}{\partial y^2} - K_r C$$
<sup>(25)</sup>

Corresponding boundary conditions are given by

$$\begin{array}{l} At \ y=0 \quad u=0, \ \theta=1+\varepsilon e^{nt}, \ C=1+\varepsilon e^{nt} \\ As \ y\to\infty \quad u=U_{\infty}, \ \theta\to0, \ C\to0 \end{array} \right\}$$

$$(26)$$
Where

Where

$$M = \frac{\sigma B_0^2 \mathcal{P}}{\rho V_0^2}, G_m = \frac{\mathcal{P} \mathcal{P} \left( C_w^* - C_\infty^* \right)}{V_0^2 U_0}, G_r = \frac{\mathcal{P} \mathcal{P} \left( T_w^* - T_\infty^* \right)}{V_0^2 U_0}, R = \frac{4\sigma T_\infty^{*^3}}{k_e k},$$

$$Sc = \frac{\mathcal{P}}{D}, Ec = \frac{U_0^2}{C_p \left( T_w^* - T_\infty^* \right)} R_a = \frac{R^* \mathcal{P} \left( C_w^* - C_\infty^* \right)}{V_0^2 \left( T_w^* - T_\infty^* \right)}, \Pr = \frac{\rho \mathcal{P} C_p}{k},$$

$$S = M + \frac{1}{K}, K_r = \frac{k_1 \mathcal{P}}{V_0^2}, \eta = \frac{\mathcal{P} Q_0}{\rho V_0^2 C_p}, \Gamma = (\Pr)^{-1} \left( 1 + \frac{4R}{3} \right), \gamma = \frac{k}{\rho \mathcal{P}}$$
(27)

The mathematical statement of the problem is now complete and embodies the solution of Eqs. (15), (16) and (17) subject to the conditions (18).

#### 3. Method of solution:

The problem posted in Esq. (15), (16) and (17) subject to the boundary conditions presented in Esq. (18) is non-linear partial differential equations and these cannot be solved in closed-form. So these equations can be reduced into the system of partial differential equations to set of ordinary differential equations, which can be solved by using multiple regular perturbation law. This is more economical and flexible from analytical point of view. Finding multiple regular perturbation method is an art rather than a science. In research it is useful to be responsive to suggestions from the physics. There is certainly no routine method appropriate to all problem, or even classes of problems. Instead one needs a determination to exploit the smallness of the parameter. Obtaining good numerical values for the solution is not the only quest of a perturbation method. One can hope that the analysis will reveal some physical insights through the simplified physics of the limiting problem. The behaviors of velocity, temperature, concentration, Skin-friction coefficient, Nusselt number and Sherwood numbers has been discussed in detail for variation of thermo physical parameters. The solution is assumed to be as

$$\begin{aligned} u &= u_0(y) + \varepsilon e^{nt} u_1(y) \\ \theta &= \theta_0(y) + \varepsilon e^{nt} \theta_1(y) \\ C &= C_0(y) + \varepsilon e^{nt} C_1(y) \end{aligned}$$

$$(28)$$

Substituting Esq. (20) in Equation (15), (16) and (17) and equating the harmonic and non-harmonic terms, and neglecting the higher order terms  $o(\varepsilon^2)$  we obtain the following pairs of equations for  $(u_0, \theta_0, C_0)$  and

$$\begin{aligned} &(u_{1},\theta_{1},C_{1}) \\ &u_{0}^{''}-S u_{0}=-S-G_{r}\theta_{0}-G_{m}C_{0} \\ &\psi u_{1}^{''}-(S+n)u_{1}=-(S+n)-G_{r}\theta_{1}-G_{m}C_{1} \\ &\Gamma \theta_{1}^{''}-(n-\eta)\theta_{1}=-2Ec \ u_{0}^{'} u_{1}^{'}-R_{a} \ C_{1} \end{aligned}$$

$$(30)$$

$$(31)$$

$$\Gamma \theta_0^{"} + \eta \theta_0 = -Ec \left( \frac{\partial u_0}{\partial y} \right) - R_a C_0$$
(32)

