µ-β-generalized α-continuous mappings in generalized topological spaces

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Abstract: In this paper, we have introduced µ-β-generalized α-continuous maps and also introduced almost µ-β-generalized α-continuous maps in generalized topological spaces by using µ-β-generalized α-closed sets (briefly µ-βGaCS). Also we have introduced some of their basic properties.

Keywords: Generalized topology, generalized topological spaces, µ-α-closed sets, µ-β-generalized α-closed sets, µ-α-continuous, µ-β-generalized α-continuous, almost µ-α-continuous, almost µ-β-generalized α-continuous.

1. Introduction

In 1970, Levin [4] introduced the idea of continuous function. He also introduced the concepts of semi-open sets and semi-continuity [3] in a topological space. Mashhour [5] introduced and studied α-continuous function in topological spaces. The notation of µ-β-generalized α-closed sets (briefly µ-βGaCS) was defined and investigated by Kowsalya. M and Jayanthi. D [2]. In this paper, we have introduced µ-β-generalized α-continuous maps and also introduced almost µ-β-generalized α-continuous maps in generalized topological spaces. Also we have investigated some of their basic properties and produced many interesting theorems.

2. Preliminaries

Let us recall the following definitions which are used in sequel.

Definition 2.1: [1] Let X be a nonempty set. A collection µ of subsets of X is a generalized topology (or briefly GT) on X if it satisfies the following:

(1) Ø, X∈ µ and
(2) If {Mᵢ : i∈ I} ⊆ µ, then ∪ᵢ∈ IMᵢ∈ µ.

If µ is a GT on X, then (X, µ) is called a generalized topological space (or briefly GTS) and the elements of µ are called µ-open sets and their complement are called µ-closed sets.
Definition 2.2: [1] Let \((X, \mu)\) be a GTS and let \(A \subseteq X\). Then the \(\mu\)-closure of \(A\), denoted by \(c_\mu(A)\), is the intersection of all \(\mu\)-closed sets containing \(A\).

Definition 2.3: [1] Let \((X, \mu)\) be a GTS and let \(A \subseteq X\). Then the \(\mu\)-interior of \(A\), denoted by \(i_\mu(A)\), is the union of all \(\mu\)-open sets contained in \(A\).

Definition 2.4: [1] Let \((X, \mu)\) be a GTS. A subset \(A\) of \(X\) is said to be

i. \(\mu\)-semi-closed set if \(i_\mu(c_\mu(A)) \subseteq A\)

ii. \(\mu\)-pre-closed set if \(c_\mu(i_\mu(A)) \subseteq A\)

iii. \(\mu\)-\(\alpha\)-closed set if \(c_\mu(i_\mu(c_\mu(A))) \subseteq A\)

iv. \(\mu\)-\(\beta\)-closed set if \(i_\mu(c_\mu(i_\mu(A))) \subseteq A\)

v. \(\mu\)-regular-closed set if \(A = c_\mu(i_\mu(A))\)

Definition 2.5:[7] Let \((X, \mu_1)\) and \((Y, \mu_2)\) be GTSs. Then a mapping \(f: (X, \mu_1) \rightarrow (Y, \mu_2)\) is called

i. \(\mu\)-Continuous mapping if \(f^{-1}(A)\) is \(\mu\)-closed in \((X, \mu_1)\) for each \(\mu\)-closed in \((Y, \mu_2)\).

ii. \(\mu\)-Semi-continuous mapping if \(f^{-1}(A)\) is \(\mu\)-semi-closed in \((X, \mu_1)\) for every \(\mu\)-closed in \((Y, \mu_2)\).

iii. \(\mu\)-pre-continuous mapping if \(f^{-1}(A)\) is \(\mu\)-pre-closed in \((X, \mu_1)\) for every \(\mu\)-closed in \((Y, \mu_2)\).

iv. \(\mu\)-\(\alpha\)-continuous mapping if \(f^{-1}(A)\) is \(\mu\)-\(\alpha\)-closed in \((X, \mu_1)\) for every \(\mu\)-closed in \((Y, \mu_2)\).

v. \(\mu\)-\(\beta\)-continuous mapping if \(f^{-1}(A)\) is \(\mu\)-\(\beta\)-closed in \((X, \mu_1)\) for every \(\mu\)-closed in \((Y, \mu_2)\).

