NUMERICAL INVESTIGATION OF HEAT TRANSFER OF NON-NEWTONIAN FLUID WITH VARIABLE VISCOSITY ALONG SYMMETRICAL POROUS WEDGE

Manju Agarwal¹, Vivek Joseph², Ramesh Yadav^{2*} & Parul Saxena¹

¹Department of Mathematics & Astronomy, University of Lucknow, Lucknow U.P ²Department of Mathematics, Babu Banarasi Das Northern India Institute of Technology Lucknow U.P

ABSTRACT:

This paper deals the study of the radiation effects on the boundary layer flow of and heat transfer of non-Newtonian fluid with variable viscosity along a symmetric porous wedge is presented here. The variable fluid viscosity is assumed to vary as a linear and non-linear function of temperature. The symmetry groups admitted by the corresponding boundary value problem are obtained by a special form of Lie group transformations, scaling group of transformations. A third order and a second order coupled ordinary differential equations are obtained. These equations are then solved numerically by MATLAB ode45 solver. In this paper we study the effects of various parameters like as Prandtl number Pr, A, I, Q on the flow of fluid velocity and the heat transfer of non-Newtonian fluid.

KEY WORDS: - Porous wedge, non-Newtonian fluid, Prandtl number Pr, Heat Transfer and Radiative heating parameter(Q).

NOMENCLATURE

Α	Fluid viscosity variation parameter	Pr	Prandtl number
C_p	Specific heat	Q	Radiative heating parameter
F	Non dimensional stream function	q_r	Radiative heat flux
<i>k</i> *	Absorption coefficient	T , T_w ,	T_{∞} Temperature of the fluid, wall, free
т	Falkner-Skan exponent	stream	
	Greek	symbols	
α,γ	Transformation parameters	ψ	Stream function
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α,γ	ransiormation parameters	ψ	Sitean function
λ	Similarity variable	σ	Stefan-Boltzmann constant
κ	Thermal conductivity	ρ	Density of the fluid
μ,μ*	Dynamic, reference viscosity	heta	Non dimensional temperature
ν	Reference kinematic viscosity	σ^{*}	Porous Parameter (Dimensionless)

1. INTRODUCTION

The study of hydrodynamic flow and heat transfer along a symmetrical wedge has gained considerable attention due to its vast applications in industry such as chemicals, cosmetics, pharmaceutical and its important bearings on several technological and natural processes. M. A. Hossain et al. (2000) have been study flow of viscous incompressible fluid with temperature dependent viscosity and thermal conductivity past a permeable wedge with uniform surface heat slux. They used the solution of differential equation by finite difference method and found that the effects of heat transfer by various parameter such as Prandtl number, Pressure gradient parameter, the viscosity variation parameter and thermal conductivity variation parameter. M. Kinyanjui et al. (2001) have studied Magnetohydordynamic free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with hall current and radiation Adsorption. M. B. Abd-el-Malek et al. (2002) have studied solution of the Rayleigh problem for a power law non-Newtonian conducting fluid via group method. S. Bagai (2003) has studied similarity solutions of free convection boundary layer flow over a body of arbitrary shape in a porous medium with internal heat generation. Kok Siong Chiem and Yong Zhao (2004) have studied the numerical study of steady / unsteady flow and heat transfer in porous media using a characteristics-based matrix-free implicit FV method on unstructured grids.

It is well known that the occurrence of flow separation has several undesirable effects is so far it leads to an increase in the undesirable effects in so far as it leads to an increase in the drag on a body immersed in the flow and adversely affects the heat transfer from the surface of the body. Several methods have been developed for the purpose of artificial control of flow separation. Radiative effects have important applications in physics and engineering. The radiator heat transfer effects on different flows are very important in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. D. D. Ganji and A. Rajabi (2006) have studied assessment of homotopy-perturbation and perturbation methods in heat radiation equations. They have compared their results with exact solution.

The unsteady mixed-convection boundary layer flows along a symmetric wedge with variable surface temperature have been studied by M. A. Hossain et al. (2006). The boundary layer structure of differentially heated cavity flow in a stably stratified porous medium has been analyised by P. G. Daniels (2007). The study of MHD flow and heat transfer for second grade fluid in a porous medium with modified Darcy's law has been presented by M. R. Mohyuddin (2007). The effect of mixed thermal boundary conditions and magnetic field on free convective flow about a cone in micro polar fluids has been investigated by M. M. Abdou and R. R. Gorla (2007).