$$C_0 = S_C K_r C_0 = 0 \tag{33}$$

$$C_1 - SC(K_r + h)C_1 = 0 \tag{34}$$

Where 
$$\psi = 1 - n\gamma$$

The corresponding boundary conditions can be written as

$$u_{0} = 0, u_{1} = 0 \ \theta_{0} = 1, \theta_{1} = 1, C_{0} = 1 \ C_{1} = 1, at \ y = 0$$

$$u_{0} = 1, u_{1} = 1, \theta_{0} \to 0, \theta_{1} \to 0, C_{0} \to 0, C_{1} \to 0 \ as \ y \to \infty$$
(35)

Here primes denote differentiation w.r.t. y the equations (21) – (24) are still coupled and non-linear, whose exact solutions are not possible. So we expand  $u_0, u_1 \theta_0 \theta_1$ 

First we solve equation (33) and equation (34) by using equation (35). Then we get

$$C_{0} = e^{-(\sqrt{Sc(Kr+n)})y}$$

$$C_{1} = e^{-(\sqrt{Sc(Kr+n)})y}$$

$$(36)$$

$$(37)$$

Now using multi parameter perturbation technique and assuming *Ec* <<1.

$$u_{0} = u_{00} + Ecu_{01} + 0(\varepsilon)^{2}$$
  

$$\theta_{0} = \theta_{00} + Ec\theta_{01} + 0(\varepsilon)^{2}$$
  

$$u_{1} = u_{10} + Ecu_{11} + 0(\varepsilon)^{2}$$
  

$$\theta_{1} = \theta_{10} + Ec\theta_{11} + 0(\varepsilon)^{2}$$
(38)

By using equations (30) in equations (21), (22), (23), (24) and equating the coefficients of like powers of Ec neglecting those of  $(Ec)^2$  and  $0(\epsilon)^2$ , we get the following set of differential equation

$$\begin{split} u_{00}^{*} - Su_{00} &= -S - G_{*} \partial_{00} - G_{m} C_{0} & (39) \\ u_{01}^{*} - Su_{01} &= -G_{*} \partial_{01} & (40) \\ \psi u_{10}^{*} - (S + n)u_{10} &= -(S + n) - G_{*} \partial_{10} - G_{m} C_{1} & (41) \\ \psi u_{11}^{*} - (S + n)u_{11} &= -G_{*} \partial_{11} & (42) \\ \Gamma \partial_{10}^{*} - (n - \eta)\partial_{10} &= -R_{n} C_{1} & (43) \\ \Gamma \partial_{00}^{*} + \eta \partial_{00} &= -R_{n} C_{0} & (43) \\ \Gamma \partial_{00}^{*} + \eta \partial_{00} &= -R_{n} C_{0} & (44) \\ \Gamma \partial_{00}^{*} + \eta \partial_{00} &= -R_{n} C_{0} & (46) \\ \Gamma he corresponding boundary conditions are \\ at y = 0; \quad u_{00} = 0, u_{01} = 0, u_{11} = 0, \partial_{00} = 1, \quad \partial_{01} = 0, \partial_{10} = 1, \quad \partial_{11} = 0 \\ As y \to \infty \ u_{00} = 1, \ u_{01} = 0, \ u_{10} = 1, u_{11} = 0, \partial_{00} = 0, \partial_{01} = 0, \partial_{01} = 0, \partial_{11} = 0 \\ Solve (39) - (46) subject to boundary Condition (47), we get \\ u_{00} = 1 + N_{3} \ e^{-dy} + N_{4} \ e^{-fy} + A_{4} \ e^{-dy} \\ u_{11} = N_{12} e^{-dy} + N_{12} e^{-2fy} + N_{13} e^{-(d+f)y} + N_{15} e^{-(f+Q)y} + N_{16} e^{-(d+Q)y} \\ + N_{17} e^{-dy} + A_{3} e^{-Qy} \\ u_{11} = N_{29} e^{-(d+m)y} + N_{30} e^{-(d+1)y} + N_{31} e^{-(d+q)y} + N_{32} e^{-(f+f)y} + N_{33} e^{-(f+f)y} \\ \end{split}$$