Definition 2.6:[6] Let \((X, \mu_1)\) and \((Y, \mu_2)\) be GTSs. Then a mapping \(f: (X, \mu_1) \rightarrow (Y, \mu_2)\) is called

i. almost \(\mu\)-Continuous mapping if \(f^{-1}(A)\) is \(\mu\)-closed in \((X, \mu_1)\) for every \(\mu\)-regular closed set \(A\) of \((Y, \mu_2)\).

ii. almost \(\mu\)-semi continuous mappings if \(f^{-1}(A)\) is \(\mu\)-semi closed in \((X, \mu_1)\) for every \(\mu\)-regular closed set \(A\) of \((Y, \mu_2)\).

iii. almost \(\mu\)-pre-continuous mappings if \(f^{-1}(A)\) is \(\mu\)-pre closed in \((X, \mu_1)\) for every \(\mu\)-regular closed set \(A\) of \((Y, \mu_2)\).

iv. almost \(\mu\)-\(\alpha\)-continuous mapping if \(f^{-1}(A)\) is \(\mu\)-\(\alpha\)-closed in \((X, \mu_1)\) for every \(\mu\)-regular closed set \(A\) of \((Y, \mu_2)\).

v. almost \(\mu\)-\(\beta\)-continuous mapping if \(f^{-1}(A)\) is \(\mu\)-\(\beta\)-closed in \((X, \mu_1)\) for every \(\mu\)-regular closed set \(A\) of \((Y, \mu_2)\).

3. \(\mu\)-\(\beta\)-generalized \(\alpha\)-continuous mappings in topological spaces

In this section we have introduced \(\mu\)-\(\beta\)-generalized \(\alpha\)-continuous mappings in generalized topological spaces and discussed some of their basic properties.
Definition 3.1: A mapping \( f: (X, \mu_1) \to (Y, \mu_2) \) is called a \( \mu\)-\( \beta\)-generalized \( \alpha\)-continuous (briefly \( \mu\)-\( \beta\)\( \alpha\)-continuous) if \( f^{-1}(A) \) is a \( \mu\)-\( \beta\)-generalized \( \alpha\)-closed set (briefly \( \mu\)-\( \beta\)\( \alpha\)CS) in \( (X, \mu_1) \) for each \( \mu\)-closed set \( A \) in \( (Y, \mu_2) \).

Example 3.2: Let \( X = Y = \{a, b, c\} \) with \( \mu_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\} \) and \( \mu_2 = \{\emptyset, \{a, b\}, Y\} \). Let \( f: (X, \mu_1) \to (Y, \mu_2) \) be a mapping defined by \( f(a) = a, f(b) = b, f(c) = c \). Now, \( \mu\)-\( \beta\)O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}. Let \( A = \{c\} \), then \( A \) is a \( \mu\)-closed set in \( (Y, \mu_2) \). Then \( f^{-1}(\{c\}) \) is a \( \mu\)-\( \beta\)\( \alpha\)CS in \( (X, \mu_1) \). Hence \( f \) is a \( \mu\)-\( \beta\)\( \alpha\)-continuous mapping.

Theorem 3.3: Every \( \mu\)-continuous mapping is a \( \mu\)-\( \beta\)\( \alpha\)-continuous mapping but not conversely in general.

Proof: Let \( f: (X, \mu_1) \to (Y, \mu_2) \) be a \( \mu\)-continuous mapping. Let \( A \) be \( \mu\)-closed set in \( (Y, \mu_2) \). Since \( f \) is a \( \mu\)-continuous mapping, \( f^{-1}(A) \) is a \( \mu\)-closed set in \( (X, \mu_1) \). Since every \( \mu\)-closed set is a \( \mu\)-\( \beta\)\( \alpha\)CS, \( f^{-1}(A) \) is a \( \mu\)-\( \beta\)\( \alpha\)CS in \( (X, \mu_1) \). Hence \( f \) is a \( \mu\)-\( \beta\)\( \alpha\)-continuous mapping.

