Assessment Of Wheel–Rail Contact Behavior Under Varying Profile Geometries Using FEM And Hertzian Contact Theory

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Abstract: The operation of railway transportation relies on the wheel–rail interaction, which provides both support and guidance for railway vehicles, ensuring safe and smooth movement. When loads are transmitted through the wheels, contact stresses develop at the wheel–rail interface. These stresses can cause wear, deformation, and damage to both components, leading to increased maintenance and replacement costs. The primary objective of this study is to evaluate the stresses generated due to variations in contact geometry resulting from changes in wheel and rail profile characteristics, including rail profile radius, wheel profile radius, and wheel taper. To investigate the influence of profile geometry, seven different profile radius values were selected for the R7T wheel and UIC60 rail, using material properties specified by European standards. Two analytical approaches were employed to determine contact stresses and maximum contact pressure: the Hertzian Contact Theory (HCT) and the Finite Element Method (FEM). A three-dimensional finite element model of the wheel–rail system was developed, and loading conditions were applied to evaluate stress distribution within the contact region. The results indicate that for a wheel radius of 330 mm and a rail radius of 300 mm, the equivalent stress values were almost closer. Likewise, for wheel and rail radii of 360 mm and 300 mm, the equivalent stress values were approximately nearer to one another. The findings of this research provide valuable insights into wheel–rail contact behaviour and may contribute to the improved design and optimization of railway systems.

Keywords: Wheel-rail Profile geometry, Contact pressure and Fatigue life.

1. Introduction

The wheel–rail contact problem has attracted significant attention due to its direct influence on railway safety, ride quality, and maintenance costs. Numerous studies have investigated contact stress distribution, contact pressure, wear mechanisms, and dynamic wheel–rail interactions using analytical, numerical, and experimental approaches. Monfared [1] analyzed contact stresses in rolling bodies using the Finite Element Method (FEM) and demonstrated that the assumption of an elliptical contact patch provides results that are in closer agreement with analytical solutions than circular or rectangular contact assumptions. Satish et al. [2] examined the effect of material properties on wheel–rail contact behavior and reported that stress and strain distributions are strongly influenced by the Young's modulus-to-yield strength ratio (E/Y). Tamas et al. [3] investigated frictional wheel–rail contact under stationary, sticking, and sliding conditions using FEM and evaluated contact pressure, contact area, and equivalent von Mises stress distributions. Several researchers have focused on the influence of wheel and rail geometries on contact mechanics. Srivastava et al. [4] employed Hertzian theory and FEM to study the effect of wheel and rail profile topology on contact stresses and pressure distributions. Sharma et al. [5] extended this work by incorporating quasi-Hertzian formulations and Kalker's theory to evaluate the influence of lateral wheel movement on contact zones and frictional behavior. Similarly, Sladkowski and Sitarz [6] analyzed wheel–rail interactions using finite element simulations and quasi-Hertzian approaches, providing insights into stress distributions for various wheel and rail profiles. Dynamic wheel–rail interactions have also been extensively studied. Sowndarya and Ratna Kiran [7] developed a dynamic FEM model to evaluate contact forces, stresses, strains, and pressures under realistic loading conditions.

Natsumi and Yoshiaki [8] proposed a three-dimensional wheel–rail contact model integrated with multibody train dynamics to assess vehicle safety under strong crosswind conditions. Their results highlighted the importance of wheel unloading ratios and vehicle vibration characteristics in high-speed railway operations. Comprehensive reviews of wheel–rail contact theories have been presented by Sajjad et al. [9], covering Hertzian and non-Hertzian contact models, as well as tangential contact formulations such as Kalker's theory, FASTSIM, CONTACT, and Polach's model. These studies emphasized the assumptions and limitations associated with different modeling approaches. Qian et al. [10] developed a three-dimensional elastic–plastic finite element model to investigate rolling contact fatigue (RCF) in high-speed railway wheels, considering realistic wheel and rail geometries and transient contact conditions. The effects of wheel–rail contact parameters on vehicle stability and critical speed have been explored by Yeon and Bum [11], who demonstrated the significance of creep force characteristics and suspension parameters in determining vehicle stability. Polach and Nicklisch [12] investigated the relationship between wheel profile wear, equivalent conicity, and dynamic vehicle behavior, showing that profile degradation significantly affects wheel–rail interaction characteristics. Ashofteh and Mohammaddnia [13] performed elastic–plastic stress analysis of wheel–rail contact using ABAQUS and reported that dynamic loading conditions can induce stresses exceeding the elastic limit of wheel materials.

Although substantial research has been conducted on wheel–rail contact mechanics, limited studies have systematically investigated the combined influence of wheel profile radius, rail profile radius, and wheel taper on contact stress distributions using both Hertzian Contact Theory (HCT) and three-dimensional finite element analysis. Therefore, the present study aims to evaluate the effect of profile geometry variations on wheel–rail contact stresses and contact pressures by comparing analytical and numerical approaches, thereby providing useful insights for the design and optimization of railway wheel–rail systems.

Based on the literature reviewed, it is observed that no previous study has investigated the contact stress behaviour of an R7T wheel material in conjunction with a UIC60 rail material while considering variations in wheel profile radius, rail profile radius, and wheel taper profile. This gap in the existing body of knowledge forms the basis of the present investigation. Srivastava et al. [6] conducted a contact stress analysis using a wheel radius of 1098 mm, with both the wheel and rail composed of UIC60 material. In contrast, the present study employs a wheel radius of 460 mm, with R7T as the wheel material and UIC60 as the rail material. The contact stress analysis is performed using both Hertzian contact theory and Finite Element Analysis (FEA) to evaluate the wheel–rail contact behaviour under the specified conditions.