$$+N_{34}e^{-(l+q)y} + N_{35}e^{-(Q+m)y} + N_{36}e^{-(l+Q)y} + N_{37}e^{-(q+Q)y} + N_{38}e^{-my} + A_6e^{-qy}$$

$$\theta_{00} = (1 - N_1) \ e^{-dy} + N_1 \ e^{-fy}$$
(52)

$$\theta_{10} = (1 - N_2) \ e^{-my} + N_2 \ e^{-ly}$$
(53)

$$\theta_{01} = N_5 e^{-dy} + N_6 e^{-2fy} + N_7 e^{-2Qy} + N_8 e^{-(d+f)y} + N_9 e^{-(l+Q)y} + N_{10} e^{-(d+Q)y} + A_2 e^{-dy}$$
(54)

$$\theta_{11} = N_{20}e^{-(d+m)y} + N_{21}e^{-(d+l)y} + N_{22}e^{-(d+q)y} + N_{23}e^{-(d+f)y} + N_{24}e^{-(f+l)y} + N_{24}e^{-(f+l)y} + N_{25}e^{-(f+q)y} + N_{26}e^{-(Q+m)y} + N_{27}e^{-(l+Q)y} + N_{28}e^{-(q+Q)y} + A_5e^{-my}$$
(55)

# **3.1.** Velocity (u), Temperature (θ) and Concentration (C):

$$u = \begin{pmatrix} 1 + N_{3}e^{-dy} + N_{4}e^{-dy} + EcN_{11}e^{-dy} + EcN_{12}e^{-2dy} + EcN_{13}e^{-2dy} \\ + EcN_{14}e^{-(d+f)y} + EcN_{15}e^{-(f+Q)y} + EcN_{16}e^{-(d+Q)y} + EcN_{17}e^{-dy} + EcA_{5}e^{-2dy} \\ + EcN_{14}e^{-(d+f)y} + EcN_{29}e^{-(d+m)y} + EcN_{29}e^{-(d+m)y} + EcN_{29}e^{-(d+m)y} \\ + N_{31}e^{-(d+q)y} + EcN_{32}e^{-(d+f)y} + EcN_{33}e^{-(f+l)y} + EcN_{34}e^{-(f+q)y} \\ + EcN_{5}e^{-(Q+m)y} + EcN_{5}e^{-(d+f)y} + EcN_{5}e^{-(d+g)y} + EcN_{7}e^{-2dy} + EcN_{6}e^{-(\sqrt{S+n})y} \end{pmatrix} \\ \theta = \begin{pmatrix} (1 - N_{1})e^{-dy} + N_{1}e^{-fy} + EcN_{5}e^{-dy} + EcN_{6}e^{-2fy} + EcN_{7}e^{-2dy} + EcN_{8}e^{-(d+f)y} \\ + EcN_{9}e^{-(f+Q)y} + EcN_{10}e^{-(d+g)y} + EcN_{2}e^{-(d+g)y} \\ + EcN_{9}e^{-(f+Q)y} + EcN_{10}e^{-(d+g)y} + EcA_{2}e^{-dy} \end{pmatrix} \\ + \varepsilon e^{nt} \begin{pmatrix} (1 - N_{2})e^{-my} + N_{2}e^{-ly} + EcN_{2}e^{-(f+q)y} + EcN_{2}e^{-(d+l)y} + EcN_{2}e^{-(d+g)y} \\ + EcN_{23}e^{-(d+f)y} + EcN_{24}e^{-(f+l)y} + EcN_{25}e^{-(f+q)y} + EcN_{26}e^{-(d+g)y} \\ + EcN_{27}e^{-(l+Q)y} + EcN_{28}e^{-(f+Q)y} + EcN_{28}e^{-(d+g)y} \\ + EcN_{27}e^{-(l+Q)y} + EcN_{28}e^{-(f+Q)y} + EcN_{28}e^{-(d+g)y} \end{pmatrix}$$
(57)
$$C = e^{-fy} + \varepsilon e^{m}e^{-ty} \qquad (58)$$

