Optimal Two-Warehouse Inventory level for Deteriorating Items with Variation Time

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Abstract: The proposed model represents the optimal inventory level for different interval time with two-warehouse demand, optimal total cost, special consideration the offering rate is less than demand rate within first time interval and takes opposite that within second time interval with these assumptions the deterioration occurs for deteriorating items, the finite horizon planning, without shortage cost, inventory level is non-zero before the replenishment. Sensitivity analysis for the proposed model was represented the many values lies in range of the deterioration rate, the represented figures explained the performance of optimal inventory level and optimal total cost for required time, the difference between the optimal total cost and actual total cost was proposed.

I. INTRODUCTION

The inventory control system with deterioration rate was represented by researchers as Bassok, Anupindi and Akella (1999) proposed Single period multiproduct inventory models with situation. Bose, Goswami, and Chaudhri (1995) developed an EOQ model for deteriorating items with linear time dependent demand rate and shortages under inflation and time discounting. Das, Maity, and Maiti (2007) formulated the two warehouse supply chain model under possibility necessary credibility measures. Goh, Greenberg, and Matsuo (1993) studied two-stage perishable inventory models. Haneveld, Teunter (1992) investigated the effects of discounting and demand rate variability on the EOQ. Lee and Hsu (2009) proposed model as two warehouse production model for deteriorating inventory items with time dependent demands. Philip (1974) assumed Weibull distribution deterioration to developed generalized EOQ model for deteriorating items. Rong, Mahapatra and Maiti (2008). Sana, Chaudhuri (2008), Shinn, Hwang (1996) developed Joint price and lot size determination under conditions of permissible delay in payment and quantity discount for freight cost. In real life deteriorating items required model to determine the optimal size of the inventory level considers the offering and demand rates according to the seasonal consume the items have deterioration when offering rate is more than demand rate and opposite that to minimize the total cost of deteriorating items over the variant times when minimum inventory level is non-zero to satisfied the demand especially the almost stock used advanced software to manage the inventory in stock and alarm the manager of stock when inventory level equal 10% of total quantity for the new replenishment. The proposed model consider the inventory level when the quantity of deteriorating items not equal zero with several values of deterioration rate over the variant times of finite planning horizon for any required time the planned model assumed that two-warehouse inventory and demand rate.

II. MATERIAL AND METHODS

2.2. Assumptions and notions

2.2.1. Assumptions

In this paper the mathematical model is developed with the following assumptions

1) Planning horizon is finite.
2) Replenishment rate is infinite.
3) Single item inventory control.
4) Demand and deterioration rate are constant.
5) The offering rate for items is less than demand rate within [0,t₁].
6) The offering rate for items is more than demand rate within [t₁,T].
7) Deteriorating occurs as soon as the items are received into inventory within [0, T].
8) Shortage is not allowed.
9) The lead time is zero.
10) The inventory level at the end of planning horizon is variation will be non-zero.
11) The total relevant cost consists of fixed ordering, purchasing and holding cost.
2.3. Notation

- $D_1$: The demand rate quantity in period $[0, t_1]$.
- $D_2$: The demand rate quantity in period $[t_1, T]$.
- $b_1$: The first level of inventory in $[0, t_1]$.
- $b_2$: The second level of inventory in $[t_1, T]$.
- $C$: The present value of purchasing cost.
- $T_{CA}$: The total fixed ordering cost during $[0, T]$.
- $I_h$: The holding cost during $[0, T]$.
- $T_{CH1}$: The total holding cost during $[0, t_1]$.
- $T_{CH2}$: The total holding cost during $[t_1, T]$.
- $T_{CP1}$: The total purchasing cost during $[0, t_1]$.
- $T_{CP2}$: The total purchasing cost during $[t_1, T]$.
- $T_{C1}$: The total relevant cost during $[0, t_1]$.
- $T_{C2}$: The total relevant cost during $[t_1, T]$.