2. Stress Analysis of Wheel-rail Contact: Hertz developed his influential work on contact mechanics, to calculate the contact area and the normal pressure distribution between the contact bodies. Hertz developed his influential work on contact mechanics, which calculated the contact area and the normal pressure distribution carried by it. The K. L. Johnson, further uses HCT to develop the contacts between two solids. He proves that when two non-conforming solids are brought into contact they touch initially at a single point or along a line. When the load is applied at the point of the first contact they deform and touch each other over a finite area which is small compared with the dimension of the two bodies. According to Bharat Bhushan Figure 1 shows the pressure distribution and contact area when two spheres are in contact due to applied compressive vertical load in both directions.

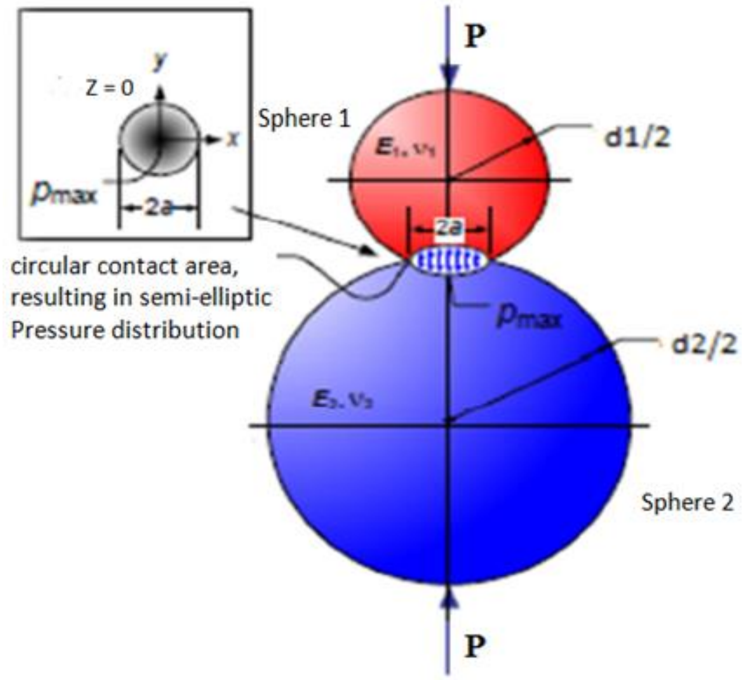


Figure 1. Two spheres held in contact by means of the compressive force, P.

Figure 2. illustrates two bodies in touch at the first point of contact. A coordinate system is defined at the first point of contact with its origin and vertical z-axis pointing upwards while lateral y-axis and longitudinal x-axis are within the tangent plane where the contact area is formed after loading.

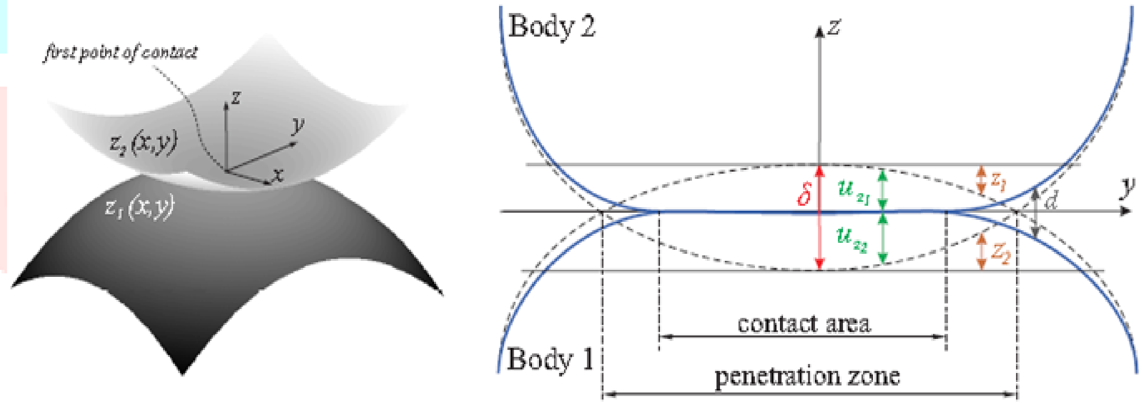


Figure 2. Coordinate system during contact (b) Hertzian contact terminologies.

Where: x -is the coordinate in the rolling direction

y -is the coordinate in the lateral direction, and

z -is the coordinate in the direction normal to the plane of contact, pointing into the wheel

Hence, the Hertzian Contact Theory (HCT) can be applied for modeling the contact between wheel and rail contact mechanics based on the following key assumptions:

- The kinematic equations and the materials' elastic properties are linear.
- Wheel and rail material exhibits completely elastic behavior (no plastic deformation).
- Wheel and rail material is homogeneous and its properties are isotropic.
- The contact surfaces are perfectly smooth and there is no friction between them.
- Contact surfaces curvature is constant within the contact patch.
- Contact surfaces radii of curvature are much larger than the dimensions of the contact patch.

The HCT assumption leads to an elliptical contact area and a semi-ellipsoid contact pressure distribution in the contact region.

Based on these assumptions, Hertz proposed the solution for the determination of elliptical contact area and pressure distribution between two bodies in contact. The distance between each contacting surface is given by:

$$z_1 = A_1x^2 + B_1y^2 \quad (1)$$

$$z_2 = A_2x^2 + B_2y^2 \quad (2)$$

Where: x and y are the coordinates of the point inside the contact area in the XY-plane.

