An In-Depth Analysis Of Weibull Distribution's Application In Wind Energy: A Retrospective Study

[1]Flowery Francis

[1]Assistant Professor of Statistics,
[1]Department of Statistics,
[1]Sri. C. Achutha Menon Govt. College, Thrissur, Kerala, India.

Abstract: This Study synthesizes fifteen seminal works spanning from 1977 to 2015, each challenging conventional methodologies in wind energy analysis and unveiling nuanced insights crucial for sustainable energy development. From scrutinizing wind speed probability distributions in locations like La Ventosa, Mexico, to optimizing wind energy planning for offshore wind farm design, these studies offer invaluable insights into wind behavior complexities and their implications for renewable energy generation. By proposing novel frameworks and tailored methodologies, researchers emphasize precision and innovation in wind energy analysis, paving the way for more informed decision-making and sustainable energy solutions globally.

***Index Terms* -** Geographical contexts, Renewable energy, Statistical modeling, Sustainable energy solutions, Weibull distribution, Wind dynamics, Wind energy analysis.

I. INTRODUCTION

This compilation encapsulates a rich tapestry of fifteen seminal works spanning nearly four decades, from 1977 to 2015, that collectively constitute a profound exploration of wind energy analysis. As the global imperative for sustainable energy solutions intensifies, researchers have embarked on a journey to decipher the intricacies of wind dynamics, challenging entrenched methodologies and paradigms to unearth nuanced insights pivotal for the advancement of renewable energy. These studies, ranging from meticulous examinations of wind speed probability distributions in diverse geographical landscapes to the strategic optimization of wind energy planning for offshore ventures, embody the relentless pursuit of precision and innovation in unlocking the vast potential of wind power. Through rigorous statistical analyses, sophisticated modeling approaches, and insightful comparisons, researchers have not only deepened our comprehension of wind behavior but have also forged pathways for more enlightened decision-making in the realm of renewable energy on a global scale. This introduction sets the stage for an illuminating exploration of groundbreaking research that continues to shape the trajectory of wind energy analysis and propel the frontiers of sustainable energy solutions.

The article is organized as follows : Section 2 explores the Weibull Distribution, Section 3 explores the Density Function of Weibull Distribution, investigating its properties and applications. Following this, Section 4 offers a comprehensive review of significant research endeavours focused on exponential aspects of the distribution. This includes discussions on extensions, techniques for parameter estimation, and real-world applications. Finally, Section 5 provides a concluding summary of the findings presented in the article

II. UNVEILING THE SIGNIFICANCE AND DIVERSE APPLICATIONS OF THE WEIBULL DISTRIBUTION

The Weibull distribution stands as a pivotal mathematical construct with widespread applications across numerous disciplines, notably in reliability engineering, survival analysis, and beyond. Its significance lies in its ability to capture the variability in lifetimes of systems, products, or components, especially when failure rates exhibit non-constant patterns over time. By accurately modeling parameters such as mean time to failure (MTTF) and reliability functions, it serves as the backbone for reliability assessments, aiding in predictive analytics for failure probabilities and maintenance scheduling. Furthermore, its adaptability extends to survival analysis, where it provides crucial insights into survival probabilities and hazard rates, essential for understanding disease progression or product longevity. In the realm of quality control, the Weibull distribution offers invaluable tools for analyzing failure modes and setting quality standards. Its utility expands further into extreme value analysis, medical research, and financial modeling, contributing to risk assessment, asset lifetime estimation, and forecasting. In software engineering, it becomes instrumental in software reliability analysis, enhancing testing strategies and system robustness. In essence, the Weibull distribution emerges as an indispensable statistical tool, offering a versatile framework for comprehensively understanding and analyzing the temporal dynamics of diverse systems and phenomena across various domains.

III. Exploring the Density Function of Weibull Distribution

The probability density function (PDF) of the Weibull distribution is given by:

$$f(x;\lambda,k) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$$

Where:

- x is the variable of the distribution (usually representing time or size),
- $\lambda > 0$ is the scale parameter, and
- $k > 0$ is the shape parameter.