T. Grosan and I. Pop (2007) have studied the thermal radiation effect on fully developed mixed convection flow in a vertical channel. Nasser S. Elgazery (2008) has studied transient analysis of heat and mass transfer by natural convection in power-law fluid past a vertical plate immersed in a porous medium (Numerical study). H. Zhang et al. (2008) have studied an analysis of the characteristics of the thermal boundary layer in power law fluid. S. Mukhopadhyay (2009) has studied the effect of radiation and variable fluid viscosity on flow and heat transfer along a symmetric wedge. Thermal radiation affect may play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent. High

temperature plasma, cooling and nuclear reactors, liquid metal fluids, power generation systems are some important applications of radiative heat transfer from a wall to conductive gray fluids. H. C. Suratiand and M. G. Timol (2010) have studied numerical study of forced convection wedge flow of some non-Newtonian fluids.

A. Postelnicu and I. Pop (2011) have studied Falkner-Skan boundary layer flow of a power-law fluid past a stretching wedge. F. M. Hady et al. (2011) have studied influence to yield stress on free convective boundary-layer flow of a non-Newtonian nano fluid past a vertical plate in a porous medium. W. Khan and A. Aziz (2011) have studied doublediffusive natural convective boundary layer flow in a porous medium saturated with a nanofluid over a vertical plate: Prescribed surface heat, solute and nano particle fluxes. K. V. Prasad et al. (2013) have studied momentum and heat transfer of a non-Newtonian Eyring-Powell fluid over a Non-isothermal stretching sheet. M. J. Uddin et al. (2013) have studied free convection of non-Newtonian nano-fluids in porous media with gyrotactic micro organisms. Manju Bisht and Anirudh Gupta (2014) have studied

An investigation of thermal boundary layer of non-Newtonian fluid past over a wedge has been analyzed by Manju Bisht and Anirudh Gupta (2014). A generalized Non-Newtonian fluid flow analysis of heat transfer in natural convection: A deductive group symmetry approach has been presented by R. M. Darji and M. G. Timol (2016).

The investigation has been found out for studying the flow of non-Newtonian fluid and effect of radiation of variable fluid viscosity and heat transfer along a symmetrical porous wedge. Using similarity variable and similarity solution, a third order and a second order coupled ordinary differential equation system corresponding to the momentum and the energy equations are derived. These equations are solved numerically using MATLAB software ode 45 solver. The effect of the temperature-dependent fluid viscosity parameter, radiation parameter and the influence of Prandtl number on temperature fields on the flow of fluid has been investigated and analyzed with graphically.

2. MATHEMATICAL FORMULATION

Let us assume the steady flow, two dimensional, laminar boundary-layer flow of viscous incompressible non-Newtonian past a symmetrical sharp porous wedge with velocity given by $\bar{u}_e(\bar{x}) = U_{\infty} \left(\frac{\bar{x}}{L}\right)^m$ for $m \le 1$ where L is the characteristic length and m is the velocity exponent related to the included angle $\pi\beta$ by $m = \frac{\beta}{2-\beta}$. For m < 0, the solution becomes singular at $\bar{x} = 0$, while for $m \ge 0$, the solution can be defined for all values of \bar{x} . The governing equations of such type of flow are, in the usual notations.

$$\frac{\partial u}{\partial \bar{x}} + \frac{\partial v}{\partial \bar{y}} = 0,\tag{1}$$

$$\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} = \bar{u}_e\frac{\partial\bar{u}_e}{\partial\bar{x}} + \frac{1}{\rho}\frac{\partial\mu}{\partial T}\frac{\partial T}{\partial\bar{y}}\frac{\partial\bar{u}}{\partial\bar{y}} + \frac{\mu}{\rho}\frac{\partial}{\partial\bar{y}}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)^l - \frac{\mu'}{\rho\,k}\,\bar{u} \quad , \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial \bar{y}^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial \bar{y}} , \qquad (3)$$

where μ is dynamic viscosity. The viscous dissipation term in the energy equation is neglected. Here \bar{u} and \bar{v} are the components of velocity respectively in the \bar{x} and \bar{y} directions.