The distance between the undeformed surfaces (before loading), $z = |z_1| + |z_2|$, is called separation.

$$z(x, y) = Ax^2 + By^2$$

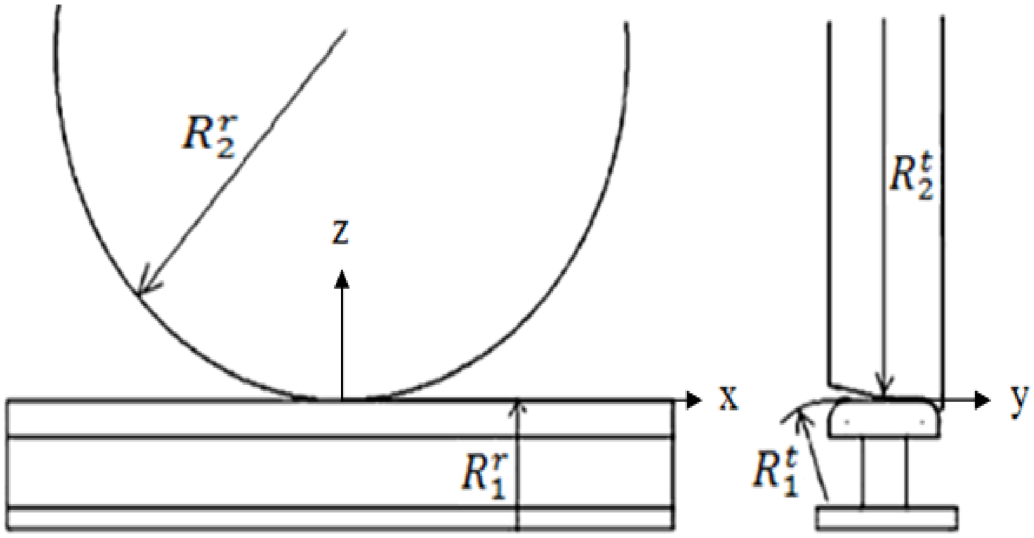


Figure 3. Wheel–rail configuration showing different principal relative radii of curvature.

Where,

R_1^r - The radius of the rail along rolling which is infinity in this case

R_1^t - The transverse radius of curvature of the rail profile, in the plane of cross section

R_2^r - The rolling radius of curvature of the wheel.

R_2^t -The transverse radius of curvature of the wheel profile in the plane of cross section which goes to infinity for a conical wheel, and

ψ - conicity (wheel taper angle)

S1002 Wheel Contact Phase Profile (Source: PR NF EN 13715, European Standard, 2015) Generally, depending on the above standards we have the following properties for wheel-rail materials. Contact load, F = 83000 N Young's modulus of elasticity E1 = 210 GPa for rail Young's modulus of elasticity E2 = 200 Gpa for wheel Poisson's ratio ν1 = ν2 = 0.3 for both rail and wheel Ultimate tensile strength σut = 960 MPa for rail and σut = 880 MPa for wheel 2Yield strength σy = 0.5σut for both wheel and rail Rolling radius curvature of wheel, $R2\ r$ = 460 mm The maximum operational speed of the train = 80 Km/hr Axle load is <17 , The mass of the wheel = 412 kg, The total weight of the wheel = 83 KN.

The K. L. Johnson, further used Hertzian Contact Theory (HCT) to develop the contacts between two solids. He proved that when two elastic non-conforming solids are brought into contact the HCT can be applied for modeling the contact between wheel and rail assuming elliptical shape for the contact area and a semi ellipsoid contact pressure distribution in the contact region. The distance between each contacting surface is given by: $Z_1=A_1x_2+B1y_2$ …(1), $Z_2=A_2x_2+B_2y_2$… (2) As the contacting surfaces of the bodies before loading are represented by Z_1(x, y) and Z_2 (x, y) for the first and the second body, respectively. The total distance between the undeformed surfaces (before loading), $Z=|Z_1|+|Z_2|$, is given as: $Z(x,y)=Ax_2+By_2$… (3) From wheel-rail configuration the radii curvatures are related as; $A+B=1/2\ (1/R1\ r+1/R1\ t+1/R2\ r+1\ R2\ t)$… (4)

$|B-A|=1/2\ [(1/R1\ r-1/R1\ t)2+(1/R2\ r-1/R2\ t)2+2(1/R1\ r-1/R1\ t)(1/R2\ r-1/R2\ t)$ $\cos 2\psi]$ 1 2⁄ ….(5) Where A and B are geometrical constants, $R1\ r$, $R1\ t$, $R2\ r$ and $R2\ t$ are radii of curvatures of the profile as shown in figure 3, and ψ is the angle between the normal planes of the radii of curvatures. The constants A and B depend on the magnitudes of the principal curvatures of the surfaces in contact and on the angle between the planes of principal curvatures of the two surfaces 'θ'. This angle is given by: $\cos\theta=|B-A|\ A+B$.

3.Finite Element Analysis:

The assembled three-dimensional analysis of wheel-rail contact model aims to study the contact stress and fatigue life for different geometries. The finite element analysis software 'ANSYS' has been used to carry out this analysis, in a symmetric condition. ANSYS mechanical is a finite element analysis tool for structural analysis, including linear, non-linear and dynamic studies. This computer simulation product provides finite elements to model behavior and supports material models and equation solvers for a wide range of mechanical design problems.

ANSYS structural software is the unique concerns of pure structural simulations without the need of extra tools. In this thesis, FEA of contact stress for two rolling bodies is studied by finite element method. Hence, the wheel and rail contact problem can be formulated as a rolling contact problem between two non-linear profiles in the presence of friction.