# 3.2. Shear stress, Nusselt number and Sherwood number:

The Skin-friction at the plate, which in the non-dimensional form is given by

$$C_{f} = \left[\frac{\tau_{w}}{\rho U_{0}V_{0}}\right] = \left[\frac{\partial u^{*}}{\partial y^{*}}\right]_{y=0}$$
(59)

The rate of heat transfer coefficient, which in the non-dimensional form in terms of the Nusselt number  $\lceil \partial T^* \rceil$ 

is given by 
$$N_u = -x \frac{\left[\frac{\partial y^*}{\partial y^*}\right]_{y=0}}{\left[T_w^* - T_\infty^*\right]} \implies N_u \operatorname{Re}_x^{-1} = -\left[\frac{\partial \theta}{\partial y}\right]_{y=0}$$
 (60)

The rate of mass transfer coefficient, which in the non-dimensional form in terms of the Sherwood number, is given by

$$S_{h} = -x \frac{\left[\frac{\partial C^{*}}{\partial y^{*}}\right]_{y=0}}{\left[C_{w}^{*} - C_{\infty}^{*}\right]} \implies S_{h} \operatorname{Re}_{x}^{-1} = -\left[\frac{\partial c}{\partial y}\right]_{y=0}$$
(61)

Given the velocity field in the boundary later, we can now calculate the skin friction at the wall as

$$\tau_{W} = \frac{\partial u}{\partial y}\Big|_{y=0} = \begin{bmatrix} -mN_{3} - fN_{4} - QA_{1} - dEcN_{11} - fEcN_{12} - 2QEcN_{13} - (d+f)EcN_{14} \\ -(f+Q)EcN_{15} - (d+Q)EcN_{16} - dEcN_{17} - QEcA_{3} \end{bmatrix} + \varepsilon e^{nt} \begin{bmatrix} -mN_{18}lN_{4} - qA_{4} - (d+m)EcN_{29} - (d+l)EcN_{30} - (d+q)EcN_{31} \\ -(d+f)EcN_{32} - (f+l)EcN_{33} - (f+q)EcN_{34} - (Q+m)EcN_{35} \\ -(l+Q)EcN_{36} - (q+Q)EcN_{37} - mEcN_{38} + EcA_{6}e^{-qy} \end{bmatrix}$$
(62)

We calculate the heat transfer coefficient in terms of Nusselt number as follows.

$$N_{u} = \frac{\partial \theta}{\partial y}\Big|_{y=0} = \begin{bmatrix} -d1 - N_{1}) - fN_{1} - dEcN_{5} - 2fEcN_{6} - 2QEcN_{7} - (d+f)EcN_{8} \\ -(f+Q)EcN_{9} - (d+Q)EcN_{10} - dEcA_{2} \end{bmatrix} + \varepsilon e^{nt} \begin{bmatrix} -m(1 - N_{2}) - lN_{2} - (d+m)EcN_{20} - (d+l)EcN_{21} - (d+q)EcN_{22} \\ -(d+f)EcN_{23} - (d+l)EcN_{24} - (f+q)EcN_{25} - (Q+m)EcN_{26} \\ -(l+Q)EcN_{27} - (q+Q)EcN_{28} - mEcA_{5} \end{bmatrix}$$
(63)

Similarly the mass transfer coefficient in terms of Sherwood number, as follows

$$S_{h} = \frac{\partial c}{\partial y}\Big|_{y=0} = -f - \varepsilon e^{nt}l$$
(64)