2.4. Parameters

- $T$: The length of the finite planning horizon.
- $I_1(t)$: The inventory level at time $[0, t_1]$.
- $I_2(t)$: The inventory level at time $[t_1, T]$.
- $t_1$: The time at which the inventory level reduce to $b_1$.
- $T$: The time at which the inventory level improved to $b_2$.
- $\Theta$: The constant deteriorating rate units/unit time during $[0, T]$.

III. MATHEMATICAL MODEL

Let $I_1(t)$ is the inventory level at any time $t$, $0 \leq t \leq t_1$, lessening inventory level to $b_1$ due to demand and deterioration rate in keeping with the assumption. The first order differential equation that describes the instantaneous state of $I_1(t)$ over the open interval $[0, t_1]$ is given by:

$$\frac{dI_1(t)}{dt} + \Theta I_1(t) = -D_1, 0 \leq t \leq t_1, \quad 0 \leq \Theta \leq 1, \quad I_0(t) = e^{-\Theta t} \quad (1)$$

Let $I_2(t)$ is the inventory level at any time $t$, $t_1 \leq t \leq T$, augment inventory level to $b_2$ due to demand and deterioration rate in line with the assumption. The first order differential equation that describes the instantaneous state of $I_2(t)$ over the open interval $[t_1, T]$ is specified by:

$$\frac{dI_2(t)}{dt} + \Theta I_2(t) = -D_2, t_1 \leq t \leq T, \quad 0 \leq \Theta \leq 1, \quad I_0(t_1) = e^{-\Theta t_1} \quad (2)$$

$$I_1(t) = I_0(t) \left[ t_1 \cdot D_1 e^{\Theta t_1} - b_1 t_1 \right] = \frac{D_1}{\Theta} \left( e^{\Theta t_1} - 1 \right) - b_1 t_1 e^{\Theta t_1}, \quad I_0(t) = e^{\Theta t_1} \quad (3)$$

$$I_1(0) = \frac{D_1}{\Theta} \left( e^{\Theta t_1} - 1 \right) - b_1 t_1 e^{\Theta t_1}$$

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Fig.1 Graphical demonstration of two warehouse inventory control diagram
\[ b_1 \approx \frac{D_1}{\Theta}(e^{\theta(t_1-t)} - 1), t_1 \approx 0 \]
\[ \frac{dI_2(t)}{dt} + \Theta I_2(t) = -D_1 t_1 \leq t \leq T, 0 \leq \Theta \leq 1. I_0(t_1) = e^{-\theta t_1} \]
\[ I_2(t) = I_0(t_1)[(e^{\int_{t_1}^{T} D_2 e^{\theta t} dt} - b_1 (T - t_1)] = \frac{D_2}{\Theta}(e^{\theta(T-t_1)} - 1) - b_1 (T - t_1)e^{\theta t_1} \]
\[ b_2 \approx \frac{D_2}{\Theta}(e^{\theta(T-t_1)} - 1), T \approx t_1 \]

3.1. Fixed ordering cost

The fixed ordering cost in the length of finite horizon \([0, T]\)

\[ TC_A = A \] (5)

3.2. Purchasing cost

According to fig. 1 of inventory level the purchasing cost of

\[ TC_{P1} = \frac{CD_1(e^{\theta t_1}-1)}{\Theta} - Cb_1 t_1 e^{\theta t_1} \] (6)

\[ TC_{P2} = \frac{CD_2(e^{\theta(T-t_1)}-1)}{\Theta} - Cb_1 (T - t_1) e^{\theta t_1} \] (7)

3.3. Holding cost excluding interest cost

We locate the average inventory quantity to obtain holding cost

\[ TC_{h1} = I_h \int_0^{t_1} I_1(t) dt \]
\[ = I_h \int_0^{t_1} \left[ \frac{D_1}{\Theta}(e^{\theta(t_1-t)} - 1) - b_1 t_1 e^{\theta t_1} \right] dt \]
\[ = \left[ \frac{I_h D_1}{\Theta^2} (e^{\theta t_1} - \theta t_1 - 1) - I_h b_1 (t_1 e^{\theta t_1} - \frac{e^{\theta t_1}}{\theta^2} + \frac{1}{\theta^2}) \right] \] (8)

\[ TC_{h2} = I_h \int_{t_1}^{T} I_2(t) dt \]
\[ = I_h \int_{t_1}^{T} \left[ \frac{D_2}{\Theta^2} (e^{\theta(T-t_1)} - 1) - b_1 (T - t_1) e^{\theta t_1} \right] dt \]
\[ = \left[ \frac{I_h D_2}{\Theta^2} (e^{\theta(T-t_1)} - \theta(T - t_1) - 1) - I_h b_1 \left( \frac{T^2}{2} - t_1 T + \frac{t_1^2}{2} \right) e^{\theta t_1} \right] \] (9)