Example 3.4: Let \( X = Y = \{a, b, c, d\} \) with \( \mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\} \) and \( \mu_2 = \{\emptyset, \{a, b\}, Y\} \). Let \( f: (X, \mu_1) \to (Y, \mu_2) \) be a mapping defined by \( f(a) = a, f(b) = b, f(c) = c, f(d) = d \). Now, \( \mu\)-\( \beta\)O(X) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, X\}. Let \( A = \{d\} \), then \( A \) is a \( \mu\)-closed set in \( (Y, \mu_2) \). Then \( f^{-1}(\{d\}) \) is a \( \mu\)-\( \beta\)\( \alpha\)CS in \( (X, \mu_1) \), but not \( \mu\)-closed as \( c_p(f^{-1}(A)) = \{d\} \neq f^{-1}(A) \). Hence \( f \) is a \( \mu\)-\( \beta\)\( \alpha\)-continuous mapping, but not a \( \mu\)-continuous mapping.

Theorem 3.5: Every \( \mu\)-\( \alpha\)-continuous mapping is a \( \mu\)-\( \beta\)\( \alpha\)-continuous mapping in general.

Proof: Let \( f: (X, \mu_1) \to (Y, \mu_2) \) be a \( \mu\)-\( \alpha\)-continuous mapping. Let \( A \) be any \( \mu\)-closed set in \( (Y, \mu_2) \). Since \( f \) is a \( \mu\)-\( \alpha\)-continuous mapping, \( f^{-1}(A) \) is a \( \mu\)-\( \alpha\)-closed set in \( (X, \mu_1) \). Since every \( \mu\)-\( \alpha\)-closed set is a \( \mu\)-\( \beta\)\( \alpha\)CS, \( f^{-1}(A) \) is a \( \mu\)-\( \beta\)\( \alpha\)CS in \( (X, \mu_1) \). Hence \( f \) is a \( \mu\)-\( \beta\)\( \alpha\)-continuous mapping.

Remark 3.6: A \( \mu\)-pre-continuous mapping is not a \( \mu\)-\( \beta\)\( \alpha\)-continuous mapping in general.

Example 3.7: Let \( X = Y = \{a, b, c\} \) with \( \mu_1 = \{\emptyset, \{a, b\}, X\} \) and \( \mu_2 = \{\emptyset, \{b, c\}, Y\} \). Let \( f: (X, \mu_1) \to (Y, \mu_2) \) be a mapping defined by \( f(a) = a, f(b) = b, f(c) = c \). Now, \( \mu\)-\( \beta\)O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}. Let \( A = \{a\} \), then \( A \) is a \( \mu\)-closed set in \( (Y, \mu_2) \). Then \( f^{-1}(\{a\}) \) is a \( \mu\)-pre-closed set in \( (X, \mu_1) \) as \( c_p(i_p(f^{-1}(A))) = c_p(i_p(\{a\})) = \emptyset \subseteq f^{-1}(A) \), but not \( \mu\)-\( \beta\)\( \alpha\)CS as \( \alpha c_p(f^{-1}(A)) = X \notin U = \{a, b\} \) in \( (X, \mu_1) \). Hence \( f \) is a \( \mu\)-pre-continuous mapping, but not a \( \mu\)-\( \beta\)\( \alpha\)-continuous mapping.

Remark 3.8: A \( \mu\)-\( \beta\)-continuous mapping is not a \( \mu\)-\( \beta\)\( \alpha\)-continuous mapping in general.

