ANALYSING MOIFLPP USING HEPTAGONAL INTUTIONISTIC FUZZY NUMBER

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Abstract : In this paper, Heptagonal Intutionistic fuzzy number is introduced which deals with the membership and non membership function. This paper provides an algorithm for solving a multi-objective Intuitionistic fuzzy linear programming problem with data as HpIFNs. Here, the multi-objective Linear programming problem is converted into a single objective linear programming and the problem is defuzzified using HpIFNs. Then it is converted into equivalent crisp linear problems, and are solved using simplex method. A numerical example is provided to show the efficiency of the methodology.

Index Terms: **Heptagonal fuzzy numbers (HpFN), Heptagonal Intutionistic fuzzy numbers (HpIFN), Arithmetic operation, MOIFLPP**.

I. INTRODUCTION

In real time, situations are not crisp and they are hesitant in nature. This happens in situations like predicting the routine measure of communication systems, mathematical modeling for engineering problems, biological problems and computer systems. The parameters involved in such mathematical models is not exact and as a result it leads to qualms and therefore it is imperative to represent such qualms in parametric characterization which can be attained through fuzzy numbers. The fuzzy set theory was first introduced by Zadeh [9] in 1965 as an extension of the classical notion of set which deals with degree of membership of elements in a set. Operations on fuzzy numbers was discussed by Dubois and Parade [5] then Atanassov generalized and introduced intuitionistic concepts in fuzzy environment [1-3] which deals with membership, non– membership and indeterminacy. Arithmetic operations on Heptagonal Fuzzy Numbers was discussed by Mohammed Shapique[6] and Rathi K, Balamohan [7]. An ideal approach for obtaining the optimal compromise solution to a multiple objective linear programming problem was discussed in [4],[8],[10].

II. PRELIMINARIES

2.1.Heptagonal Intuitionstic Fuzzy Number [6]

A Heptagonal intuitionistic fuzzy number is specified by $\tilde{A}^{I}_{Hp} = \{(a_1, a_2, a_3, a_4, a_5, a_6, a_7), (a'_1, a'_2, a'_3, a'_4, a'_5, a'_6, a'_7)\}$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a'_1, a'_2, a'_3, a'_4, a'_5, a'_6, a'_7$ are real numbers such that $a'_1 \le a_1 \le a'_2 \le a_2 \le a'_3 \le a_3 \le a'_4 \le a_4 \le a'_5 \le a_5 \le a'_6 \le a_6 \le a'_7 \le a_7$ and its membership and non membership are given by

$$\mu_{\tilde{A}_{Hp}^{I}}(x)=\begin{cases}0 & \text{for } x<a_1\\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & \text{for } a_1\le x\le a_2\\ \frac{1}{2} & \text{for } a_2\le x\le a_3\\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & \text{for } a_3\le x\le a_4\\ \frac{1}{2}+\frac{1}{2}\left(\frac{a_5-x}{a_5-a_4}\right) & \text{for } a_4\le x\le a_5\\ \frac{1}{2}, & \text{for } a_5\le x\le a_6\\ \frac{1}{2}\left(\frac{a_7-x}{a_7-a_6}\right), & \text{for } a_6\le x\le a_7\\ 0, & \text{otherwise}\end{cases}$$

$$\gamma_{\tilde{A}_{Hp}^{I}}(x)=\begin{cases}1 & \text{for } x<a_1'\\ 1-\frac{1}{2}\left(\frac{x-a_1'}{a_2'-a_1'}\right), & \text{for } a_1'\le x\le a_2'\\ \frac{1}{2} & \text{for } a_2'\le x\le a_3'\\ \frac{1}{2}\left(\frac{x-a_3'}{a_4'-a_3'}\right), & \text{for } a_3'\le x\le a_4'\\ 0 & \text{for } x=a_4'\\ \frac{1}{2}\left(\frac{a_5'-x}{a_5'-a_4}\right) & \text{for } a_4'\le x\le a_5'\\ \frac{1}{2} & \text{for } a_5'\le x\le a_6'\\ 1-\frac{1}{2}\left(\frac{a_7'-x}{a_7'-a_6'}\right), & \text{for } a_6'\le x\le a_7'\\ 1, & \text{otherwise}\end{cases}$$