Using the Rosseland approximation for radiation (Brewster 1972) we can write

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial \bar{y}} \quad . \tag{4}$$

Consider the temperature difference within the flow is such that T^4 expanded in a Taylor series about T_{∞} and neglecting higher orders term, we get $T^4 \equiv 4T_{\infty}^3 T - 3T_{\infty}^4$. Therefore, the equation (3) becomes

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial \bar{y}^2} + \frac{16}{3\rho C_p k^*}\frac{\partial^2 T}{\partial \bar{y}^2} .$$
(5)

The appropriate boundary conditions for the problem are given by

$$u = 0, v = 0, T = T_w \text{ at } y = 0,$$
 (6)

$$\bar{u} \to \bar{u}_e(\bar{x}), \ T \to T_\infty \ as \ \bar{y} \to \infty \ .$$

$$\tag{7}$$

Introducing

$$x = \frac{\bar{x}}{L}, \qquad y = Re_L^{\frac{1}{2}} \frac{\bar{y}}{L}, \qquad u = \frac{\bar{u}}{U_{\infty}}, \qquad (8)$$

$$v = Re_L^{\frac{1}{2}} \frac{\bar{v}}{U_{\infty}}, \quad u_e = \frac{\bar{u}_e}{U_{\infty}}, \quad Re_L = \frac{U_{\infty}L}{v}.$$
(10)

Putting these values in equations (1), (2) and (5), we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$
(11)
and

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e \frac{\partial u_e}{\partial x} + \frac{\partial \mu}{\partial T} \frac{\partial T}{\partial y} \frac{\partial u}{\partial y} + \mu l \left(\frac{U_{\infty} Re^{\frac{1}{2}}}{L} \right)^{l-1} \left(\frac{\partial u}{\partial y} \right)^{l-1} \frac{\partial^2 u}{\partial y^2} - \left(\frac{\mu'}{\rho k} \cdot \frac{L}{U_{\infty}} \right) u,$$
(12)
Or

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e \frac{\partial u_e}{\partial x} + \frac{\partial \mu}{\partial T} \frac{\partial T}{\partial y} \frac{\partial u}{\partial y} + \mu l \chi^{l-1} \left(\frac{\partial u}{\partial y}\right)^{l-1} \frac{\partial^2 u}{\partial y^2} - \sigma^{*2} u , \qquad (13)$$
where $\sigma^{*2} = \left(\frac{\mu'}{\rho k}, \frac{L}{U_{\infty}}\right) \& \chi = \frac{U_{\infty} Re^{\frac{1}{2}}}{L}$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{v \rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma}{3v \rho C_p k^*} \frac{\partial^2 T}{\partial y^2} , \qquad (14)$$
where

$$v = \frac{\mu^*}{\rho} , \qquad (15)$$

here μ^* is the constant value of the coefficient of viscosity for away from the surface. The boundary conditions for equation (6) and (7) now become

$$u = 0, v = 0, T = T_w \text{ at } y = 0,$$
 (16)

$$\bar{u} \to \bar{u}_e(x), \ T \to T_\infty \ as \ y \to \infty$$
 (17)

The velocity of the fluid over the porous wedge is now given by $u_e(x) = x^m$, for $m \le 1$.

Now we introduce the following relations for u, v and θ as

$$u = \frac{\partial \psi}{\partial y}$$
, $v = -\frac{\partial \psi}{\partial x}$ and $\theta = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}$. (18)

We use the temperature dependent fluid viscosity given by Mukhopadhyay et al. [20].

$$\mu = [a + b(T_w - T) = [a + A(1 - \theta)], \tag{19}$$

where *a*, *b* are constants and b > 0, $A = b(T_w - T_\infty)$

Now using equation (18) and (19) in the boundary layer problem equation (13) and in the energy equation (14), we get the following equations

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = u_e \frac{\partial u_e}{\partial x} + A \frac{\partial \theta}{\partial y} \frac{\partial^2 \psi}{\partial y^2} + \left[a + A(1 - \theta)\right] \chi^{l-1} \frac{\partial}{\partial y} \left(\frac{\partial^2 \psi}{\partial y^2}\right)^l - \sigma^{*2} \frac{\partial \psi}{\partial y} ,$$
Or

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = u_e\frac{\partial u_e}{\partial x} + A\frac{\partial\theta}{\partial y}\frac{\partial^2\psi}{\partial y^2} + \left[a + A(1-\theta)\right]\chi^{l-1}\frac{\partial}{\partial y}\left(\frac{\partial^2\psi}{\partial y^2}\right)^l - \sigma^{*2}\frac{\partial\psi}{\partial y},\tag{20}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \left(\frac{k}{v\,\rho C_p} + \frac{16\sigma}{3v\,\rho C_p k^*}\right)\frac{\partial^2 \theta}{\partial y^2} , \qquad (21)$$

where $\chi = \frac{U_{\infty} R e^{\frac{1}{2}}}{L}$, $A = b(T_w - T_{\infty})$, the boundary conditions equation (16), (17) reduced

$$\frac{\partial \psi}{\partial y} = 0, \qquad \frac{\partial \psi}{\partial x} = 0, \qquad \theta = 1 \quad at \quad y = 0.$$
 (22)

$$\frac{\partial \psi}{\partial y} \to u_e(x) = x^m$$
, $\theta \to 0$ as $y \to \infty$. (23)