This distribution is often used to model reliability data, particularly in engineering, and is characterized by its flexibility in capturing different shapes of failure rates over time. The shape parameter \(k \) determines the shape of the distribution:

- When $k < 1$, the hazard function (failure rate) decreases over time, indicating "infant mortality" where early failures are more likely.
- When $k = 1$, the distribution reduces to an exponential distribution.
- When $k > 1$,the hazard function increases over time, indicating "wear-out" failures where older units are more likely to fail.

The scale parameter λ scales the distribution along the time axis.

IV. Previous Year Studies: The Role of Weibull Distribution in Wind Energy Analysis

- In their groundbreaking study, O.A. Jaramillo and M.A. Borja challenge established paradigms in wind speed analysis by meticulously examining data from La Ventosa, Oaxaca, Mexico. Their findings, published in "Wind speed analysis in La Ventosa, Mexico: a bimodal probability distribution case" (2004) [7], reveal that the traditional two-parameter Weibull function fails to accurately represent wind speed distributions in the region. Instead, they propose a bimodal probability distribution function (PDF) as a more fitting model, highlighting its significance for wind power generation. By dissecting data across various temporal and directional categories, they expose the limitations of prevailing methodologies and advocate for a context-specific approach. Their research not only sheds light on the nuances of wind behavior in La Ventosa but also carries profound implications for renewable energy projects worldwide. As the global pursuit of sustainability intensifies, Jaramillo and Borja's work serves as a rallying cry for precision and innovation in harnessing wind energy potential.

- In his illuminating exploration, Joseph P. Hennessey Jr. delves into the complexities of wind power statistics, shedding light on critical aspects often overlooked in conventional analyses. Published under the title "Some Aspects of Wind Power Statistics" (1977) [6], Hennessey addresses fundamental challenges in understanding wind energy potential, particularly in relation to wind speed distributions. Through meticulous examination, he unveils the intricate relationship between mean wind speed and the cube of wind speed, elucidating how the Weibull probability density function serves as a robust model for such distributions. By extending this model to encompass the distribution of the cube of wind speed, Hennessey introduces a powerful framework for computing essential metrics such as mean and standard deviation of total wind power density, usable wind power density, and operational wind power density. Applying this framework to data from Oregon wind power sites, situated in rugged terrain, he demonstrates the efficacy of the Weibull model in capturing the intricacies of wind power dynamics. Hennessey's insights compel a reevaluation of existing wind power studies, emphasizing the importance of incorporating comprehensive statistical analyses beyond mere mean wind power density. In essence, his work underscores the imperative of leveraging sophisticated statistical tools to unlock the full potential of wind energy resources, thus paving the way for more accurate and informed decision-making in renewable energy endeavors.

- In their comprehensive study, J.A. Carta, P. Ramírez, and S. Velázquez embark on a meticulous examination of wind speed probability density functions (PDFs) and their implications for wind energy analysis, with a focus on the unique context of the Canary Islands. Published under the title "A review of wind speed probability distributions used in wind energy analysis: Case studies in the Canary Islands" (2009) [1] the authors survey a plethora of PDFs proposed in scientific literature, assessing their applicability across diverse wind regimes, from high frequencies of null winds to bimodal distributions. Through an extensive analysis, they explore the flexibility and utility of these PDFs in capturing the intricacies of wind behavior, considering methods such as the method of moments (MM), maximum likelihood method (MLM), and least squares method (LSM) for parameter estimation. Furthermore, the authors scrutinize statistical tests employed to evaluate the goodness of fit between observed wind data and theoretical distributions, emphasizing the importance of the coefficient of determination R^2 as a metric for judging suitability. While acknowledging the widespread use of the two-parameter Weibull distribution (W.pdf) in wind energy literature, the authors caution against its generalized application, citing its limitations in representing certain wind regimes encountered in nature. Instead, they advocate for a nuanced approach, wherein the selection of PDFs is tailored to specific wind conditions to minimize estimation errors in wind energy conversion systems (WECS). This discerning analysis culminates in the compilation of a valuable catalogue of PDFs, offering practitioners a comprehensive toolkit for accurate wind energy estimation and planning in the Canary Islands and beyond. Through their meticulous scrutiny and insightful conclusions, Carta, Ramírez, and Velázquez propel the field of wind energy analysis forward, paving the way for more nuanced and informed decision-making in renewable energy endeavors.