#### 4. Results and Discussion:

The formulation of the chemical reaction, Radiation absorption, thermal diffusion and thermal radiation, heat source and viscous dissipation effects on MHD convective Rivlin-Ericksen flow and mass transfer of an incompressible, viscous fluid along a semi infinite vertical porous stationary plate in a porous medium has been performed in the preceding section. This enables us to carry out the numerical calculation for the distribution of the velocity, temperature and concentration across the boundary layer for various values of the parameters. In

the present study we have to select t=1.0, n=0.5,  $\epsilon=0.03$ , A=0.5, Up=0 while Kr, Pr, Gr, Gm,  $\eta$ , k, M, Ra,  $\psi$ and R are varied over a range, which listed in the figure legends. The variation in velocity profile with y for different values in Modified Grashof number is shown in Figure 1: The modified Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases as Grashof number Gm increases. The variation in velocity profile with y for various values in Grashof numbers are shown in Figure 2: This figure reflects that with increase in Gr there is increase in fluid velocity due to improvement of the buoyancy force. The variety of velocity and concentration profiles with y for various esteems in chemical reaction parameter Kr is appeared in the Figure 3: and Figure 17: these figures mirror that with increment in Kr there is diminishing in fluid velocity and concentration. For different value of permeability of the porous medium K, the velocity profile is plotted in Figure 4: Here we find that as the values of permeability of the porous medium K increases then it leads to the velocity increase. For different values of the Prandtl number Pr, the Velocity and Temperature profile is plotted in Figure 5: and Figure 12: Here we find that as the values of Prandtl number Pr increase then the both Velocity and Temperature profiles decreases. To be realistic, the numerical values of Prandtl number Pr are chosen as Pr=0.71, and Pr=7.00, which correspond to air and water at 20 degree Celsius's respectively. The influence of Magnetic field parameter M on velocity profiles is as shown in the Figure 6: by keeping supplementary parameters in rest. The presence of a magnetic field in an electrically conducting fluid introduces a force called Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as considered in the present problem. This type of resistive force tends to slow down the flow field. Also, it is observed that the velocity of fluid decreases with increasing of magnetic parameter. The effect of R on the velocity and temperature profiles is shown in Figure 7: and Figure 13: It shows velocity profiles increases with an increase in R but a reverse effect is found in the case of Temperature. For different value of Sc on the velocity is shown in the Figure 8: and Figure 16: Here we find that as the values of Schmidt number Sc increase then it leads to velocity and concentration decreases. The values of the Schmidt number are chosen to represent the presence of various species Oxygen (0.60), Carbon dioxide (Sc=0.94), the values of other parameters are chosen arbitrarily. The numerical estimations of the rest of the parameters are picked self-assertively. The variation in velocity profile with y for different values in Rivlin-Ericksen fluid parameter  $\psi$  shown in the Figure 9: Here we found velocity increases with the increase of Rivlin-Ericksen parameter. Figure 10: and Figure 15: depicts the effect of Eckert number Ec on the velocity temperature profiles. It is revealed that velocity scores increases and diminishing in temperature with the increasing of the Eckert number Ec. Eckert number physically is a measure of frictional heat in the system. In the Figure 11: the temperature decreases with increase of heat generator parameter  $\eta$ . For different values of Ra on the temperature show in the Figure 14: here the values of Ra increase then it leads to temperature decreases. Numerical values of the Skin-friction coefficient  $\tau_w$  Nusselt number  $N_u$  Sheer wood number  $S_h$  are shown below  $t=1, n=0.5, \epsilon=0.03$  From the Table: 1 depict the effect of the Prandtl number Pr Schmidt number Sc, heat generation parameter ( $\xi$ ), Eckert number (*Ec*), radiation absorption(Ra) ,heat source parameter, viscoelastic parameter( $\psi$ ), chemical reaction (Kr) and reaction rate R on the Skin friction coefficient  $\tau_w$  and Nusselt number respectively. From the talble-1 it is observed from this table that as Pr, Sc, K, Kr Ra and n increases then it leads to skin friction decreases but it was shown that reveres effect was occurred in case of Gr, Gm,  $\psi$ , Ra and Ec. From the talble-2 it is observed from this table that as pr, Ra and  $\eta$  increases then it gives a response Nusselt number decreases. But Nusselt number increases with an increase in Sc, M, R, Ra and Ec. From the Table: 3 shows that the effect of the Schmidt number Sc and chemical reaction Kr for different values on Sherwood  $S_h$ . It is observed from the table that as Sherwood  $S_h$  decreases with increase of Sc and Kr.