Example 3.9: Let \( X = Y = \{a, b, c\} \) with \( \mu_1 = \{\emptyset, \{a, b\}, X\} \) and \( \mu_2 = \{\emptyset, \{b, c\}, Y\} \). Let \( f: (X, \mu_1) \to (Y, \mu_2) \) be a mapping defined by \( f(a) = a, f(b) = b, f(c) = c \). Now, \( \mu\)-\( \beta\)O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}. Let \( A = \{a\} \), then \( A \) is a \( \mu\)-closed set in \( (Y, \mu_2) \). Then \( f^{-1}(\{a\}) \) is a \( \mu\)-\( \beta\)-closed in \( (X, \mu_1) \) as \( i_p(c_p(i_p(f^{-1} \{a\}))) = X \notin U = \{a, b\} \) in \( (X, \mu_1) \). Hence \( f \) is a \( \mu\)-\( \beta\)\( \alpha\)-continuous mapping, but not a \( \mu\)-\( \beta\)\( \alpha\)-continuous mapping.
(A))) = \sum_{\mu(a)}(\mu((a))) = \emptyset \subseteq f^{-1}(A)$, but not a $\mu$-$\beta$G$\alpha$CS as $\alpha e_{\mu}(f^{-1}(A)) = X \not\subseteq U = \{a, b\}$ in $(X, \mu_1)$. Hence $f$ is a $\mu$-$\beta$-continuous mapping, but not a $\mu$-$\beta$G$\alpha$-continuous mapping.

In the following diagram, we have provided relation between various types of $\mu$-continuous mappings.

![Diagram showing the relationship between different types of $\mu$-continuous mappings]

**Theorem 3.10:** A mapping $f$: $(X, \mu_1) \to (Y, \mu_2)$ is $\alpha$-$\mu$-$\beta$G$\alpha$-continuous mapping if and only if the inverse image of every $\mu$-open set in $(Y, \mu_2)$ is a $\mu$-$\beta$G$\alpha$OS in $(X, \mu_1)$.

**Proof:** 

**Necessity:** Let $U$ be a $\mu$-open set in $(Y, \mu_2)$. Then $Y-U$ is a $\mu$-closed set in $(Y, \mu_2)$. Since $f$ is $\alpha$-$\mu$-$\beta$G$\alpha$-continuous mapping, $f^{-1}(Y-U) = X - f^{-1}(U)$ is a $\mu$-$\beta$G$\alpha$CS in $(X, \mu_1)$. Hence $f^{-1}(U)$ is a $\mu$-$\beta$G$\alpha$OS in $(X, \mu_1)$.

**Sufficiency:** Assume that $f^{-1}(V)$ is a $\mu$-$\beta$G$\alpha$OS in $(X, \mu_1)$ for each $\mu$-open set $V$ in $(Y, \mu_2)$. Let $V$ be any $\mu$-closed set in $(Y, \mu_2)$. Then $Y-V$ is $\mu$-open in $(Y, \mu_2)$. By assumption, $f^{-1}(Y-V) = X - f^{-1}(V)$ is $\alpha$-$\mu$-$\beta$G$\alpha$OS in $(X, \mu_1)$ which implies that $f^{-1}(V)$ is a $\mu$-$\beta$-generalized $\alpha$-closed set in $(X, \mu_1)$. Hence $f$ is $\alpha$-$\mu$-$\beta$G$\alpha$-continuous mapping.

**Theorem 3.11:** If $f$: $(X, \mu_1) \to (Y, \mu_2)$ is a $\mu$-$\beta$G$\alpha$-continuous mapping and $g$: $(Y, \mu_2) \to (Z, \mu_3)$ is a $\mu$-continuous mapping then $g \circ f$: $(X, \mu_1) \to (Z, \mu_3)$ is a $\mu$-$\beta$G$\alpha$-continuous mapping.

**Proof:** Let $V$ be any $\mu$-closed set in $(Z, \mu_3)$. Since $g$ is a $\mu$-continuous mapping, $g^{-1}(V)$ is a $\mu$-closed set in $(Y, \mu_2)$. Since $f$ is a $\mu$-$\beta$G$\alpha$-continuous mapping, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a $\mu$-$\beta$G$\alpha$CSin $(X, \mu_1)$. Therefore $g \circ f$ is a $\mu$-$\beta$G$\alpha$-continuous mapping.