- In their seminal work, L. Van Der Auwera, F. De Meyer, and L. M. Malet delve into the intricacies of wind power density estimation through the lens of the Weibull three-parameter model. Published under the title "The Use of the Weibull Three-Parameter Model for Estimating Mean Wind Power Densities" (1980) [14], the authors present a comprehensive analysis of this probability density function and its implications for wind energy assessment. Through meticulous examination of wind speed observations, they demonstrate the superior performance of the three-parameter Weibull model in fitting empirical wind speed frequency data compared to conventional density functions with fewer parameters. Moreover, the authors highlight the profound impact of the hypothesized probability density function on wind power density estimations, underscoring the critical role of model selection in accurate assessment. In a quest for deeper understanding, they investigate the variation of the model's parameters with height, revealing the absence of a simple height dependence. This nuanced exploration not only enhances our understanding of wind power dynamics but also underscores the importance of sophisticated modeling approaches in renewable energy research. Van Der Auwera, De Meyer, and Malet's pioneering work paves the way for more refined and reliable methods of wind power estimation, contributing to the advancement of sustainable energy solutions worldwide.

- In a significant contribution to wind energy research, Atsu S.S. Dorvlo investigates wind speed distribution modeling in Oman, employing the Weibull distribution across four distinct locations. Published under the title "Estimating Wind Speed Distribution" (2002) [4], Dorvlo explores the estimation of scale and shape parameters through three methods: the Chi-square method, method of moments, and regression method. Through meticulous analysis, Dorvlo discerns that the Chi-square method yields the most accurate fit to the distribution of wind data across the locations studied. Furthermore, the study unveils significant temporal variability, with both scale and shape parameters exhibiting wide-ranging fluctuations over the months. This revelation underscores the dynamic nature of wind patterns in Oman and emphasizes the importance of robust statistical methods in accurately characterizing wind speed distributions. Dorvlo's findings not only enhance our understanding of wind dynamics in Oman but also offer valuable insights for wind energy planning and resource assessment in the region and beyond.

- In a pioneering study, D. Weisser undertakes a comprehensive wind energy analysis of Grenada, West Indies, utilizing the Weibull density function to estimate wind energy potential. Published under the title "A Wind Energy Analysis of Grenada: An Estimation Using the 'Weibull' Density Function" (2003) [15], Weisser's research sheds light on the significance of incorporating diurnal variations in wind energy potential, particularly in the context of electricity load management. Drawing on historic recordings of mean hourly wind velocity, the analysis underscores the nuanced nature of wind speed probabilities, which can vary significantly throughout the day and night. Critically, Weisser highlights the limitations of wind energy assessments based solely on average daily or seasonal wind speeds, emphasizing the potential for significant underestimation or overestimation of wind power potential when diurnal wind patterns are neglected. This insight carries profound implications for renewable energy planning and underscores the necessity of accounting for temporal variability in wind energy assessments. Weisser's study not only enriches our understanding of wind dynamics in Grenada but also provides valuable guidance for optimizing wind energy utilization and integration into the electricity grid, offering a pathway towards more efficient and sustainable energy systems in the region.

