

BUCKLING OF RE-INFORCED ANISOTROPIC CYLINDRICAL SHELLS WITH RINGS AND STRINGERS SUBJECT TO SHEAR LOAD IN THE CASE OF TORSION OF A LONG TUBE.

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(1) INTRODUCTION:

Anisotropic shell consists of composite materials such as Boron-Epoxy, Glass Epoxy, re-inforced plastics and whiskers. They are used in many advanced structural applications and quite often in cylindrical shells.

Buckling problem of isotropic material are known from literature on shells e.g. Flugge^[1](1973). The buckling problem of anisotropic cylindrical shells has occupied the interest of many researchers such as Tasi^[2], Cheng & Kuenzi^[3], Hess T.E^[4], Thielemann, Schnell and Fisher^[5], Tasi, Fellmann and Strang^[6], Cheng & Ho^[7,8], Lie and Cheng^[9], De,A^[10-14].

The re-inforced anisotropic shell is the most important one. The shell of uniform thickness re-inforced by closely spaced rings or stringers or both.(fig. 1)

Object of this paper is to investigate the solution of the differential equation of the buckling problem of re-in forced anisotropic shells with rings and stringers or both for shells with shear load in the case of torsion of a long tube. The condition of neutral equilibrium and the critical load and the critical value of the torque are found out in this case. The corresponding result for gridwork cylindrical shells are deduced as a special case which are found to be identical with the previous result.

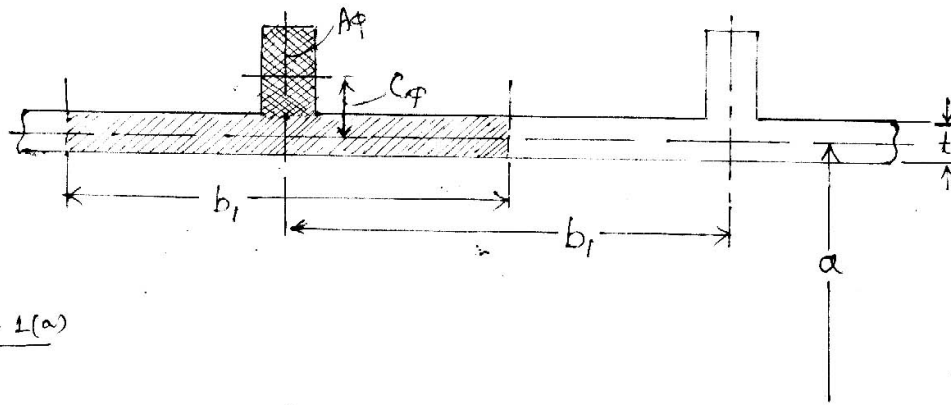


Fig. 1(a)

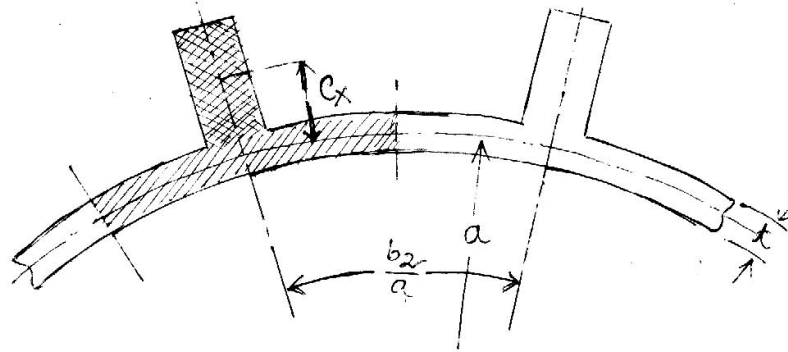


Fig. 1(b)

(2). BASIC EQUATION:

Fig.1(a) shows the section $\phi = \text{constant}$ through a re-inforced shell i.e shell with rings and stringers and fig.1(b) shows the sections $x = \text{constant}$ through the reinforced cylindrical shell