2.2 Alpha Cut In Heptagonal Intuitionistic Fuzzy Number

If $\tilde{A}_{Hp}^{I}=\{(a_1,a_2,a_3,a_4,a_5,a_6,a_7),(a_1',a_2',a_3',a_4',a_5',a_6',a_7')\}$ is a HpIFN, we will let $\tilde{A}_{Hp(\alpha)}^{I}=\{A_\alpha^-,A_\alpha^+\}$ where $(A_\alpha^-,A_\alpha^+)=(2\alpha(a_2-a_1)+a_1,\ a_7-2\alpha(a_7-a_6)),(\ a_3'+2\alpha(a_4'-a_3'),a_5'-2\alpha(a_5'-a_4'))$ be the closed interval which is a α-cut for $\tilde{A}_{Hp}^{I}$ in $0\le\alpha\le 0.5$ and $(A_\alpha^-,A_\alpha^+)=(\ 2(\alpha-1)(a_4-a_3)+a_4,\ a_4-2(\alpha-1)(a_5-a_4)),(2(1-\alpha)(a_2'-a_1')+a_1',a_7'-2(1-\alpha)(a_7'-a_6'))$ be the closed interval which is a α-cut for $\tilde{A}_{Hp}^{I}$ in $0.5\le\alpha\le 1$

2.3 Multi-Objective Intuitionistic Fuzzy Linear Programming Problem[4]

Let F (S) be the set of all Heptagonal Intutionistic fuzzy numbers.

The model

$$\max \tilde{z}_1,\tilde{z}_2,\ldots\ldots,\tilde{z}_k$$

Where $\tilde{z}_k=\sum_{j=1}^{n}\tilde{c}_j^{\,k}x_j,k=1,2,3,\ldots\ldots,k$

Subject to $\sum_{i=1}^{n}\tilde{a}_{ij}x_j\le\tilde{b}_i,i=1,2,3,\ldots\ldots,m,\quad x_j\ge 0,\ j=1,2,3,\ldots\ldots,n.$

where $\tilde{c}_j^k, \tilde{a}_{ij}, \tilde{b}_i \in$ F (S) and x_j are fuzzy variables is called multi-objective LPP

III. THEOREM

If $\tilde{A}_{Hp}^{I} = \{(a_1, a_2, a_3, a_4, a_5, a_6, a_7),(a_1', a_2', a_3', a_4', a_5', a_6', a_7')\}$ and

$\tilde{B}_{Hp}{}^{I} = \{(b_1, b_2, b_3, b_4, b_5, b_6, b_7),(b_1', b_2', b_3', b_4', b_5', b_6', b_7')\}$ are two heptagonal intutionistic fuzzy numbers then

$\tilde{A}_{Hp}{}^{I} + \tilde{B}_{Hp}{}^{I}$ is also HpIFN by Co – ordinate Method

Proof

We define $\tilde{A}_{Hp}{}^{I} + \tilde{B}_{Hp}{}^{I} = \tilde{C}_{Hp}{}^{I} = \phi(\tilde{A}, \tilde{B})$ and $\psi(\tilde{A}, \tilde{B})$

From the definition 2.2, Consider for $0 \le \alpha \le 0.5$

$$x_1 = 2\alpha(a_2 - a_1) + a_1,\ y_1 = 2\alpha(b_2 - b_1) + b_1 \qquad X_1 = 2(1-\alpha)(a_2' - a_1') + a_1', Y_1 = 2(1-\alpha)(b_2' - b_1') + b_1'$$
$$x_2 = a_7 - 2\alpha(a_7 - a_6), y_2 = b_7 - 2\alpha(b_7 - b_6) \qquad X_2 = a_7' - 2(1-\alpha)(a_7' - a_6'), Y_2 = b_7' - 2(1-\alpha)(b_7' - b_6')$$

Now we form the coordinate of the vertices $c_1 = (x_1, y_1), c_2 = (x_1, y_2), c_3 = (x_2, y_1), c_4 = (x_2, y_2),$

$$d_1 = (X_1, Y_1), d_2 = (X_1, Y_2), d_3 = (X_2, Y_1), d_4 = (X_2, Y_2),$$
$$\phi(c_1) = (x_1 + y_1), \phi(c_2) = (x_1 + y_2), \phi(c_3) = (x_2 + y_1), \phi(c_4) = (x_2 + y_2),$$
$$\psi(d_1) = (X_1 + Y_1), \psi(d_2) = (X_1 + Y_2), \psi(d_3) = (X_2 + Y_1), \psi(d_4) = (X_2 + Y_2),$$
$$u = [\min \phi(c_1), \phi(c_2), \phi(c_3), \phi(c_4), \max \phi(c_1), \phi(c_2), \phi(c_3), \phi(c_4)] \ = [\phi(c_1), \phi(c_4)],$$
$$u = [a_1 + b_1 + 2\alpha(a_2 + b_2 - a_1 - b_1), a_7 + b_7 - 2\alpha(a_7 + b_7 - a_6 - b_6)]$$