3.4. Optimal inventory level and optimal time

3.4.1. Optimal inventory level and optimal time \([0, t_1]\)

To optimal inventory level and optimal its time by minimizing the total cost

\[ TC_t = TC_A + TC_{h1} + TC_{P1} \] (10)

By subsisting Eq. (5, 6, 8) in Eq. (10)

Then

\[ TC_t = A + \left[ Cb_1 - Cb_1 t_1 e^{\theta t_1} + \left( \frac{I_h D_1}{\Theta^2} (e^{\theta t_1} - \theta t_1 - 1) - I_h b_1 \left( t_1 e^{\theta t_1} - \frac{e^{\theta t_1}}{\theta^2} + \frac{1}{\theta^2} \right) \right) \right] \]

Based on Taylor’s series about \( t_1 = 0 \) then,

\[ t_1 = \frac{h_1}{D_1} \]

\[ TC_t = A + \left[ Cb_1 - Cb_1 \left( 1 + \theta \frac{b_1}{D_1} \right) - I_h b_1 \left( \frac{b_1}{D_1} \right)^2 \right] \] (11)

Deviating Eq. (11) with respect to \( b_1 \)
\[
\frac{dT_C}{db_1} = (C - \frac{2Cb_1}{D_1} - \frac{3\theta Cb_1^2}{D_1^2} - \frac{3lb_1^2}{D_1^2}) = 0
\]

\[
b_1^* = \frac{D_1}{3} \left[ (C^2 + 3(C\theta + lb_1)^2) \frac{1}{\theta} \right] = q_1
\]

\[
t_1^* = \frac{b_1^*}{D_1}
\]

3.4. Economic order quantity during \([t_1, T]\)

To optimal inventory level and optimal its time by minimizing the total cost

\[TC_2 = TC_A + TC_{b2} + TC_{P2} (14)\]

By subsisting Eq. (5, 7, 9) in Eq. (14)

Then

\[TC_2 = A + \left[ CD(e^{\theta(T-t_1)} - 1) - Cb_1(T-t_1)e^{\theta t_1} + \frac{D_2}{\theta} \left( e^{\theta(T-t_1)} - \theta(T-t_1) - 1 \right) - \frac{l_b D_2}{\theta b_1} \left( \frac{T^2}{2} - t_1 T + \frac{t_1^2}{2} \right) e^{\theta t_1} \right] (15)\]

Similarly the Taylor’s series for \(e^{\theta(T-t_1)}\) about \(T = t_1\) then.

\[T = \frac{b_2 D_1}{D_2(D_1 - b_1)} + \frac{b_1}{D_1}\]

\[TC_2 = A + \left[ C D_2 - Cb_1 \left( \frac{b_2 D_1}{D_2(D_1 - b_1)} + \frac{b_1}{D_1} \right) e^{\theta t_1} + \frac{l_b}{\theta} \left( b_2 - \frac{D_2 b_1}{D_1 - b_1} \right) - l_b b_1 \left( \frac{b_2^2 D_1^2}{2D_2^2(D_1 - b_1)^2} \right)(1 + \frac{\theta b_1}{D_1}) \right] (16)\]

Deviating Eq. (16) with respect to \(b_2\)

\[\frac{dT_C}{db_2} = (C - \frac{Cb_1 D_1 (1 + \frac{\theta b_1}{D_1})}{D_2(D_1 - b_1)} + \frac{l_b}{\theta} - \frac{l_b D_1}{\theta(D_1 - b_1)} - \frac{l_b b_1 b_2 D_1^2}{D_2^2(D_1 - b_1)^2}) (17)\]