The upper surface of rail is selected as target surface and the curved surface of the wheel is selected as a contact surface. The nodes lying on the axis of the wheel and rail are restricted to move in the radial and vertical direction. Also, the bottom of the rail and two opposite side faces are fixed in all the directions. According to Roya Sadat Ashofteh and Ali Mohammadnia [10], mostly the distance between two bogies that supports the rail from the bottom is 600 mm. Considering two bogies in this thesis rail is modeled with restriction to an overall length of 1200 mm, due to this fixed supports are taken at the bottom and on both sides of the rail.

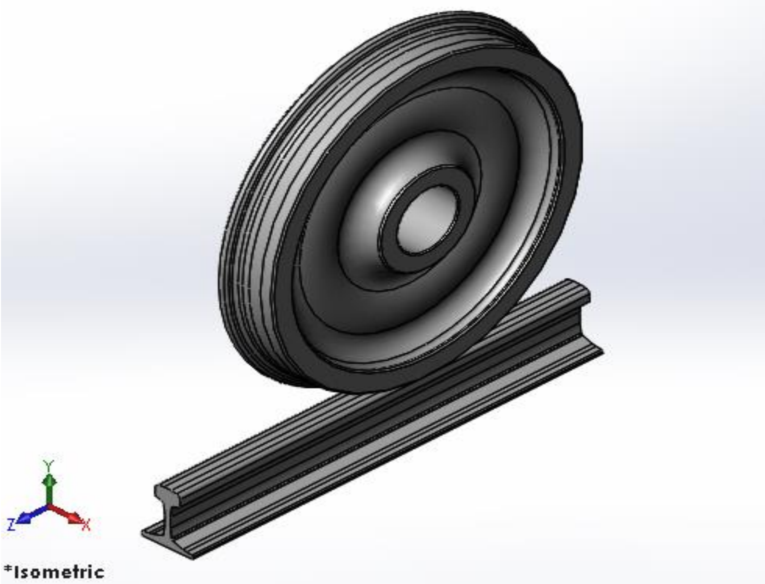


Figure 4. Assembly of wheel and rail using solid work

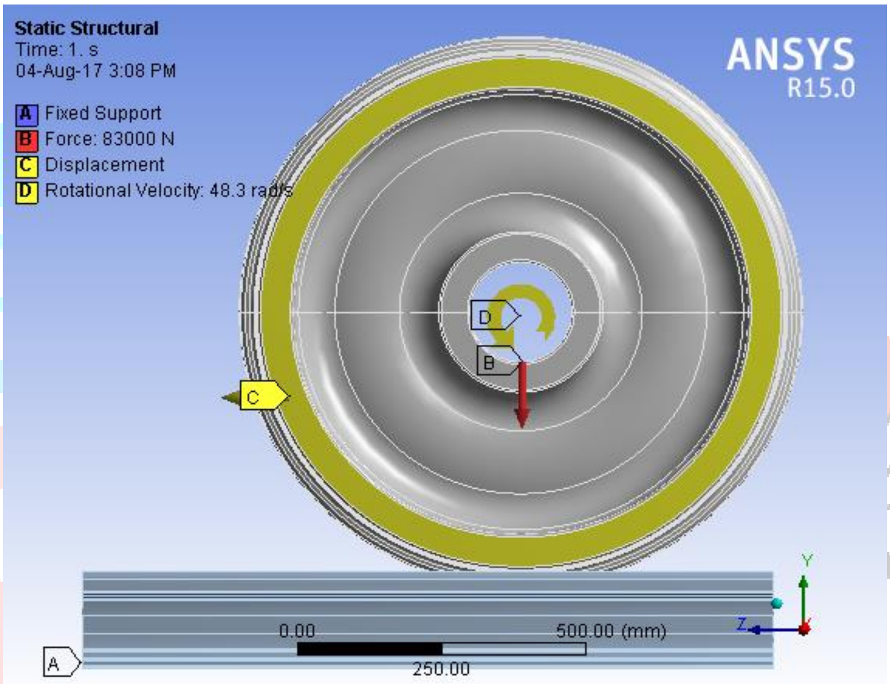


Figure 5. Loading condition

4. Results and Discussion:

The analyses are carried out by using Hertz contact theory and FEA. The analyses include the effect of change of geometrical parameters such as wheel taper profile and radii of curvature of both wheel and rail to determine the maximum contact pressure, equivalent stress and number of cycles to failure during the wheel slides over the rail.

4.1 Results

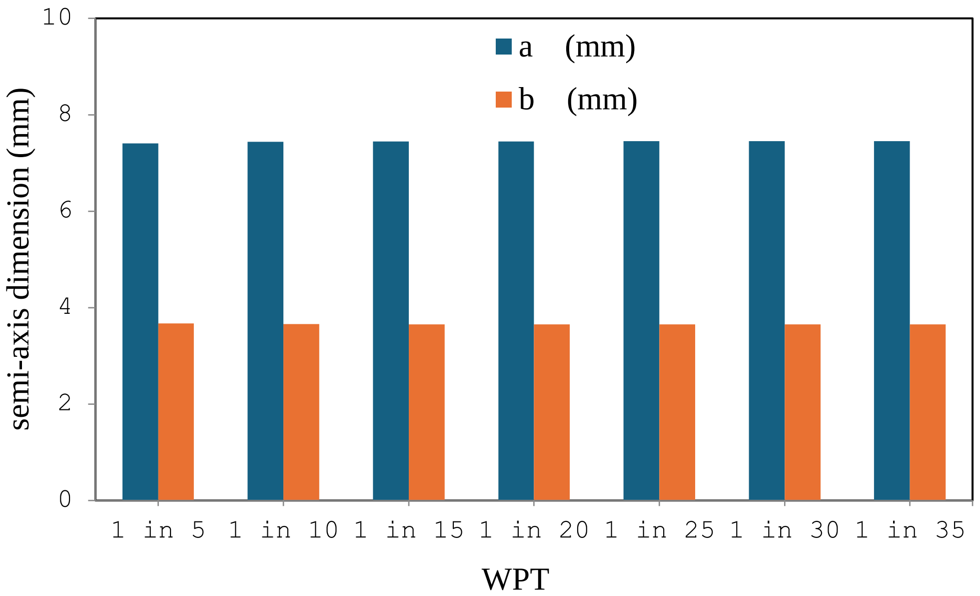
4.1.1 Hertzian Contact Theory Results

The effect of geometrical parameters (WPT, wheel profile radii, and rail profile radii) for different contact parameters like semi-major axis 'a', semi-minor axis 'b', maximum contact pressure 'p_o' and von Mises stress 'σ´' are calculated.