We choose the middle plane of the wall as the middle plane of the entire shell. The equations of equilibrium in the case of buckling of cylindrical shell vide Flugge^[1](1973)(p-448) are given by,

$$a N_x' + a N_{\phi x} \dot{} - pa(\ddot{u} - w') - Pu'' - 2Tu' \dot{} = 0 \quad \text{-----(1-a)}$$

$$a \dot{N}_{\phi} + a N_{x\phi}' - \dot{M}_{\phi} - M_{x\phi}' - pa(\ddot{v} + \dot{w}) - Pv'' - 2T(v' \dot{} + w') = 0 \quad \text{-----(1-b)}$$

$$\ddot{M}_{\phi} + \dot{M}_{x\phi}' + M_{\phi x}' \dot{} + M_x'' + a N_{\phi} + pa(u' - \dot{v} + \ddot{w}) + Pw'' - 2T(v' - w' \dot{}) = 0 \quad \text{-----(1-c)}$$

where ()' and ($\dot{}$) denotes $a \frac{\partial}{\partial x}(\dots)$ and $\frac{\partial}{\partial \phi}(\dots)$ respectively.

The shell being simultaneously subject to three simple loads -

- (i) a uniform normal pressure on its wall, $p_r = -p$ (Fig.2)
- (ii) an axial compression applied at the edges, the force per unit of circumference being P
- (iii) a shear load applied at the same edges so as to produce a torque in the cylinder. The shearing force (shear flow) is T.

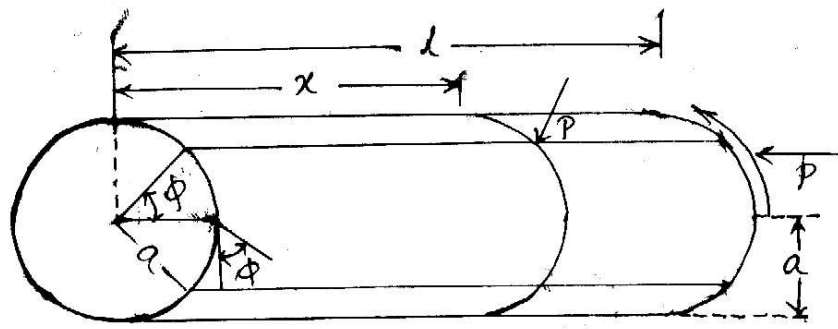


Fig. 2(a)

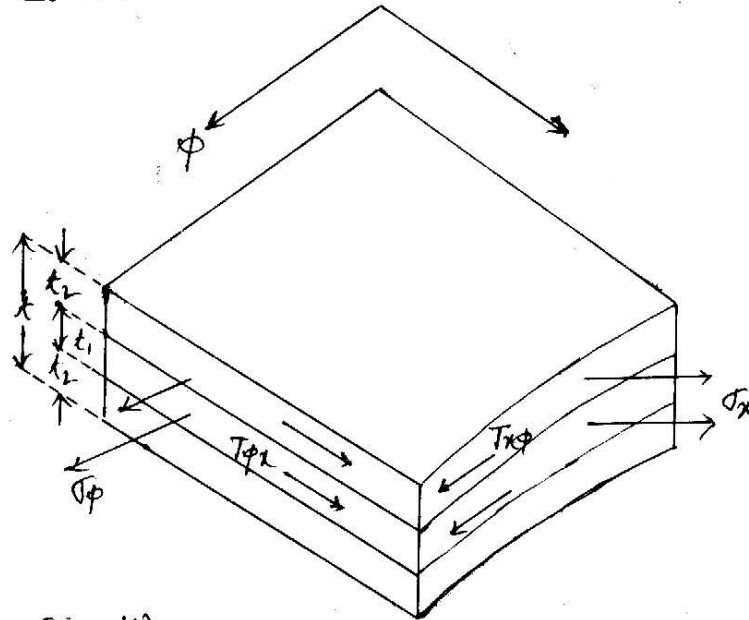


Fig. 2(b)

Substituting the values of elastic laws given by Flugge^[1](1973)(p-309) equation (5.116) in (1) we get after proper simplification,

$$u'' + A_1 \ddot{u} + A_2 v' + A_3 w' - k_1 (A_4 w''' - A_5 w'') - q_1 (\ddot{u} - w') - q_2 u'' - 2q_3 u' = 0 \quad \dots\dots(2-a)$$

$$A_6 u' + \ddot{v} + A_7 v''' + \dot{w} + k_1 [A_8 (\ddot{v} + \dot{w} - \ddot{w}) - A_9 w'' - A_{10} \ddot{w}] - A_{11} [q_1 (\ddot{v} + \dot{w}) + q_2 v'' + 2q_3 (v' + w')] = 0 \quad \dots\dots(2-b)$$