- In a meticulous investigation, S.H. Pishgar-Komleh, A. Keyhani, and P. Sefeedpari delve into the intricate wind dynamics of Firouzkooh region in Iran, aiming to assess its wind power generation potential. Published under the title "Wind speed and power density analysis based on Weibull and Rayleigh distributions (a case study: Firouzkooh county of Iran)" (2015) [9], the study spans a decade from 2001 to 2010, analyzing wind speed data over three-hour intervals. Notably, the authors uncover consistent trends in wind speeds across different years, with May 2010 recording the highest mean wind speed and June 2002 the lowest. A detailed diurnal analysis reveals peak wind speeds occurring from 6 am to 3 pm, providing crucial insights into temporal wind patterns. Employing Weibull and Rayleigh distribution functions, the study identifies both as effective tools for fitting wind speed data, yielding comparable coefficient of determination values (R^2) of 0.97. Furthermore, wind power density estimations based on mean and root mean cube speed approaches indicate Firouzkooh's classification as a Class 4 region, conducive to wind turbine establishment, with average values of 203 and 248 W m^{-2} $year^{-1}$, respectively. The study culminates in a wind rose diagram, elucidating prevailing wind directions predominantly within the sector between 180° and 270° clockwise from North. This comprehensive analysis not only enriches our understanding of wind dynamics in Firouzkooh but also provides valuable insights for harnessing its wind energy potential, paving the way for sustainable energy development in the region.trends in wind speeds across different years, with May 2010 recording the highest mean wind speed and June 2002 the lowest. A detailed diurnal analysis reveals peak wind speeds occurring from 6 am to 3 pm, providing crucial insights into temporal wind patterns. Employing Weibull and Rayleigh distribution functions, the study identifies both as effective tools for fitting wind speed data, yielding comparable coefficient of determination values (R^2) of 0.97. Furthermore, wind power density estimations based on mean and root mean cube speed approaches indicate Firouzkooh's classification as a Class 4 region, conducive to wind turbine establishment, with average values of 203 and 248 W m^{-2} $year^{-1}$, respectively. The study culminates in a wind rose diagram, elucidating prevailing wind directions predominantly within the sector between 180° and 270° clockwise from North. This comprehensive analysis not only enriches our understanding of wind dynamics in Firouzkooh but also provides valuable insights for harnessing its wind energy potential, paving the way for sustainable energy development in the region.

- In a segment of a comprehensive wind energy examination, Károly Tar employs statistical methods to dissect the time series of monthly average wind speed measured between 1991 and 2000 at seven Hungarian meteorological stations. Published under the title "Some statistical characteristics of monthly average wind speed at various heights" (2008) [13], the study delves into the empirical distribution of measured monthly average wind speeds, aiming to identify universal patterns independent of orography. Through a meticulous analysis, Tar approximates the empirical distribution using theoretical distributions, notably highlighting the versatility of the Weibull distribution in modeling wind speed distributions. Leveraging the Weibull distribution, the study extends its analysis to generate the distribution of monthly average wind speeds across different levels from the anemometer altitude. Subsequently, Tar calculates averages for the entire period and fits a power function, thereby demonstrating a correlation between Hellmann's wind profile law and the Weibull distribution. This insightful exploration not only sheds light on the statistical characteristics of monthly average wind speed across various heights but also uncovers underlying correlations between wind profile laws and distribution functions, offering valuable insights for wind energy research and application.

- In a bid to address Turkey's burgeoning energy demands and bolster its renewable energy portfolio, Ali Naci Celik embarks on a comprehensive investigation into the wind energy potential of the southern region. Published under the title "A statistical analysis of wind power density based on the Weibull and Rayleigh models at the southern region of Turkey" (2004) [3], Celik underscores the imperative of harnessing renewable resources like wind to meet the nation's escalating energy needs while mitigating environmental impact. Focusing on Iskenderun, situated on Turkey's Mediterranean coast, Celik leverages 1-year measured hourly time-series wind speed data to conduct a rigorous statistical analysis of wind energy potential. Deriving probability density distributions from the time-series data, Celik identifies distributional parameters and fits two probability density functions—the Weibull and Rayleigh models—on a monthly basis to the measured probability distributions. Through this meticulous analysis, Celik unveils key insights into the wind energy potential of the region, shedding light on the efficacy of both models in characterizing wind power density. This study not only enriches our understanding of wind dynamics in southern Turkey but also provides valuable guidance for leveraging wind energy as a sustainable and reliable power source in the region's energy transition journey.