a) Effect of variation in wheel profile taper (WPT) on contact parameter

By varying the values of WPT from 1 in 5 to 1in 35 and making the other parameters constant, i.e. $R_1^r = \infty$, $R_1^t = 300mm$, $R_2^r = 460mm$ and $R_2^t = 330mm$.

For example: taking WPT, $\psi = 1 in\ 5 = 11.3099°$



6. Effect of wheel profile taper (WPT) on the dimensions ‘a’ and ‘b’

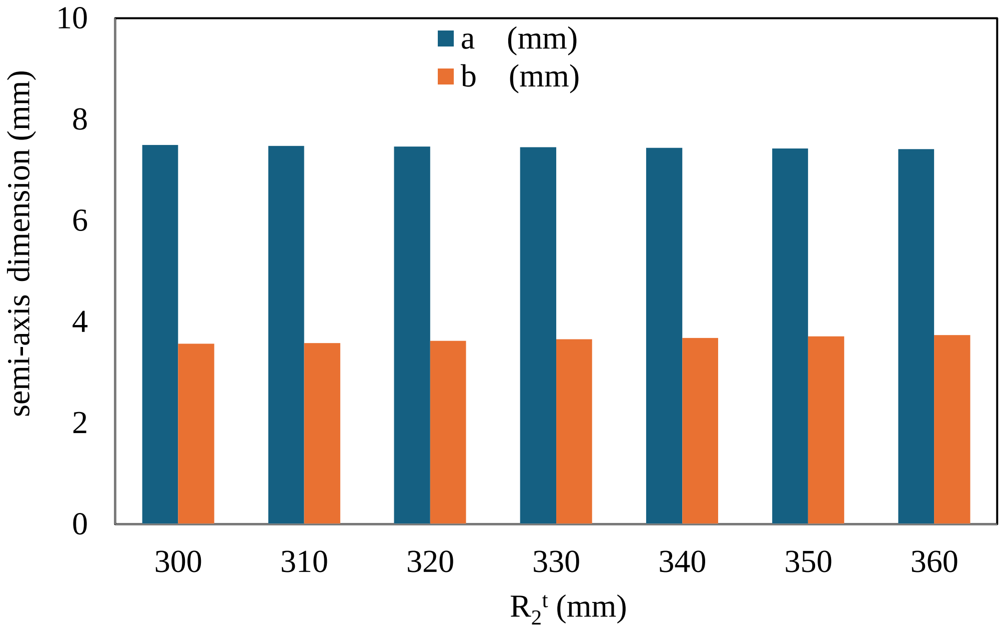


Figure 7. Effect of rail profile radii (R_1^t) on the dimensions ‘a’ and ‘b’

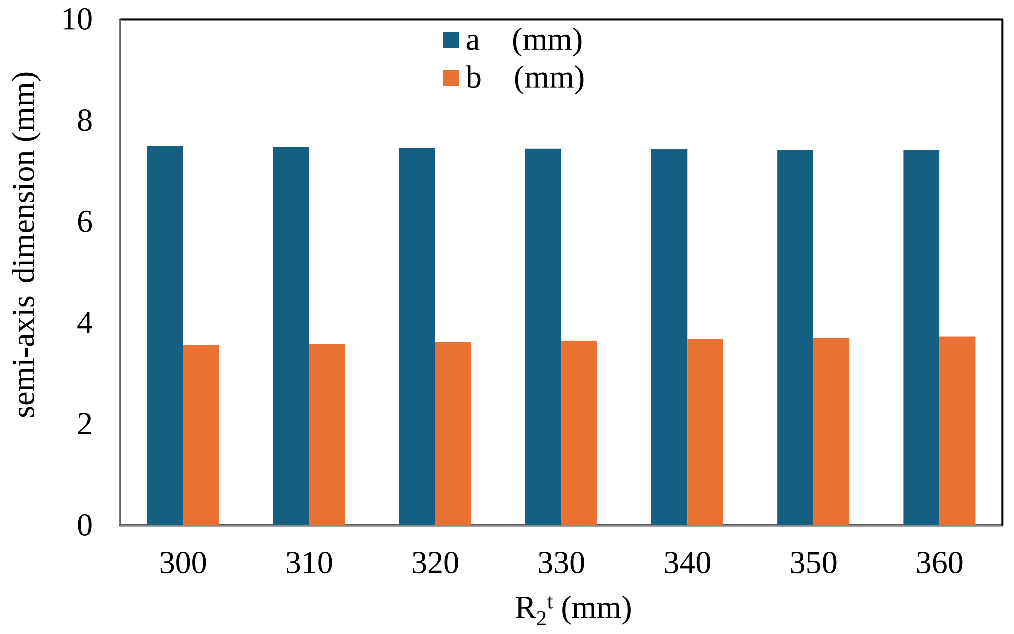


Figure 8. Effect of wheel profile radii (R_2^t) on the dimensions 'a' and 'b'

The relationship between the effect of geometrical parameters (WPT, wheel profile radii, and rail profile radii) and creep forces (longitudinal and lateral) are represented in figure 9, 10 and 11 respectively.