**Theorem 3.12:** If $f$: $(X, \mu_1) \to (Y, \mu_2)$ is a $\mu$-continuous mapping and $g$: $(Y, \mu_2) \to (Z, \mu_3)$ is a $\mu$-continuous mapping then $g \circ f$: $(X, \mu_1) \to (Z, \mu_3)$ is a $\mu$-$\beta$G$\alpha$-continuous mapping.
Proof: Let V be any $\mu$-closed set in $(Z, \mu_3)$. Since $g$ is a $\mu$-continuous mapping, $g^{-1}(V)$ is a $\mu$-closed set in $(Y, \mu_2)$. Since $f$ is a $\mu$-continuous mapping, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a $\mu$-closed set in $(X, \mu_1)$. Since every $\mu$-closed set is $a_\mu$-$\beta G\alpha CS$, $(g \circ f)^{-1}(V)$ is $a_\mu$-$\beta G\alpha CS$. Therefore $g \circ f$ is a $\mu$-$\beta G\alpha$-continuous mapping.

Theorem 3.13: If $f: (X, \mu_1) \to (Y, \mu_2)$ is a $\mu$-$\alpha$-continuous mapping and $g: (Y, \mu_2) \to (Z, \mu_3)$ is a $\mu$-continuous mapping then $g \circ f: (X, \mu_1) \to (Z, \mu_3)$ is a $\mu$-$\beta G\alpha$-continuous mapping.

Proof: Let V be any $\mu$-closed set in $(Z, \mu_3)$. Since $g$ is a $\mu$-continuous mapping $g^{-1}(V)$ is a $\mu$-closed in $(Y, \mu_2)$. Since $f$ is a $\mu$-$\alpha$-continuous mapping, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a $\mu$-$\alpha$-closed in $(X, \mu_1)$. Since every $\mu$-$\alpha$-closed set is $a_\mu$-$\beta G\alpha CS$, $(g \circ f)^{-1}(V)$ is a $\mu$-$\beta G\alpha CS$ in $(X, \mu_1)$. Therefore $g \circ f$ is a $\mu$-$\beta G\alpha$-continuous mapping.

4. ALMOST $\mu$-$\beta$-GENERALIZED $\alpha$-CONTINUOUS MAPPINGS

In this section we have introduced almost $\mu$-$\beta$-generalized $\alpha$-continuous mappings in generalized topological spaces and studied some of their basic properties.

Definition 4.1: A mapping $f: (X, \mu_1) \to (Y, \mu_2)$ is called an almost $\mu$-$\beta$-generalized $\alpha$-continuous mapping (briefly almost $\mu$-$\beta G\alpha$-continuous) if $f^{-1}(A)$ is a $\mu$-$\beta$-generalized $\alpha$-closed set (briefly $\mu$-$\beta G\alpha CS$) in $(X, \mu_1)$ for each $\mu$-regular closed set $A$ in $(Y, \mu_2)$.

Example 4.2: Let $X = Y = \{a, b, c\}$ with $\mu_1 = \emptyset, \{a\}, \{b\}, \{a, b\}, X$ and $\mu_2 = \emptyset, \{c\}, \{a, b\}, Y$. Let $f: (X, \mu_1) \to (Y, \mu_2)$ be a mapping defined by $f(a) = a, f(b) = b, f(c) = c$. Now, $\mu$-$\beta O(X) = \emptyset, \{a\}, \{a, b\}, \{b, c\}, \{a, c\}, X$. Let $A = \{c\}$, then $A$ is a $\mu$-regular closed set in $(Y, \mu_2)$. Then $f^{-1}(\{c\})$ is $\mu$-$\beta G\alpha CS$ in $(X, \mu_1)$. Hence $f$ is an almost $\mu$-$\beta G\alpha$-continuous mapping.

Theorem 4.3: Every almost $\mu$-continuous mapping is an almost $\mu$-$\beta G\alpha$-continuous mapping but not conversely in general.