$$A_{12} u' + \dot{v} + w - k_1 [A_8 (\ddot{v} + 2\ddot{w}) + A_{13} u''' - A_{10} \ddot{w} - A_{14} w'' - A_{11} w'''] + A_{11} [q_1 (u' - \dot{v} + \ddot{w}) + q_2 w'' - q_3 (v' - w')] = 0 \quad \dots\dots(2-c)$$

$$\begin{aligned} \text{where, } A_1 &= \frac{D_{x\phi}}{D_x}, \quad A_2 = \frac{D_v + D_{x\phi}}{D_x}, \quad A_3 = \frac{D_v}{D_x}, \quad A_4 = \frac{aS_x}{K_x}, \\ A_5 &= \frac{K_{\phi,x}}{K_x}, \quad A_6 = \frac{D_v + D_{x\phi}}{D_\phi}, \quad A_7 = \frac{D_{x\phi}}{D_\phi}, \quad A_8 = \frac{aS_\phi D_x}{K_x D_\phi}, \quad \dots\dots(3) \\ A_9 &= \frac{D_x (K_v + K_{x\phi})}{K_x D_\phi}, \quad A_{10} = \frac{K_\phi D_x}{K_x D_\phi}, \quad A_{11} = \frac{D_x}{D_\phi}, \quad A_{12} = \frac{D_v}{D_\phi}, \\ A_{13} &= \frac{aS_x D_x}{K_x D_\phi}, \quad A_{14} = \frac{2K_v + K_{\phi x} + K_{x\phi}}{K_x D_\phi} \end{aligned}$$

$$\text{And } k_1 = \frac{K_x}{a^2 D_x}, \quad q_1 = \frac{pa}{D_x}, \quad q_2 = \frac{P}{D_x}, \quad q_3 = \frac{T}{D_x} \quad \dots\dots(4)$$

The equations (2) describes the buckling of equations for the buckling of re-inforced anisotropic shells with rings and stringers. In these case the parameters defined by (4) are small quantities.

For k_1 , it is obvious since we are interested in thin shells, where $t \ll a$. The three load parameters q_1 , q_2 , q_3 are approximately the elastic strains caused by the corresponding basic loads, and since all our theory is based on the assumption that such strains are small compared with unity, we shall neglect the parameters q_1 , q_2 , q_3 compared with 1 whenever the opportunity comes.

(3). SOLUTION FOR SHELLS WITH SHEAR LOAD

When $q_1 = q_2 = 0$, $q_3 \neq 0$, then there exists a solution, which is applicable in this case, of the form,

$$\begin{aligned} u &= A' \sin\left(\frac{\lambda x}{a} + m\varphi\right) \\ v &= B' \sin\left(\frac{\lambda x}{a} + m\varphi\right) \dots\dots\dots(5) \\ w &= C' \cos\left(\frac{\lambda x}{a} + m\varphi\right) \end{aligned}$$

$$\text{where } \lambda = \frac{n\pi a}{l} \text{ (n being an integer)}$$

It may be used for any combination of loads p, P and T. The terms of u, v, w and of their derivatives are found on lines $\frac{\lambda x}{a} + m\varphi = \text{const.}$, winding around the cylinder(fig.3). It is therefore not possible to satisfy reasonable boundary conditions on lines $x = \text{const.}$ and the solution (5) can not be used to deal with the cylinders length. We shall use it here to study the buckling of infinitely long cylinder subject to a torsion T only.

When we introduce the expression (5) in (2) we get a set of linear equations in A' , B' , C' :

$$A' [\lambda^2 + A_1 m^2 - 2q_3 \lambda m] + B' [A_2 \lambda m] + C' [A_3 \lambda + k_1 (A_4 \lambda^3 - A_5 \lambda m^2)] = 0 \dots\dots\dots(6-a)$$

$$A' [A_{12} \lambda + A_{13} \lambda^3 k_1] + B' [m + k_1 A_8 m^3 - 2A_{11} q_3 \lambda] + C' [1 + k_1 (2A_8 m^2 + A_{10} m^4 + A_{14} \lambda^2 m^2 + A_{11} \lambda^4) - A_{11} (2q_3 \lambda m)] = 0 \dots\dots\dots(6-b)$$

$$A' [A_{12} \lambda + A_{13} \lambda^3 k_1] + B' (m + k_1 A_8 m^3 - 2A_{11} q_3 \lambda) + C' \{1 + k_1 (2A_8 m^2 + A_{10} m^2 + A_{14} \lambda^2 m^2 + A_{14} \lambda^2 m^2 + A_{11} \lambda^4) - 2A_{11} q_3 \lambda m\} = 0 \dots\dots\dots(6-c)$$

The equations (6) are three linear equations with the buckling amplitudes A' , B' , C' as unknowns and with the brackets as coefficients.