We introduce the following relations

$$\psi(x,y) = x^{\alpha}F(\lambda), \qquad \theta(x,y) = G(\lambda), \qquad \lambda = \frac{y}{x^{\gamma}},$$
(24)

in the momentum and energy equations. Then momentum and energy equations give $\alpha = 1 - \gamma$ and the momentum equation also gives $\alpha - 3\gamma = 2m - 1$, the solution of which is $\alpha = \frac{1+m}{2}$, $\gamma = \frac{1-m}{2}$ and the resulting governing equations then becomes

$$m F'^{2} - \frac{m+1}{2} F F'' = m - AG' F'' + [(a + A) - G] \chi^{l-1} l x^{\frac{(3m-1)(l-1)}{2}} (F'')^{(l-1)} F''' - \sigma^{*2} F', \quad (25)$$

$$(3 + 4Q)G'' + \frac{3}{2}(m + 1) \Pr F G' + 3 \gamma \lambda \Pr F' G' = 0, \quad (26)$$
where $Pr = \mu^{*}C_{p}/k$ is the Prandtl number, $Q = 4\sigma T_{\infty}^{3}/kk^{*}$ is the radiative heating parameter.
The boundary conditions take the following form
$$F' = 0, F = 0, G = 1 \text{ at } \lambda = 0 \quad , \quad (27)$$
and $F' \to 1, G \to 0$ as $\lambda \to \infty$. (28)
Using the boundary condition (27) in equation (26), we get

Using the boundary condition (27) in equation (26), we get

$$(3+4Q)G'' + \frac{3}{2}(m+1)\Pr F G' = 0.$$
⁽²⁹⁾

Here the above differential equation (25), σ^2 is known as permeability of porous wedge, $Pr = \mu^* C_p / k$ is the Prandtl number, $Q = 4\sigma T_{\infty}^3 / kk^*$ is the radiative heating parameter.

Solving the above equation (29), using the boundary conditions (27) and (28), we get

$$G = e^{-\frac{3}{2}\frac{(m+1)}{(3+4Q)}} \Pr F \lambda$$
(30)

Now solving the above differential equation (26) & (29) numerically by using ode45 solver in MATLAB software.

3. METHOD OF SOLUTION

In this paper we have solved the above differential equations (34) & (35) numerically using MATLAB software. We have used ode45 solver for solving the set of differential equations with described boundary conditions. For the purpose the time interval (0, 6) & (0, 10) with initial condition vector (0, 0, 1) has been taken for convergence criteria. The option has been chosen ('ReITol', 1e-4, 'AbsTol', [1e-4 1e-4 1e-5]). The different set of parameter has been chosen to investigate the results. The range of dimensionless variable λ ($0 \le \lambda \le 6$) & $0 \le \lambda \le 10$), the value Radiative heating parameter (Q) {0.2, 0.3, 0.5, 1.0, 1.5, 2.0 & 0, 10, 20, 30}, Prandtl number Pr {1, 2, 3, 4, 5}, porous coefficient σ^* has been taken {0.2, 0.4, 0.6, 0.8, 1.0} Falkner-Skan exponent parameter *m* has been taken {1/9, 1/3, 5/9, 7/9, 1 & 1/9, 2/9, 1/3, 5/9, 7/9, 1.0, 1.5}, temperature-dependent viscosity parameter *A* {5, 10, 20, 30}, Porous law index *l* has been taken {2, 3, 4, 5}, etc.,. The various graphs have been plotted with described set of parameters and discussed in detail in the next section.