Here

\[b_2^* = \frac{D_2^2(D_1 - b_1) - b_1}{l_b b_1 D_1^2 \left( 1 + \frac{\theta b_1}{D_1} \right)^2} \left[ C - \frac{Cb_1 D_1 (1 + \frac{\theta b_1}{D_1})}{D_2(D_1 - b_1)} + \frac{l_b}{\theta} - \frac{l_b D_1}{\theta(D_1 - b_1)} \right] = q_2\]

\[T^* = \frac{b_2^* D_1}{D_2(D_1 - b_1)} + \frac{b_1}{D_2} = l_2 (18)\]

Lemma:

1) \((l_1, q_1)\) is optimal solution intended for \(TC_1\).

2) \((l_2, q_2)\) is optimal solution designed for \(TC_2\).

Proof:

Since

\[\frac{dT_C}{db_1} = (C - \frac{2Cb_1}{D_1} - \frac{3\theta Cb_1^2}{D_1^2} - \frac{3lb_1^2}{D_1^2}) (19)\]

Deviating the Eq. (19) with respect to \(b_1\)

\[\frac{d^2TC_1}{db_1^2} \mid _{(l_1,q_1)} = \left( -\frac{2C}{D_1} + \frac{6\theta Cb_1^*}{D_1^2} + \frac{6lb_1^*}{D_1^2} \right) \times 0 \frac{2C}{D_1} + \frac{6\theta Cb_1^*}{D_1^2} + \frac{6lb_1^*}{D_1^2} > 0\]

Lemma (1) is holding.
\[
\frac{dTC_2}{db_2} = (C - \frac{Cb_1D_1(1 + \frac{b_1}{D_1})}{D_2(D_1 - b_1)} + \frac{I_h}{0}) - \frac{I_hD_1}{0(D_1 - b_1)} - \frac{I_hb_1b_2D_1^2(1 + \frac{b_1}{D_1})}{D_2^2(D_1 - b_1)^2}
\]

Similarly we found out that
\[
\frac{\frac{d^2TC_2}{db_2^2}}{1(D_2^2(D_1 - b_1)^2)} < 0
\]

Lemma (2) is holding.

IV. Sensitivity analysis

The conjecture of the parameters of total cost in for the two inventory models as follows:

Example 1

\[D_1 = 350 \text{ unite}, I_h = 3\$ \text{ per unite}, C_1 = 300\$ \text{ per unite}, A = 200\$ \text{ per unite}\]

Table 1. The sensitivity analysis for first interval

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<tr>
<th>(\theta)</th>
<th>(b_1^*)</th>
<th>(t_1^*)</th>
<th>(TC_1^*)</th>
<th>(TC_1)</th>
<th>Difference</th>
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Example 2.

\[D_2 = 500 \text{ unite}, I_h = 3\$ \text{ per unite}, C_2 = 300\$ \text{ per unite}, A = 200\$ \text{ per unite}\]

Table 2. The sensitivity analysis for second interval

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IV. RESULTS AND DISCUSSION
The output of planned model within two class intervals founded out that, according fig.2 when the offering rate less than demand rate the first inventory level was decreasing when deterioration rate increased, fig.3 illustrate that the deterioration rate was decreasing when first optimal time increased this is support the possibility in real life with deteriorating items, fig.4 exemplify the gap between actual total and optimal costs was high that made the proposed model is applicable to achieve optimal total cost under any deterioration rate lies [0.06,1] because at second time the planned model is not fitted when low risk as deterioration risk for item that makes the suggested model is more significance with high risk.

Fig2. Graph depiction optimal first time inventory level VS deterioration rate

Fig3. Graph illustration optimal first time VS deterioration rate

Fig4. Graph sign the optimal total and actual costs VS deterioration rate within first time

Fig5. Graph representation optimal second inventory level VS deterioration rate within second time

Fig6. Graph representation optimal second time optimal second time

Fig7. Graph representation optimal total and actual total costs VS deterioration rate

V. ACKNOWLEDGMENTS
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REFERENCES