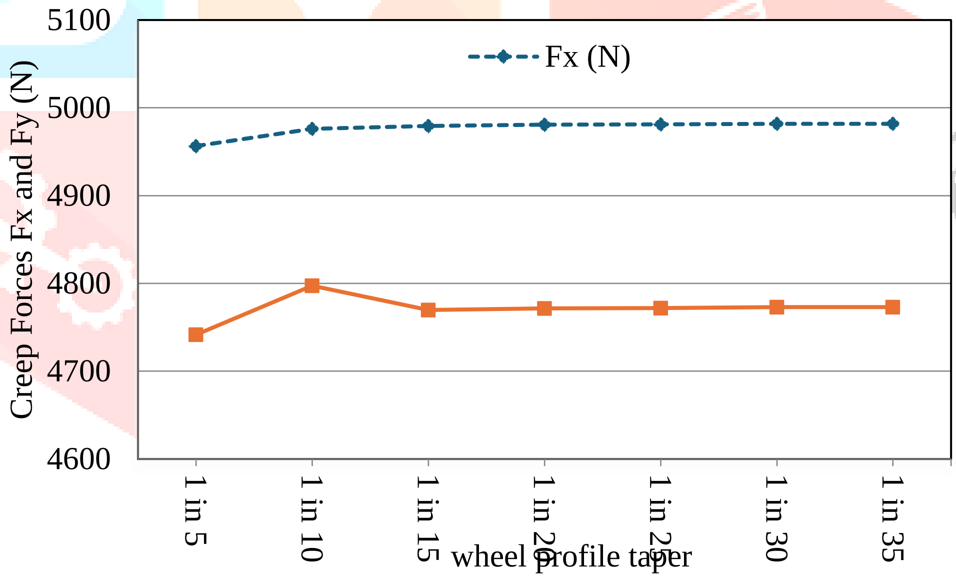


Figure 9. The relationship between longitudinal and lateral creep forces with wheel profile taper.

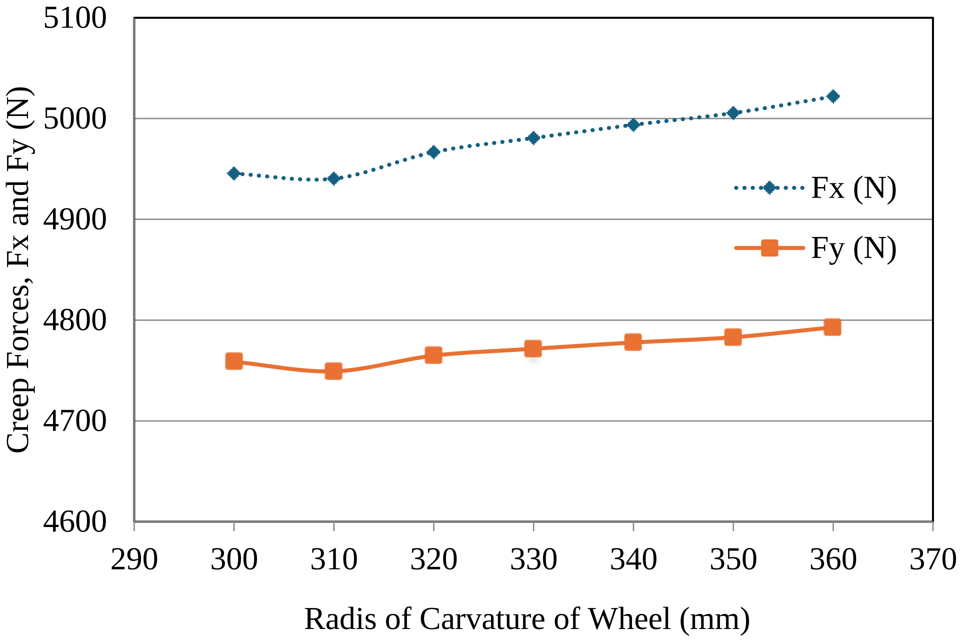


Figure 10. The relationship between longitudinal and lateral creep forces with radius of curvature of wheel

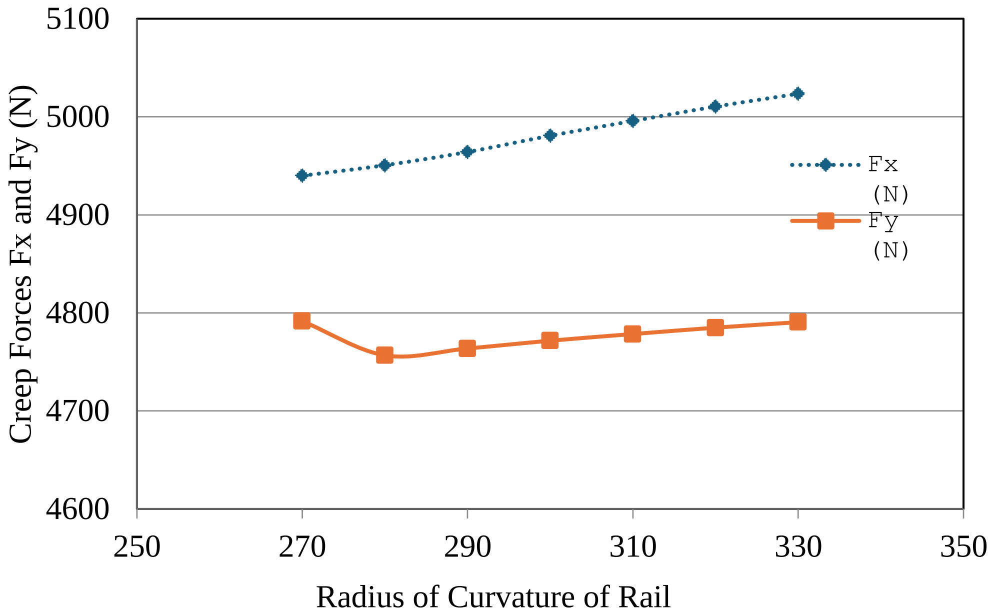


Figure 11. The relationship between longitudinal and lateral creep forces with radius of curvature of wheel.