## 5. Validation of the results:

Comparison of the accuracy of the present results with the previous results														
	Mohammed Ibrahim [] when $\gamma=0$ , Ra=0 and R=0													
Previous	Our present results													
Increasing	II	т	С	τ	N	c	II	т	C	Ŧ	N	ç		
parameter	U	1	C	$\iota_w$	1 <b>v</b> <sub>w</sub>	$\mathbf{S}_h$	U	1	C	$\iota_w$	1 <b>v</b> <sub>w</sub>	$\mathcal{S}_h$		
Ec	Inc	Inc		Inc	Dec		Inc	Dec		Inc	Dec			
М	Dec	Inc		Inc	Inc		Dec			Inc				
Pr	Dec	Dec		Dec				Dec		Dec	Dec			
Sc	Dec		Dec	Dec		Inc			Dec	Dec		Dec		
K	Inc			Inc			inc			Dec				
Kr	Dec		Dec	Dec		Inc	Dec	Dec	Dec	Dec		Dec		
Gm	Inc			Inc			Inc			Inc				
Gr	Inc			Inc			Inc			Inc				
n 🧹	Dec		Dec	Dec	Inc		Inc	Inc		Inc	Inc			



Figure 1: Variation of Velocity against y for different values of Gm







Figure 5: Variation of Velocity against y for different values of Pr



Figure 6: Variation of Velocity against y for different values of M



Figure 8: Variation of Velocity against y for different values of Sc

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Figure 11: Variation of Temperature against y for different values of  $\eta$ 



Figure 12: Variation of Temperature against y for different values of  $\eta$ 



Figure 14: Variation of Temperature against y for different values of Ra



Figure 17: Variation of Concentration against y for different values of  $\eta$ 

Table 2: Effect of various parameters on Nussle									
	-			num	nber	-			
PR	Sc	М	R	Ra	Ec	η	$N_{_W}$		
0.71	0.60	0.2	1	1	0.03	0.03	-5.7100		
5	0.60	0.2	1	1	0.03	0.03	-32.5679		
7	0.60	0.2	1	1	0.03	0.03	-48.7963		
0.71	0.42	0.2	1	1	0.03	0.03	-6.1663		
0.71	0.81	0.2	1	1	0.03	0.03	-5.4365		
0.71	0.94	0.2	1	1	0.03	0.03	-5.3300		
0.71	0.60	0.4	1	1	0.03	0.03	-5.7099		
0.71	0.60	0.6	1	1	0.03	0.03	-5.7098		
0.71	0.60	0.8	1	1	0.03	0.03	-5.7097		
0.71	0.60	0.2	2	1	0.03	0.03	-4.2493		
0.71	0.60	0.2	З	1	0.03	0.03	-3.5203		
<b>0</b> .71	0.60	0.2	4	1	0.03	0.03	-3.0685		
0.7 <mark>1</mark>	0.60	0.2	1	0.2	0.03	0.03	-4.9645		
0.7 <mark>1</mark>	0.60	0.2	1	0.4	0.03	0.03	-5.1509		
0.71	0.60	0.2	1	0.6	0.03	0.03	-5.3372		
0.7 <mark>1</mark>	0.60	0.2	1	1	0.01	0.03	-5.7132		
0.7 <mark>1</mark>	0.60	0.2	1	1	0.05	0.03	-5.7068		
0.71	0.6 <mark>0</mark>	0.2	1	1	0.07	0.03	-5.7037		
0.71	0.60	0.2	1	1	0.03	0.01	-3.1775		
0.71	0.60	0.2	1	1	0.03	0.05	-7.4482		
0.71	0.60	0.2	1	1	0.03	0.07	-8.8570		

Table 3: Eff	Table 3: Effect of Sc and Kr on Sherwood												
	numbers												
Sc	Kr	Sh											
0.42	2	-0.9319											
0.60	2	-1.1138											
0.94	2	-1.3941											
0.60	2	-1.1138											
0.60	4	-1.5738											
0.60	6	-1.9270											

