ON SUPRA g*bω - CONTINUOUS MAPS IN SUPRA TOPOLOGICAL SPACES

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ABSTRACT:

In this paper, we introduce the concepts of supra g*bω - continuous maps and study their basic properties in supra topological spaces.

Keywords: supra topological spaces, supra g*bω - continuous maps, supra g*bω - irresolute maps.

1 INTRODUCTION


In this paper, we introduce the concepts of supra g*bω - continuous maps in supra topological spaces.

2 PRELIMINARIES

Definition 2.1 [2, 4] A subfamily μ of X is said to be a supra topology on X if

(i) \( X, \varphi \in \mu \)

(ii) if \( A_i \in \mu \) for all i, then \( \bigcup A_i \in \mu \)

The pair \((X, \mu)\) is called supra topological space. The elements of \( \mu \) are called supra open sets in \((X, \mu)\) and complement of a supra open set is called a supra closed set.

Definition 2.2 [2] The supra closure of a set \( A \) is defined as \( cl^\mu(A) = \bigcap \{B : B \text{ is supra closed and } A \subseteq B\} \) and the supra interior of a set \( A \) is defined as \( int^\mu(A) = \bigcup \{B : B \text{ is supra open and } A \supseteq B\} \).

Throughout this paper we shall denote by \((X, \mu)\) a supra topological space. For any subset \( A \subseteq X \), \( int^\mu(A) \) and \( cl^\mu(A) \) denote the supra interior of \( A \) and the supra closure of \( A \) with respect to \( \mu \).

We shall require the following known definitions:

Definition 2.3 [2] Let \((X, \mu)\) be a supra topological spaces. A subset \( A \) of \( X \) is called

- supra semi-open if \( A \subseteq cl^\mu(int^\mu(A)) \) and \( supra \text{ semi-closed if } int^\mu(cl^\mu(A)) \subseteq A \)
- supra pre-open if \( A \subseteq int^\mu(cl^\mu(A)) \) and \( supra \text{ pre-closed if } cl^\mu(int^\mu(A)) \subseteq A \)
- supra α-open if \( A \subseteq int^\mu(cl^\mu(int^\mu(A))) \) and supra α-closed if \( cl^\mu(int^\mu(cl^\mu(A))) \subseteq A \)
- supra regular open if \( A = int^\mu(cl^\mu(A)) \) and supra regular closed if \( A = cl^\mu(int^\mu(A)) \)
- supra b-open if \( A \subseteq cl^\mu(int^\mu(A)) \cup int^\mu(cl^\mu(A)) \) and supra b-closed if \( cl^\mu(int^\mu(A)) \cap int^\mu(cl^\mu(A)) \subseteq A \).

Let \((X, \mu)\) or simply \( X \) denote a supra topological space. For any subset \( A \subseteq X \), the intersection of all supra b-closed sets containing \( A \) is called the supra b-closure of \( A \), denoted by \( bcl^\mu(A) \). The union of all b-open sets contained in \( A \) is called the supra b-interior of \( A \), denoted by \( bint^\mu(A) \).

Definition 2.4 [2] Let \((X, \tau)\) be a topological space and \( \mu \) be a supra topology on \( X \). We call \( \mu \) a supra topology associated with \( \tau \) if \( \tau \subseteq \mu \).

Definition 2.5 [2] A set \( A \) of a supra topological space \((X, \mu)\) is called supra generalized semi closed (briefly gs^\mu - closed) if \( scl^\mu(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is supra open in \((X, \mu)\).
Definition 2.6 [3] A set $A$ of a supra topological space $(X, \mu)$ is called **supra generalized star b omega closed** (briefly, $g^*b\omega^\mu$ - closed) if $bcl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is supra $gs$ - open in $(X, \mu)$.