- In a crucial endeavor to optimize offshore wind farm planning, Eugene C. Morgan, Matthew Lackner, Richard M. Vogel, and Laurie G. Baise delve into the intricate domain of short-term wind speed modeling. Published under the title "Probability distributions for offshore wind speeds" (2011) [8], the study underscores the pivotal role of wind speed data in estimating critical engineering parameters essential for offshore wind farm design. Given the challenge of limited wind speed time series data, the study highlights the utility of probability distributions as substitutes for estimating design parameters. While the Weibull distribution is a widely-accepted model, the authors utilize 10-min wind speed time series data from 178 ocean buoy stations ranging from 1 month to 20 years to demonstrate its inadequacy compared to more complex models. Through meticulous comparison across three metrics—probability plot R^2, estimates of average turbine power output, and estimates of extreme wind speed—the study reveals that the bimodal Weibull, Kappa, and Wakeby models provide superior fits to the distribution of wind speeds. The Kappa and Wakeby distributions excel in estimating average turbine power output, crucially controlling high wind speeds, while the 2-parameter Lognormal distribution outperforms in estimating extreme wind speeds. This nuanced analysis underscores the importance of tailored model selection based on the specific engineering parameter of interest, driving forward the quest for enhanced offshore wind farm planning through rigorous empirical modeling and analysis.

- In a pioneering study, José Antonio Carta and Penélope Ramírez undertake a comprehensive analysis of hourly mean wind speed data recorded across the Canarian Archipelago. Published under the title "Use of finite mixture distribution models in the analysis of wind energy in the Canarian Archipelago" (2007) [2], the study challenges the conventional wisdom surrounding wind speed distribution modeling. Through meticulous analysis, Carta and Ramírez reveal that the typical two-parameter Weibull distribution fails to accurately capture all observed wind regimes in the region. Instead, they introduce the Singly Truncated from below Normal Weibull mixture distribution (TNW-pdf) and a two-component mixture Weibull distribution (WW-pdf), both of which offer superior fits for both unimodal and bimodal wind speed frequency distributions. Utilizing the least squares method and the Levenberg–Marquardt algorithm for parameter estimation, the study showcases the flexibility and accuracy of these mixture distributions in representing diverse wind regimes. While the TNW-pdf accounts for the frequency of null winds, the WW-pdf and W-pdf do not, making it particularly suitable for regions with high percentages of null wind speeds. However, the study notes that the calculation of the TNW-pdf is slower. Based on the results obtained, Carta and Ramírez conclude that these finite mixture distribution models offer highly flexible and adaptable frameworks for wind speed studies not only in the Canarian archipelago but also in regions with similar characteristics. This groundbreaking research not only enhances our understanding of wind dynamics in complex terrain but also paves the way for more accurate and reliable wind energy assessments worldwide.

- E. Simiu, N.A. Heckert, J.J. Filliben, and S.K. Johnson contribute to the ongoing discourse surrounding extreme wind load estimation methods by evaluating the suitability of the Gumbel distribution for modeling extreme dynamic pressures. Published under the title "Extreme wind load estimates based on the Gumbel distribution of dynamic pressures: an assessment" (2001) [11], the study addresses the debate regarding whether fitting a Gumbel distribution to the time series of extreme dynamic pressures or to the time series of extreme wind speeds themselves is more appropriate. Contrary to previous assertions, the authors argue that the use of time series of extreme dynamic pressures is not justified, particularly when wind speed data at small intervals are not approximately Rayleigh-distributed. Through meticulous analysis, including probability plot correlation coefficient (PPCC) analyses of multiple datasets, the study demonstrates that the fit of reverse Weibull distributions to largest yearly wind speeds significantly outperforms the fit of Gumbel distributions to corresponding largest yearly dynamic pressures. The authors interpret these findings as lacking convincing support for the hypothesis that the Gumbel distribution should serve as a model of extreme dynamic pressures. This critical reassessment sheds light on the limitations of current methodologies and underscores the need for further research to enhance the accuracy and reliability of extreme wind load estimation techniques.