Collecting α we get $\mu^L{}_{\tilde{c}}(v) = \dfrac{u - (a_1 + b_1)}{2(a_2 + b_2 - a_1 - b_1)}$ $\qquad \mu^R{}_{\tilde{c}}(v) = \dfrac{(a_7 + b_7) - u}{2(a_7 + b_7 - a_6 - b_6)}$

$$v = [\min \psi(d_1), \psi(d_2), \psi(d_3), \psi(d_4), \max \psi(d_1), \psi(d_2), \psi(d_3), \psi(d_4)] \ = [\psi(d_1), \psi(d_4)],$$
$$v = [a_1' + b_1' + 2(1-\alpha)(a_2' + b_2' - a_1' - b_1'), a_7' + b_7' - 2(1-\alpha)(a_7' + b_7' - a_6' - b_6')]$$

Collecting α we get $\gamma_{\tilde{c}}{}^{L}(v) = 1 - \left[\dfrac{v - (a_1' + b_1')}{2(a_2' + b_2' - a_1' - b_1')}\right]$ $\gamma_{\tilde{c}}{}^{R}(v) = 1 - \left[\dfrac{(a_7' + b_7') - v}{2(a_7' + b_7' - a_6' - b_6')}\right]$

Similarly, from the definition 2.2 we get μ and γ for $0.5 \le \alpha \le 1$ as

$$\mu^L{}_{\tilde{c}}(v) = \frac{u - (a_4 + b_4)}{2(a_4 + b_4 - a_3 - b_3)} + 1 \qquad \mu^R{}_{\tilde{c}}(v) = \frac{(a_4 + b_4) - u}{2(a_5 + b_5 - a_4 - b_4)} + 1$$

$$\gamma_{\tilde{c}}{}^{L}(v) = \frac{v - (a_3' + b_3')}{2(a_4' + b_4' - a_3' - b_3')} \qquad \gamma_{\tilde{c}}{}^{R}(v) = \frac{(a_5' + b_5') - v}{2(a_5' + b_5' - a_4' - b_4')}$$

Hence the membership and non membership function obtained using co-ordinate method is also HpIFN.

IV. RANKING OF HEPTAGONAL INTUTIONISITC FUZZY NUMBER

The ranking of Heptagonal intuitionistic fuzzy number $\tilde{A}_{Hp}^{I} = \{(a_1, a_2, a_3, a_4, a_5, a_6, a_7),(a_1', a_2', a_3', a_4', a_5', a_6', a_7')\}$ maps the set of all fuzzy numbers to a set of a real numbers defined as

$$R(\tilde{A}_{Hp}^{I}) = Max[Mag_\mu(\tilde{A}_{Hp}^{I}), Mag_\gamma(\tilde{A}_{Hp}^{I})] \longrightarrow \quad (4.1)$$

where $Mag_\mu(\tilde{A}_{Hp}^{I}) = \left(\dfrac{3a_1 + 3a_2 + 11a_3 + 14a_4 + 11a_5 + 3a_6 + 3a_7}{12}\right)$ and

$$Mag_\gamma(\tilde{A}_{Hp}^{I}) = \left(\frac{3a_1' + 3'a_2 + 11a_3' + 14a_4' + 11a_5' + 3a_6' + 3a_7'}{12}\right).$$

V. Numerical Example

In a foundary, moulding department produces moulds in two different process conventional and automation with a profit of Rs. 3 / kg and Rs. 4 / kg in summer season and Rs.6 and Rs.8 in winter season . Production time of conventional is 20 moulds / hr and automation is 30 moulds / hr. To perform this process number of labours required is 10 and 6 respectively. Similarly the electricity usage in conventional method is 10 units / hr and in automation is 20 units / hr. The problem is formulated into FLPP and an optimum solution is obtained.