3 $g^*b\omega^\mu$ - CONTINUOUS MAPS

Definition 3.1 Let $(X, \tau)$ and $(Y, \sigma)$ be two topological spaces and $\mu$ be an associated supra topology with $\tau$. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is supra **generalized star b omega - continuous** (briefly, $g^*b\omega^\mu$ - continuous) if $f^\dagger(U)$ is $g^*b\omega^\mu$ - closed in $(X, \mu)$ for each closed set $U$ in $(Y, \sigma)$.

Theorem 3.2 Every continuous map is $g^*b\omega^\mu$ - continuous.

**Proof:** Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be continuous. Let $U$ be a closed set in $(Y, \sigma)$. Then $f(U)$ is closed in $(X, \tau)$. Since $\mu$ is associated with supra topology $\tau$ then $\tau \subseteq \mu$. Therefore $f^\dagger(U)$ is closed in $(X, \mu)$ and it is $g^*b\omega^\mu$ - closed, $f$ is $g^*b\omega^\mu$ - continuous.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.3 Let $X = Y = \{a, b, c\}$ with the topologies $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\mu = \{\emptyset, Y, \{a\}, \{a, b\}, \{a, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then $f$ is $g^*b\omega^\mu$ - continuous but not continuous, since $\{c\}$ is $g^*b\omega^\mu$ - closed but $f^\dagger(\{c\}) = \{a\}$ is not closed.

Theorem 3.4 Every supra continuous map is $g^*b\omega^\mu$ - continuous.

**Proof:** Obvious.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.5 In example 3.3, $f$ is $g^*b\omega^\mu$ - continuous but not supra continuous, since $\{c\}$ is $g^*b\omega^\mu$ - closed but $f^\dagger(\{c\}) = \{a\}$ is not supra closed.

Theorem 3.6 The following are equivalent for a map $f : (X, \tau) \rightarrow (Y, \sigma)$ and $\mu$ be an associated supra topology with $\tau$.

(a) $f$ is $g^*b\omega^\mu$ - continuous
(b) $f^\dagger(A)$ is $g^*b\omega^\mu$ - open for each open set $A$ in $(Y, \sigma)$.

Theorem 3.7 If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $g^*b\omega^\mu$ - continuous then $f(g^*b\omega^\mu cl(A)) \subseteq cl(f(A))$ for every subset $A$ of $(X, \tau)$.

Proof: Since $A \subseteq f^\dagger(f(A))$, we have $A \subseteq f^\dagger(cl(f(A)))$. Now $cl(f(A))$ is a closed set in $(Y, \sigma)$ and hence $f^\dagger(cl(f(A)))$ is a $g^*b\omega^\mu$ - closed set containing $A$. Consequently, $g^*b\omega^\mu cl(A) \subseteq f^\dagger(cl(f(A)))$. Therefore $f(g^*b\omega^\mu cl(A)) \subseteq f^\dagger(cl(f(A))) \subseteq cl(f(A))$.

Theorem 3.8 The following are equivalent for a map $f : (X, \tau) \rightarrow (Y, \sigma)$.

(a) For every subset $A$ of $(X, \tau)$, $f(g^*b\omega^\mu cl(A)) \subseteq cl(f(A))$.
(b) For every subset $B$ of $(Y, \sigma)$, $g^*b\omega^\mu cl(f^\dagger(B)) \subseteq f^\dagger(cl(B))$.

**Proof:** Suppose that (a) holds and let $B = f(A)$ where $A$ is a subset of $X$. Then $g^*b\omega^\mu cl(A) \subseteq g^*b\omega^\mu cl(f^\dagger(B)) \subseteq f^\dagger(cl(B))$. Therefore $f(g^*b\omega^\mu cl(A)) \subseteq cl(B) = cl(f(A))$.

Conversely, suppose that (b) holds and let $B = f(A)$ where $A$ is a subset of $X$. Then $g^*b\omega^\mu cl(A) \subseteq g^*b\omega^\mu cl(f^\dagger(B)) \subseteq f^\dagger(cl(B))$. Therefore $f(g^*b\omega^\mu cl(A)) \subseteq cl(f(A))$.