- In their insightful study, A. Garcia, J.L. Torres, E. Prieto, and A. de Francisco delve into the fitting of wind speed distributions using data from 20 locations in Navarre. Published under the title "Fitting wind speed distributions: a case study" (1998) [5], the study focuses on hourly mean wind speed data and explores the potential models represented by frequency curves. Leveraging the Weibull and Lognormal models, the authors estimate annual parameters for both distributions and assess their suitability through statistical metrics. The study employs R^2 coefficients, utilizing linear regression for the Weibull distribution and nonlinear regression for the Lognormal distribution. While both approaches yield satisfactory fits, the Weibull distribution demonstrates better results overall. Furthermore, the study offers a comparison between the estimated wind speed distributions and the production for a wind farm, providing valuable insights into the practical implications of distribution selection. This comprehensive analysis not only enhances our understanding of wind dynamics in Navarre but also offers practical guidance for wind energy planning and optimization in the region and beyond.

- In their groundbreaking study, Bonfils Safari and Jimmy Gasore address the critical need for assessing wind power potential in Rwanda, with the aim of transforming rural economies and enhancing access to essential services like water and electricity. Published under the title "A statistical investigation of wind characteristics and wind energy potential based on the Weibull and Rayleigh models in Rwanda" (2010) [10], the study leverages a time series of hourly daily measured wind speed and wind direction data from five main Rwandan meteorological stations spanning from 1974 to 1993. Employing statistical methods utilizing the Weibull and Rayleigh distributions, the authors meticulously evaluate wind speed characteristics and wind power potential at a height of 10 meters above ground level, utilizing hourly monthly average data. Furthermore, the study extrapolates these characteristics for higher altitude levels, providing a comprehensive understanding of wind potential across different locations in Rwanda. The results offer invaluable insights into the distribution of wind potential, paving the way for informed decision-making regarding the installation of wind energy systems and related projects. This study not only contributes to the development of renewable energy infrastructure in Rwanda but also serves as a valuable resource for policymakers, researchers, and practitioners in the field of sustainable energy development.

- In their insightful paper, M.J.M. Stevens and P.T. Smulders tackle the crucial task of estimating the parameters of the Weibull wind speed distribution for wind energy utilization purposes. Published under the title "The Estimation of the Parameters of the Weibull Wind Speed Distribution for Wind Energy Utilization Purposes" (1979) [12], the study presents five methods for parameter estimation, out of which two are selected for wind energy evaluation studies: one utilizing Weibull probability paper and the other employing percentiles. Through meticulous comparison using data from six meteorological stations, the authors demonstrate the efficacy of the graphical method using Weibull probability paper over percentile estimators. This nuanced analysis not only enhances our understanding of parameter estimation techniques for the Weibull distribution but also provides valuable insights for optimizing wind energy evaluation studies. The findings of this study offer practical guidance for researchers and practitioners engaged in wind energy utilization projects, ultimately contributing to the advancement of renewable energy infrastructure and sustainable development initiatives.

V. Conclusion

The culmination of these twelve seminal works signifies not just a historical record of advancements in wind energy analysis but a testament to the ongoing evolution of renewable energy research. Through meticulous scrutiny and innovative methodologies, researchers have not only deepened our comprehension of wind dynamics but have also illuminated pathways towards more sustainable energy solutions. By challenging entrenched methodologies and proposing novel frameworks, these studies have demonstrated the transformative potential of renewable energy. As the world increasingly embraces sustainability, the insights garnered from these studies will play a crucial role in guiding informed decision-making and shaping the trajectory of renewable energy on a global scale. Thus, this compilation stands as a testament to the enduring commitment of researchers worldwide to harness the power of wind for a greener, more sustainable tomorrow, underscoring the significance of ongoing research in renewable energy.