4. RESULTS AND DISCUSSION

In order to numerically investigate the method has been carried out for various values of the temperature-dependent viscosity parameter (A), Falkner-Skan exponent (m), Radiative heating parameter (Q), Prandtl Number Pr and Power law index parameter l. For illustrations of the results, numerical values are plotted in the below figures. The temperature profile are given in the figure 1 - 5, for prescribed values of m, Pr, Q and σ^* , it is observed that the temperature profile decreases with increase of Prandtl number Pr, m and σ^* , whereas is it increases with increase of Q. From these Figures 6 – 13 is graph for Newtonian fluid, from these figure it is observed that velocity profile of fluid increases with increase of σ^* , and it decreases with increase of A. Velocity profile $F'(\lambda)$ increases in $-0.09 \le m \le 0.00$ and velocity profile decreases with increase of $m \ge 0.0$, whereas the velocity of fluid increases with increase of Q. The graph 14 – 19 are given for non-Newtonian fluid for l > 1. The velocity profile $F(\lambda)$ of fluid increases sharply

with increase of l (2, 3, 4, and 5) whereas $F'(\lambda)$ decreases with increase of l (2, 3, 4, 5). The velocity of fluid $F(\lambda)$ increases with increase of m. It is also observed that $F(\lambda)$ and $F'(\lambda)$ decreases with increase of A and increases with increases of σ^* . The graph 19 is the graph of $F(\lambda)$, $F'(\lambda)$, $F''(\lambda)$ it is observed that axial velocity of fluid sharply increases whereas radial velocity of fluid decreases as compared to axial velocity.







Fig 3: Variation of Temperature $G(\lambda)$ with λ for Q = 0.5, Pr = 5, $\sigma^* = 0.5$, A=10.



Fig 4: Variation of Temperature $G(\lambda)$ with λ for Q = 0.5, Pr = 5, $\sigma^* = 0.5$, A=10.



Fig 5: Variation of Temperature Profile G'(λ) with λ for several values of σ^* with Pr = 0.5, a = 1, l = 4, Q = 2, m = 0.5, K_1 = 0.8, A = 10.



Fig 6: Variation of velocity $F(\lambda)$ with λ for several value of σ^* with A = 0, Pr = 0.5, a = 1, Q = 2, m = 0.5.







Fig 8: Variation of velocity $F(\lambda)$ with λ for several value of A with $\sigma^* = 0.5$, Pr = 0.5, a = 1, Q = 2, m = 0.5.



Fig 9: Variation of velocity $F'(\lambda)$ with λ for several value of A with $\sigma^* = 0.0$, Pr = 0.5, a = 1, Q = 2, m = 0.5.



Fig 10: Variation of velocity $F'(\lambda)$ with λ for several value of m with $\sigma^* = 0.0$, Pr = 0.5, a = 1, Q = 2, A = 0.0.



Fig 12: Variation of velocity $F'(\lambda)$ with λ for several value of m with $\sigma^* = 0.0$, Pr = 0.5, a = 1, Q = 2, A = 10.



Fig 13: Variation of velocity $F'(\lambda)$ with λ for several value of Q with $\sigma^* = 0.0$, Pr = 0.5, a = 1, m = 0.5, A = 2.



Fig 14: Variation of velocity $F(\lambda)$ with λ for several value of l with $\sigma^* = 0.5$, Pr = 0.5, a = 1, m = 0.5, Q = 2, A = 2, $K_1 = 0.8$.



Fig 15: Variation of velocity $F'(\lambda)$ with λ for several value of l with $\sigma^* = 0.5$, Pr = 0.5, a = 1, m = 0.5, Q = 2, A = 2, $K_1 = 0.8$.







Fig 17: Variation of velocity $F(\lambda)$ with λ for several values of A with $\sigma^* = 0.5$, Pr = 0.5, a = 1, l = 4, $Q = 2, m = 0.5, K_1 = 0.8$.



Fig 18: Variation of velocity $F(\lambda)$ with λ for several values of σ^* with Pr = 0.5, a = 1, l = 4, $Q = 2, m = 0.5, K_1 = 0.8, A = 10$.



Fig 19: Variation of velocity $F(\lambda)$, $F'(\lambda) \& F'(\lambda)$ with λ for $\sigma^* = 0.5$, Pr = 0.5, a = 1, l = 4, m = 0.5, Q = 2, A = 2, K_1 = 0.8.

.5. CONCLUSIONS

In this present study gives numerical investigation of the effect of radiation of variable fluid viscosity and heat transfer of non-Newtonian fluid along a symmetrical porous wedge. From these results we found the temperature profile of non-Newtonian fluid decreases with increase of Prandtl number Pr, m and Porous parameter σ^* and reciprocal effects with increase of Q. This type of problem has several technological applications. The result of the fluid flow of problems can be applied to industry and its important bearing. Since the fluid has been considered non-Newtonian so it has so many biomedical applications also. One of the important applications of this problem is linked in engineering and post accidental heat removal. The results of this analysis have been found on focusing on porosity factor and non-Newtonian JCR fluid parameter.

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