As shown in figure 12 the number of cycles to failure N is almost similar for all-wheel profile taper.

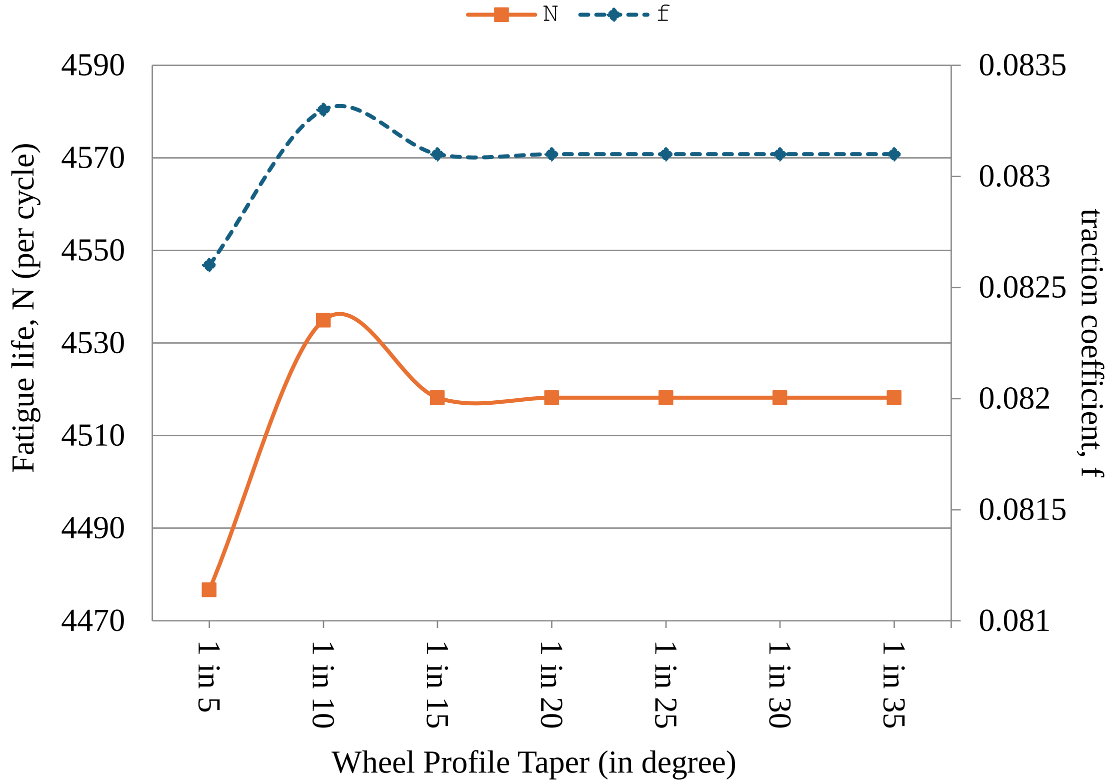
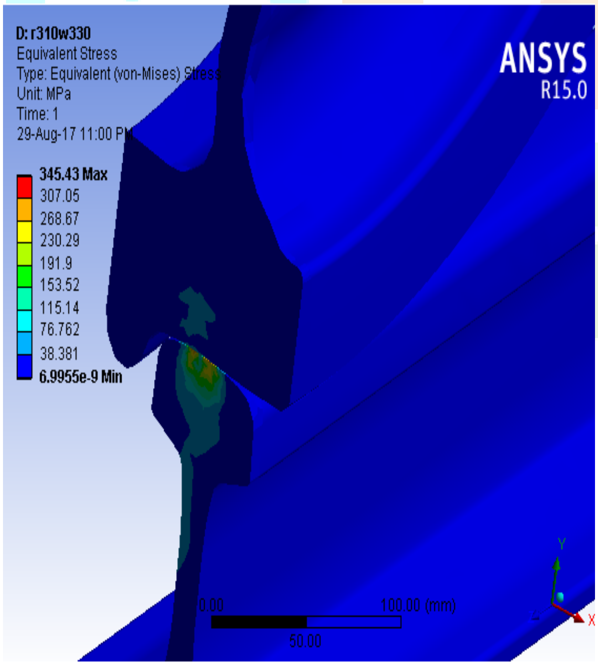
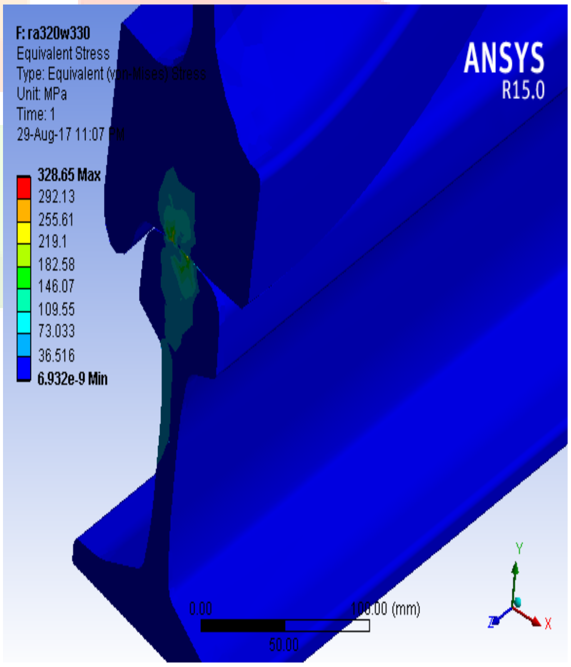
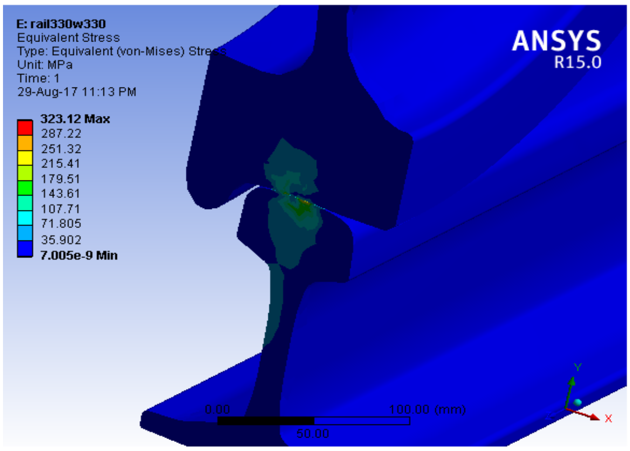
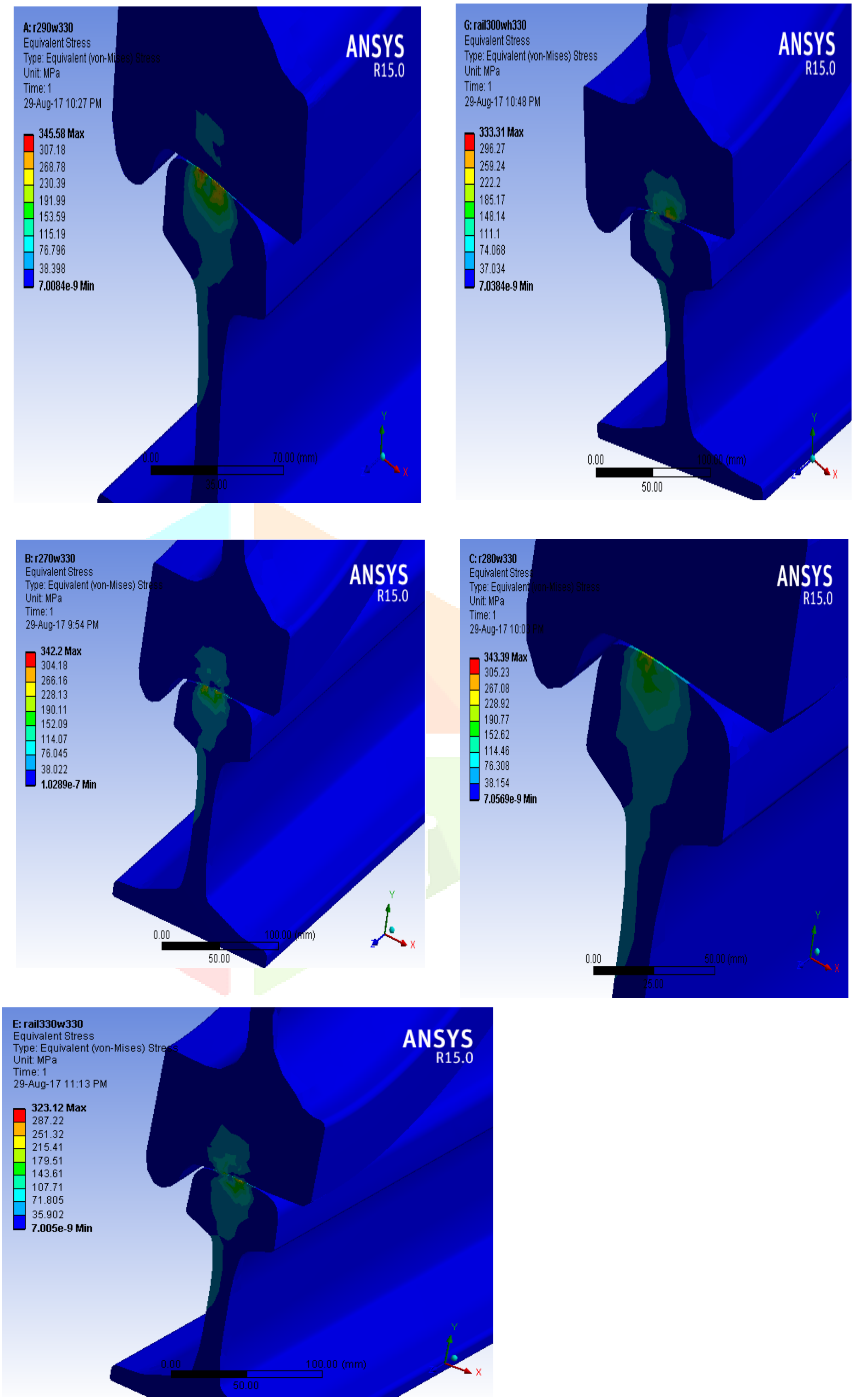


Figure 12 Fatigue life (N) and Traction coefficient (f) with wheel profile taper.









Figures 13: Equivalent (von Mises) stress for radius of curvatures of rail (a) 270, (b) 280, (c) 290, (d) 300, (e) 310, (f) 320, (g) 330 and radius of curvatures of wheel 330 (All dimensions are in mm)

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