Table	Table-1 Effect of various parameters on Skin friction											
PR	Gr	Gm	Sc	Kr	К	Ψ	R	Ra	Ec	η	$ au_{_{W}}$	
0.71	2	2	0.60	2	0.2	5	1	1	0.03	0.03	4.2035	
5	2	2	0.60	2	0.2	5	1	1	0.03	0.03	3.8764	
7	2	2	0.60	2	0.2	5	1	1	0.03	0.03	2.8739	
0.71	4	2	0.60	2	0.2	5	1	1	0.03	0.03	4.4283	
0.71	6	2	0.60	2	0.2	5	1	1	0.03	0.03	4.6531	
0.71	8	2	0.60	2	0.2	5	1	1	0.03	0.03	4.8779	
0.71	2	4	0.60	2	0.2	5	1	1	0.03	0.03	4.7766	
0.71	2	6	0.60	2	0.2	5	1	1	0.03	0.03	5.3497	
0.71	2	8	0.60	2	0.2	5	1	1	0.03	0.03	5.9227	

0.71	2	2	0.42	2	0.2	5	1	1	0.03	0.03	4.2686
0.71	2	2	0.81	2	0.2	5	1	1	0.03	0.03	4.2083
0.71	2	2	0.94	2	0.2	5	1	1	0.03	0.03	4.2043
0.71	2	2	0.60	4	0.2	5	1	1	0.03	0.03	4.1922
0.71	2	2	0.60	6	0.2	5	1	1	0.03	0.03	4.1636
0.71	2	2	0.60	8	0.2	5	1	1	0.03	0.03	4.1387
0.71	2	2	0.60	2	0.4	5	1	1	0.03	0.03	3.6779
0.71	2	2	0.60	2	0.6	5	1	1	0.03	0.03	3.4709
0.71	2	2	0.60	2	0.8	5	1	1	0.03	0.03	3.3599
0.71	2	2	0.60	2	0.2	10	1	1	0.03	0.03	4.2399
0.71	2	2	0.60	2	0.2	15	1	1	0.03	0.03	4.2751
0.71	2	2	0.60	2	0.2	20	1	1	0.03	0.03	4.3088
0.71	2	2	0.60	2	0.2	5	2	1	0.03	0.03	4.2659
0.71	2	2	0.60	2	0.2	5	3	1	0.03	0.03	4.3036
0.71	2	2	0.60	2	0.2	5	4	1	0.03	0.03	4.3307
0.71	2	2	0.60	2	0.2	5	1	0.2	0.03	0.03	4.2601
0.71	2	2	0.60	2	0.2	5	1	0.4	0.03	0.03	4.2460
0.71	2	2	0.60	2	0.2	5	1	0.6	0.03	0.03	4.2318
0.71	2	2	0.60	2	0.2	5	1	1	0.01	0.03	4.3775
0.71	2	2	0.60	2	0.2	5	1	1	0.05	0.03	5.3713
0.71	2	2	0.60	2	0.2	5	1	1	0.07	0.03	5.8683
0.71	2	2	0.60	2	0.2	5	1	1	0.03	0.01	4.3140
0.71	2	2	0.60	2	0.2	5	1	1	0.03	0.05	4.1672
0.71	2	2	0.60	2	0.2	5	1	1	0.03	0.0 <mark>7</mark>	4.1503

## Conclusion:

1) Velocity and temperature profiles decrease with the increase of Thermal radiation parameter

2) Concentration profiles decrease near the plate with the increase of chemical reaction parameter Kr and Schmidt number Sc.

3) An increase in K, Gm,  $\in$  and R leads to increase in both velocity and temperature.

4) Velocity profile decrease with the increase of Eckert number (Ec) and Magnetic field parameter M.

5) Sherwood S<sub>h</sub> profile decrease with the increase of chemical reaction parameter Kr and Schmidt number

Sc. Skin friction profile decreases with the increase of Eckert number (Ec). However, Nusselt number remains unchanged.