References

1. Carta, J. A., Ramirez, P., & Velazquez, S. (2009). A review of wind speed probability distributions used in wind energy analysis: Case studies in the Canary Islands. Renewable and sustainable energy reviews, 13(5), 933-955. https://doi.org/10.1016/j.rser.2008.05.005

2. Carta, J. A., & Ramírez, P. (2007). Use of finite mixture distribution models in the analysis of wind energy in the Canarian Archipelago. Energy Conversion and Management, 48(1), 281-291. https://doi.org/10.1016/j.enconman.2006.04.004

3. Celik, A. N. (2004). A statistical analysis of wind power density based on the Weibull and Rayleigh models at the southern region of Turkey. Renewable energy, 29(4), 593-604. https://doi.org/10.1016/j.renene.2003.07.002

4. Dorvlo, A. S. (2002). Estimating wind speed distribution. Energy Conversion and Management, 43(17), 2311-2318. https://doi.org/10.1016/S0196-8904(01)00182-0

5. Garcia, A., Torres, J. L., Prieto, E., & De Francisco, A. (1998). Fitting wind speed distributions: a case study. Solar energy, 62(2), 139-144. https://doi.org/10.1016/S0038-092X(97)00116-3

6. Hennessey Jr, J. P. (1977). Some aspects of wind power statistics. Journal of Applied Meteorology and Climatology, 16(2), 119-128. https://doi.org/10.1175/1520-0450(1977)016<0119:SAOWPS>2.0.CO;2

7. Jaramillo, O. A., & Borja, M. A. (2004). Wind speed analysis in La Ventosa, Mexico: a bimodal probability distribution case. Renewable energy, 29(10), 1613-1630. https://doi.org/10.1016/j.renene.2004.02.001

8. Morgan, E. C., Lackner, M., Vogel, R. M., & Baise, L. G. (2011). Probability distributions for offshore wind speeds. Energy Conversion and Management, 52(1), 15-26. https://doi.org/10.1016/j.enconman.2010.06.015

9. Pishgar-Komleh, S. H., Keyhani, A., & Sefeedpari, P. (2015). Wind speed and power density analysis based on Weibull and Rayleigh distributions (a case study: Firouzkooh county of Iran). Renewable and sustainable energy reviews, 42, 313-322. https://doi.org/10.1016/j.rser.2014.10.028

10. Safari, B., & Gasore, J. (2010). A statistical investigation of wind characteristics and wind energy potential based on the Weibull and Rayleigh models in Rwanda. Renewable energy, 35(12), 2874-2880. https://doi.org/10.1016/j.renene.2010.04.032

11. Simiu, E., Heckert, N. A., Filliben, J. J., & Johnson, S. K. (2001). Extreme wind load estimates based on the Gumbel distribution of dynamic pressures: an assessment. Structural Safety, 23(3), 221-229. https://doi.org/10.1016/S0167-4730(01)00016-9

12. Stevens, M. J. M., & Smulders, P. T. (1979). The Estimation of the Parameters of the Weibull Wind Speed Distribution for Wind Energy Utilization Purposes. Wind Engineering, 3(2), 132–145. http://www.jstor.org/stable/43749134

13. Tar, K. (2008). Some statistical characteristics of monthly average wind speed at various heights. Renewable and Sustainable Energy Reviews, 12(6), 1712-1724. https://doi.org/10.1016/j.rser.2007.01.014

14. Van Der Auwera, L. , De Meyer, F. , & Malet, L. M. . (1980). The Use of the Weibull Three-Parameter Model for Estimating Mean Wind Power Densities. Journal of Applied Meteorology and Climatology, 19(7), 819-825. https://doi.org/10.1175/1520-0450(1980)019<0819:TUOTWT>2.0.CO;2

15. Weisser, D. (2003). A wind energy analysis of Grenada: an estimation using the ‘Weibull’density function. Renewable energy, 28(11), 1803-1812. https://doi.org/10.1016/S0960-1481(03)00016-8