Proof: Let $f: (X, \mu_1) \to (Y, \mu_2)$ be an almost $\mu$-continuous mapping. Let $A$ be a $\mu$-regular closed set in $(Y, \mu_2)$. Since $f$ is an almost $\mu$-continuous mapping, $f^{-1}(A)$ is a $\mu$-closed set in $(X, \mu_1)$. Since every $\mu$-closed set is a $\mu$-$\beta G\alpha CS$, $f^{-1}(A)$ is a $\mu$-$\beta G\alpha CS$ in $(X, \mu_1)$. Hence $f$ is an almost $\mu$-$\beta G\alpha$-continuous mapping.

Example 4.4: Let $X = Y = \{a, b, c, d\}$ with $\mu_1 = \emptyset, \{a\}, \{c\}, \{a, c\}, X$ and $\mu_2 = \emptyset, \{a, b\}, \{a, b, c\}, Y$. Let $f: (X, \mu_1) \to (Y, \mu_2)$ be a mapping defined by $f(a) = a, f(b) = b, f(c) = c$. Now, $\mu$-$\beta O(X) = \emptyset, \{a\}, \{a, b\}, \{a, c\}, X$. Let $A = \{b\}$, then $A$ is a $\mu$-regular closed set in $(Y, \mu_2)$. Then $f^{-1}(\{b\})$ is $a_\mu$-$\beta G\alpha CS$, but not $\mu$-closed as $c_\mu(f^{-1}(A)) = c_\mu(\{b\}) = \{b, c\} \neq f^{-1}(A)$ in $(X, \mu_1)$. Hence $f$ is an almost $\mu$-$\beta G\alpha$-continuous mapping, but not an almost $\mu$-continuous mapping.
**Theorem 4.5:** Every almost $\mu$-$\alpha$-continuous mapping is an almost $\mu$-$\beta$G$\alpha$-continuous mapping in general.

**Proof:** Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be an almost $\mu$-$\alpha$-continuous mapping. Let $A$ be any $\mu$-regular closed set in $(Y, \mu_2)$. Since $f$ is an almost $\mu$-$\alpha$-continuous mapping, $f^{-1}(A)$ is a $\mu$-$\alpha$-closed set in $(X, \mu_1)$. Since every $\mu$-$\alpha$-closed set is $\alpha$-$\mu$-$\beta$G$\alpha$CS, $f^{-1}(A)$ is a $\mu$-$\beta$-generalized $\alpha$-closed set in $(X, \mu_1)$. Hence $f$ is an almost $\mu$-$\beta$G$\alpha$-continuous mapping.

**Remark 4.6:** An almost $\mu$-pre-continuous mapping is not an almost $\mu$-$\beta$G$\alpha$-continuous mapping in general.

**Example 4.7:** Let $X = Y = \{a, b, c\}$ with $\mu_1 = \{\emptyset, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{a\}, \{b, c\}, Y\}$. Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Now, $\mu$-$\beta$O$(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$. Let $A = \{a\}$, then $A$ is a $\mu$-regular closed set in $(Y, \mu_2)$. Then $f^{-1}(A)$ is a $\mu$-pre closed set as $c_\mu(i_\mu(f^{-1}(A))) = c_\mu(i_\mu(\{a\})) = \emptyset \subseteq f^{-1}(A)$, but not $\mu$-$\beta$G$\alpha$CS as $\alpha c_\mu(f^{-1}(A)) = X \not\subseteq U = \{a, b\}$ in $(X, \mu_1)$. Hence $f$ is an almost $\mu$-pre-continuous, but not an almost $\mu$-$\beta$G$\alpha$-continuous mapping.

**Remark 4.8:** An almost $\mu$-$\beta$-continuous mapping is not an almost $\mu$-$\beta$G$\alpha$-continuous mapping in general.