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Appendices

$$\begin{split} d &= i \sqrt{\frac{\xi}{\Gamma}}, f = \sqrt{ScKr}, Q = \sqrt{s}, l = \sqrt{Sc(Kr+n)}, m = \sqrt{\frac{n-\xi}{\Gamma}}, q = \sqrt{s+n} N_1 = \frac{-R_a}{\Gamma f^2 + \eta}, N_2 = \frac{-R_a}{\Gamma l^2 - (n-\eta)}, \\ N_3 &= \frac{-Gr(1-N_1)}{d^2 - S}, N_4 = \frac{-(GrN_1 + Gm)}{d^2 - S}, N_5 = \frac{-d^2N_3^2}{4\Gamma d^2 + \xi}, N_6 = \frac{-f^2N_4^2}{4\Gamma f^2 + \xi}, N_7 = \frac{-sA_1^2}{4\Gamma S + \xi}, N_8 = \frac{-2fdN_3N_4}{\Gamma(d+f)^2 + \xi}, \\ N_9 &= \frac{-2fdA_1N_4Q}{\Gamma(Q+f)^2 + \xi}, N_{10} = \frac{-2dA_1N_3Q}{\Gamma(Q+d)^2 + \xi}, N_{11} = \frac{-GrN_5}{d^2 - S}, N_{12} = \frac{-GrN_6}{4f^2 - S}, N_{13} = \frac{-GrN_7}{3S}, N_{14} = \frac{-GrN_8}{(d+f)^2 - S}, \\ N_{15} &= \frac{-GrN_9}{(Q+f)^2 - S}, N_{16} = \frac{-GrN_{10}}{(Q+d)^2 - S}, N_{17} = \frac{-GrA_2}{(d^2 - S)}, N_{18} = \frac{-Gr(1-N_2)}{\psi m^2 - (S+n)}, N_{19} = \frac{-GrN_2 - Gm}{l^2\psi - (S+n)}, \\ N_{20} &= \frac{-2dmN_3N_{18}}{\Gamma(d+m)^2 - (n-\xi)}, N_{21} = \frac{-2dlN_3N_{19}}{\Gamma(d+l)^2 - (n-\xi)}, N_{22} = \frac{-2dqN_3A_4}{\Gamma(d+q)^2 - (n-\xi)}, N_{23} = \frac{-2fmN_4N_{18}}{\Gamma(f+m)^2 - (n-\xi)}, \\ N_{24} &= \frac{-2flN_4N_{19}}{\Gamma(f+l)^2 - (n-\xi)}, N_{25} = \frac{-2dqN_4A_4}{\Gamma(f+q)^2 - (n-\xi)}, N_{26} = \frac{-2QmN_{18}A_1}{\Gamma(Q+m)^2 - (n-\xi)}, N_{27} = \frac{-2dlN_{19}A_1}{\Gamma(Q+l)^2 - (n-\xi)}, \\ N_{28} &= \frac{-2QqA_4A_1}{\Gamma(Q+q)^2 - (n-\xi)}, N_{39} = \frac{-GrN_{20}}{(d+m)^2\psi - (S+n)}, N_{30} = \frac{-GrN_{21}}{\psi (d+l)^2 - (S+n)}, N_{35} = \frac{-GrN_{26}}{\psi (d+q)^2 - (S+n)}, \\ N_{36} &= \frac{-GrN_{27}}{\psi (Q+l)^2 - (S+n)}, N_{37} = \frac{-GrN_{28}}{\psi (Q+q)^2 - (S+n)}, N_{38} = \frac{-GrN_{2}}{\psi m^2 - (S+n)}, A_1 = -(1+N_3+N_4) \end{split}$$

$$\begin{split} A_2 &= -(1 - N_5 - N_6 - N_7 - N_8 - N_9 - N_{10}), A_3 = -(N_{11} + N_{12} + N_{13} + N_{14} + N_{15} + N_{16} + N_{17}), A_4 = -(1 + N_{18} + N_{19}), A_5 &= -(N_{20} + N_{21} + N_{22} + N_{23} + N_{24} + N_{25} + N_{26} + N_{27} + N_{28}), A_6 &= -(N_{29} + N_{30} + N_{31} + N_{32} + N_{33} + N_{34} + N_{35} + N_{36} + N_{37} + N_{38}), \end{split}$$