Definition 3.9 Let $(X, \tau)$ and $(Y, \sigma)$ be two topological spaces and $\mu$ be an associated supra topology with $\tau$. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called **$g^*b\omega^\mu$ - irresolute** if $f^\dagger(U)$ is $g^*b\omega^\mu$ - closed in $(X, \mu)$ for every $g^*b\omega^\mu$ - closed set $U$ in $(Y, \sigma)$.

Theorem 3.10 A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is $g^*b\omega^\mu$ - irresolute if and only if $f(V)$ is $g^*b\omega^\mu$ - open in $(X, \mu)$ for every $g^*b\omega^\mu$ - open set $V$ in $(Y, \sigma)$.

Theorem 3.11 Every $g^*b\omega^\mu$ - irresolute map is $g^*b\omega^\mu$ - continuous.

**Proof:** Let $U$ be any closed set in $(Y, \sigma)$. Then $U$ is a $g^*b\omega^\mu$ - closed set in $(Y, \sigma)$. Since $f$ is $g^*b\omega^\mu$ - irresolute, $f^\dagger(U)$ is $g^*b\omega^\mu$ - closed in $(X, \tau)$. Since $\mu$ is associated with supra topology $\tau$ then $\tau \subseteq \mu$. Therefore $f^\dagger(U)$ is $g^*b\omega^\mu$ - closed in $(X, \mu)$. Therefore $f$ is $g^*b\omega^\mu$ - continuous.

Definition 3.12: A supra topological space $(X, \mu)$ is called $g^*b\omega^\mu T_c$ - space if every $g^*b\omega^\mu$ - closed subset of $(X, \mu)$ is closed in $(X, \mu)$.

Theorem 3.13 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $g^*b\omega^\mu$ - continuous map where $(X, \mu)$ is a $g^*b\omega^\mu T_c$ - space. Then $f$ is continuous.
Proof: Let $U$ be any closed set in $(Y, \sigma)$. Since $f : (X, \tau) \rightarrow (Y, \sigma)$ is $g^*b^\omega -$ continuous, $f^{-1}(U)$ is $g^*b^\omega -$ closed in $(X, \tau)$. Since $\mu$ is associated with supra topology $\tau$ then $\tau \subseteq \mu$. Therefore $f^{-1}(U)$ is $g^*b^\omega -$ closed in $(X, \mu)$. Since $(X, \mu)$ is a $g^*b^\omega T_c$ - space, $f^{-1}(U)$ is closed in $(X, \mu)$. Hence $f$ is continuous.

Theorem 3.14 If $(X, \mu)$ is a $g^*b^\omega T_c$ - space then $g^*b^\omega \text{cl}(B) = \text{cl}(B)$ for each subset $B$ of $(X, \mu)$.

Proof: Since $(X, \mu)$ is a $g^*b^\omega T_c$ - space, every $g^*b^\omega -$ closed set is closed. Since every closed set is $g^*b^\omega -$ closed set in $(X, \mu)$, $G^*b^\omega C(X, \mu) = C(X, \mu)$. Hence $g^*b^\omega \text{cl}(B) = \text{cl}(B)$ for each subset $B$ of $(X, \mu)$.

Theorem 3.15 If $(X, \mu)$ is a $g^*b^\omega T_c$ - space then for each $x \in X$ either $\{x\}$ is supra gs - closed or supra open.

Proof: Let $x \in X$ and suppose $\{x\}$ is not supra gs - closed in $(X, \tau)$. Then $X \setminus \{x\}$ is not supra gs - open. Hence $X$ is the only supra gs - open set containing $X \setminus \{x\}$. This implies that $X \setminus \{x\}$ is a $g^*b^\omega -$ closed set of $(X, \tau)$. Since $(X, \mu)$ is a $g^*b^\omega T_c$ - space, $X \setminus \{x\}$ is a closed set in $(X, \mu)$ or equivalently $\{x\}$ is open in $(X, \mu)$.

REFERENCE