**Example 4.9:** Let $X = Y = \{a, b, c\}$ with $\mu_1 = \{\emptyset, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{a\}, \{b, c\}, Y\}$. Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Now, $\mu$-$\beta$O$(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$. Let $A = \{a\}$, then $A$ is a $\mu$-regular closed set in $(Y, \mu_2)$. Then $f^{-1}(A)$ is a $\mu$-$\beta$-closed set as $i_\mu(c_\mu(i_\mu(f^{-1}(A)))) = i_\mu(c_\mu(i_\mu(\{a\}))) = \emptyset \subseteq f^{-1}(A)$, but not $\mu$-$\beta$G$\alpha$CS as $\alpha c_\mu(f^{-1}(A)) = X \not\subseteq U = \{a, b\}$ in $(X, \mu_1)$. Hence $f$ is an almost $\mu$-$\beta$-continuous mapping, but not an almost $\mu$-$\beta$G$\alpha$-continuous mapping. In the following diagram, we have provided the relation between various types of almost $\mu$-continuous mappings.
Theorem 4.10: A mapping $f: (X, \mu_1) \to (Y, \mu_2)$ is an almost $\mu$-$\beta G\alpha$-continuous mapping if and only if the inverse image of every $\mu$-regular open set in $(Y, \mu_2)$ is $\mu$-$\beta G\alpha OS$ in $(X, \mu_1)$.

Proof: Necessity: Let $U$ be a $\mu$-regular open set in $(Y, \mu_2)$. Then $Y-U$ is $\mu$-regular closed in $(Y, \mu_2)$. Since $f$ is an almost $\mu$-$\beta G\alpha$-continuous mapping, $f^{-1}(Y-U) = X - f^{-1}(U)$ is a $\mu$-$\beta G\alpha CS$ in $(X, \mu_1)$. Hence $f^{-1}(U)$ is a $\mu$-$\beta G\alpha OS$ in $(X, \mu_1)$.

Sufficiency: Let $V$ be any $\mu$-regular closed set in $(Y, \mu_2)$. Then $Y-V$ is $\mu$-regular open in $(Y, \mu_2)$. By hypothesis, $f^{-1}(Y-V) = X - f^{-1}(V)$ is $\alpha\mu$-$\beta G\alpha OS$ in $(X, \mu_1)$ which implies that $f^{-1}(V)$ is a $\mu$-$\beta G\alpha CS$ in $(X, \mu_1)$. Hence $f$ is an almost $\mu$-$\beta G\alpha$-continuous mapping.

Theorem 4.11: If $f: (X, \mu_1) \to (Y, \mu_2)$ is a $\mu$-continuous mapping and $g: (Y, \mu_2) \to (Z, \mu_3)$ is an almost $\mu$-continuous mapping then $g \circ f: (X, \mu_1) \to (Z, \mu_3)$ is an almost $\mu$-$\beta G\alpha$-continuous mapping.

Proof: Let $V$ be any $\mu$-regular closed set in $(Z, \mu_3)$. Since $g$ is an almost $\mu$-continuous mapping, $g^{-1}(V)$ is a $\mu$-closed set in $(Y, \mu_2)$. Since $f$ is $\mu$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a $\mu$-closed set in $(X, \mu_1)$. Since every $\mu$-closed set is $\alpha\mu$-$\beta G\alpha CS$, $(g \circ f)^{-1}(V)$ is a $\mu$-$\beta G\alpha CS$ in $(X, \mu_1)$. Therefore $g \circ f$ is an almost $\mu$-$\beta G\alpha$-continuous mapping.

Theorem 4.12: If $f: (X, \mu_1) \to (Y, \mu_2)$ is a $\mu$-$\alpha$-continuous mapping and $g: (Y, \mu_2) \to (Z, \mu_3)$ is an almost $\mu$-continuous mapping then $g \circ f: (X, \mu_1) \to (Z, \mu_3)$ is an almost $\mu$-$\beta G\alpha$-continuous mapping.

Proof: Let $V$ be any $\mu$-regular closed set in $(Z, \mu_3)$. Since $g$ is almost $\mu$-continuous, $g^{-1}(V)$ is $\mu$-closed in $(Y, \mu_2)$. Since $f$ is $\mu$-$\alpha$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\mu$-$\alpha$-closed in $(X, \mu_1)$. Since every $\mu$-$\alpha$-closed set is $\alpha\mu$-$\beta G\alpha CS$, $(g \circ f)^{-1}(V)$ is $\alpha\mu$-$\beta G\alpha CS$. Therefore $g \circ f$ is an almost $\mu$-$\beta G\alpha$-continuous mapping.

REFERENCE