Since these equations are homogeneous they admit, in general, only the solution $A' = B' = C' = 0$, indicating that the shell is not in neutral equilibrium.

A non-vanishing solutions A' , B' , C' are possible if and only if the determinant of the 9 coefficients of the equations (6) is zero.

Eliminating A' , B' , C' from the equations (6) we get,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0 \dots\dots\dots(7)$$

where a_{11} , a_{12} , a_{13} etc. are the coefficients of A' , B' , C' in the equations (6-a), (6-b), (6-c) respectively.

Whenever the condition (7) is satisfied i.e. the buckling condition is fulfilled, any two of the three equations (6) determine the ratios A'/C' and B'/C' and thus the buckling mode according to equation (5).

The coefficients of the equation (6) are linear function of k_1 and q_3 .

The determinant when expanded is therefore the polynomial of degree 3 in these parameters. Since they are very small quantities it is sufficient to keep the linear terms and to write the buckling condition as,

$$C_1 + C_2 k_1 = C_5 q_3 \quad \dots\dots\dots(8)$$

$$\text{Where,} \quad C_1 = A_7(1 - A_{12}) \lambda^4 \quad \dots\dots\dots(9-a)$$

$$C_2 = (\lambda^2 + A_1 m^2)(m^2 + A_7 \lambda^2)(A_{10} m^4 + A_{14} \lambda^2 m^2 + A_{11} \lambda^4) - [A_{15} \lambda^6 - A_{16} \lambda^4 m^2 - A_{17} \lambda^2 m^4 \\ - A_{18} m^6 + A_{19} \lambda^2 m^6 + A_{20} \lambda^4 m^4 + A_{21} \lambda^6 m^2 - A_{22} \lambda^2 m^2 - A_{23} \lambda^3 m^3 - A_{24} \lambda^5 m] \quad \dots\dots(9-b)$$

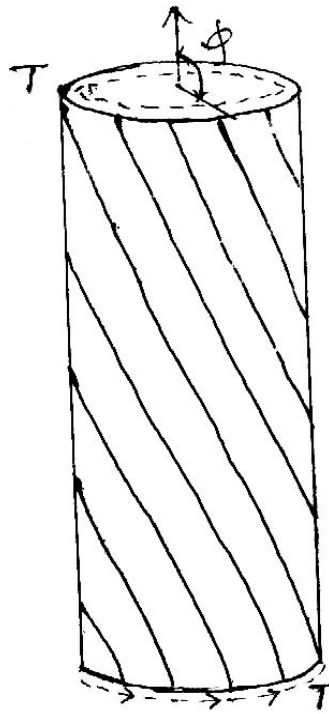
$$C_5 = 2 \lambda m A_{11} (\lambda^2 + A_1 m^2)(m^2 + A_7 \lambda^2 - 1) + \lambda m [A_{27} \lambda^2 - A_{21} \lambda^2 m^2] \quad \dots\dots\dots(9-c)$$

$$\begin{aligned} \text{where} \quad A_{15} &= A_7 (A_3 A_{13} + A_4 A_{12}) \\ A_{16} &= 2 A_7 A_8 + A_9 + A_2 A_{13} + A_2 A_9 A_{12} - A_3 A_{13} \\ &\quad + A_4 A_6 - A_4 A_{12} \\ A_{17} &= A_{10} + 2 A_1 A_7 A_8 + A_1 A_9 - 2 A_2 A_6 A_8 + \\ &\quad A_2 A_8 A_{12} + A_2 A_{10} A_{12} + A_3 A_6 A_8 \\ A_{18} &= A_1 A_{10} \\ A_{19} &= A_2 A_6 A_{10} \\ A_{20} &= A_2 A_6 A_{14} \\ A_{21} &= A_2 A_6 A_{11} \quad \dots\dots\dots(10) \\ A_{22} &= A_8 A_{12} (A_2 - A_3) \\ A_{23} &= A_5 (A_{12} - A_6) \\ A_{24} &= A_5 A_7 A_{12} \\ A_{25} &= A_{11} A_2 A_{12} \\ A_{26} &= A_7 - A_{11} - A_2 A_{11} + A_3 A_6 A_{11} + A_3 A_{11} \\ &\quad - A_3 A_{11} A_{12} - A_6 + A_{12} \\ A_{27} &= A_1 A_{11} \end{aligned}$$

We see at once that neither λ nor m can be zero because in both the cases $C_5 = 0$ and hence $q_3 = \infty$.