Solution:

Step 1: Since the profit from each process and the time availability are uncertain, the number of units to be produced on each process will also be uncertain. So we will model the problem as an intutionistic fuzzy linear programming problem and use HpIFNs for each uncertain value.

Profit for C1 which is close to 3 is modelled as {[0,0,0,1,3,5,7,9],[0,0,0,2,4,6,8]}. Similarly the other parameters are also modelled as HpIFNs taking into consideration the nature of the problem and other requirements. So a multi-objective intutionistic fuzzy LPP is formulated as.

$$\max z_1 = \tilde{3}x_1 + \tilde{4}x_2 \quad , \quad \max z_2 = \tilde{6}x_1 + \tilde{8}x_2$$

$$\text{Subject to ,} \quad \begin{matrix} \tilde{10}x_1 + \tilde{6}x_2 \le \tilde{20} \\ \tilde{10}x_1 + \tilde{20}x_2 \le \tilde{30} \end{matrix} \quad \text{and } x_1, x_2 \ge \tilde{0}$$

Where $C_1 = \tilde{3}$ ={[0,0,1,3,5,7,9], [0,0,0,2,4,6,8]}, $C_2 = \tilde{4}$ **=**{[0,0,2,4,6,8,10], [0,0,1,3,5,7,9]}

$P_1 = \tilde{6}$ ={[0,2,4,6,8,10,12], [0,1,3,5,7,9,11]}, $P_2 = \tilde{8}$ **=**{[2,4,6,8,10,12,14], [1,3,5,7,9,11,13]}

Similarly $a_{11} = \tilde{10}$, $a_{12} = \tilde{6}$, $a_{21} = \tilde{10}$, $a_{22} = \tilde{20}$, $b_1 = \tilde{20}$, $b_2 = \tilde{30}$ are also modelled as HpIFNs.

Step 2: Converting the multi-objective HpIFLP into a single objective HpIFLP .

By using Theorem 3 we get the membership value of $X_1 = C_1 + P_1$ is

$$\text{When } \alpha = 0,\ C_1 + P_1 = \begin{cases} \min(0,12,9,21)\max(0,12,9,21) \\ \min(1,9,9,17)\max(1,9,9,17) \end{cases} = \begin{cases} (0,21) \\ (1,17) \end{cases}$$

$$\alpha = 0.5,\ C_1 + P_1 = \begin{cases} \min(2,10,9,17)\max(2,10,9,17) \\ \min(5,9,9,13)\max(5,9,9,13) \end{cases} = \begin{cases} (2,17) \\ (5,13) \end{cases}$$

$$\alpha = 1,\ C_1 + P_1 = \begin{cases} \min(4,8,9,13)\max(4,8,9,13) \\ \min(9,9,9,9)\max(9,9,9,9) \end{cases} = \begin{cases} (4,13) \\ (9,9) \end{cases}$$

Similaraly the non membership value can be calculated

Therefore $X_1 = C_1 + P_1$ ={[0,2,5,9,13,17,21], [0,1,3,7,11,15,19]}

$X_2 = C_2 + P_2$ = {[2,4,6,8,12,16,20,24], [1,3,6,10,14,18,22]}

Step 3: Defuzzify the HpIFLP into crisp LPP using the above ranking function (4.1).

max z = max {mag[0,2,5,9,13,17,21], mag[0,1,3,7,11,15,19]} x_1+

max {mag[2,4,6,8,12,16,20,24] , mag[1,3,6,10,14,18,22]}

= max [37,29.75] x_1 + max [48.5, 41] x_2

Max z = 37 x_1 +48.5 x_2

Similaraly we can calculate Subject to the constraints as

$40x_1 + 24x_2 \le 80$

$40x_1 + 80x_2 \le 120$, where $x_1,\ x_2 \ge 0$

Step 4: Solving the above crisp LPP of Step 3 to get the optimum solution.

max $z = 92.79$, $x_1 = 1.57$ and $x_2 = 0.71$

On analysis of the results the foundry should produce 1.57 moulds in conventional and 0.71 moulds in automation to get the profit of Rs. 92.79. This results that conventional method is the best method.

VI. Conclusion

In this paper, Heptagonal Intutionistic fuzzy numbers has been introduced where both membership and non membership functions has been taken into consideration. This paper is useful to solve the multi-objective Linear programming problem. It also helps in defuzzification of HpIFNs. A numerical example is provided to show the efficiency of the methodology.

VIII. References

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