It is also without interest to consider negative values of λ or m . When both are negative nothing is changed in the equation (8) when either λ or m alone is negative the buckling mode (5) is altered in so far as the nodal lines (fig.3) becomes right handed screws.

One would expect that then the buckling load T must be applied in the opposite sense and this is exactly what happens. In the equation (8) the L.H.S remains the same while C_5 changes sign and hence q_3 .

**Fig - 3**

The discussion of the buckling formulae is now restricted to +ve values λ and to +ve integers m . One might think of solving for q_3 , differentiating the expression with respect to λ and m , and putting the first partial derivatives equal to zero. This would yield two algebraic equations for λ and m , and their solution (or one of them) would lead to the smallest possible q_3 . This procedure however, is rather tiresome and may be avoided. By some trial computations one may find out that any $m > 2$ yields a higher buckling load than does $m = 2$ and that λ must be chosen rather small, $\lambda \ll 1$, to obtain a low q_3 .

With this in mind we now investigate separately the two cases $m = 1$, $m = 2$.

For $m = 2$, the equation (8) gives,

$$q_3 = \left[\frac{C_1 + C_2 k_1}{C_5} \right]_{m=2}$$

When we neglect λ^2 , compared with unity we get,

$$q_3 = \frac{A_7(1 - A_{12})}{48A_1A_{11}} \lambda^3 + \frac{3A_{10}}{A_{11}\lambda} k_1$$

It is easy to find from,

$$\frac{\partial q_3}{\partial \lambda} = \frac{A_7(1 - A_{12})}{16A_1A_{11}} \lambda^2 - \frac{3A_{10}}{A_{11}\lambda^2} k_1 = 0$$

$$\text{that, } \lambda^4 = \frac{48A_1A_{10}}{A_7(1 - A_{12})} k_1 \dots\dots\dots(11)$$

This yields the lowest possible value of q_3 and is given by,

$$q_3 \big|_{\min} = 2 \cdot k_1^{3/4} \cdot \sqrt[4]{\frac{A_7(1 - A_{12})A_{10}^3}{3A_1A_{11}^4}} = 2 \cdot \sqrt[4]{\frac{A_7(1 - A_{12})A_{10}^3}{3A_1A_{11}^4}} \left(\frac{K_x}{D_x} \right)^{3/4} \cdot \frac{1}{a^{3/2}} \dots(12-a)$$

Using the last equation of (4) we may now return to the real shear load T and find its critical value given by,

$$T_{cr} = 2 \cdot \sqrt[4]{\frac{A_7(1-A_{12})A_{10}^3}{3A_1A_{11}^4}} \cdot \frac{(K_x)^{3/4} \cdot D_x^{1/4}}{a^{3/2}} \dots\dots\dots(12-b)$$

The total torque applied to the tube is given by,

$$M = T \cdot 2\pi a \cdot a$$

$\therefore$ The critical value of this torque is given by,

$$M_{cr} = 4\pi \cdot a^{1/2} \cdot D_x^{1/4} \cdot (K_x)^{3/4} \cdot \sqrt[4]{\frac{A_7(1-A_{12})A_{10}^3}{3A_1A_{11}^4}}$$

All this results have been derived for an infinitely long cylinder of re-inforced anisotropic shell with rings and stringers.

Since they do not contain any wave length we are tempted to apply them to cylinders of finite length.

However such a cylinder usually has some kind of stiffening at the end, say a bulk head requiring $w = 0$. Any such condition is in contradiction to the equation (5), and the additional constraint imposed by the bulkhead will increase the buckling load beyond the one given by the preceding formulae.

We may expect that the difference is not too big, if the cylinder is rather long.

When $m = 1$, the equation (8) gives,

$$q_3 = \left[\frac{C_1 + C_2 k_1}{C_5} \right]_{m=1}$$

$$\therefore q_3 = \frac{A_7(1-A_{12})\lambda^4 + k_1 P}{\lambda^3 [2A_{11}(\lambda^2 + A_1)A_7 + (A_{27} - A_{21})]}$$

where $P = [(\lambda^2 + A_1)(1 + A_7\lambda^2)(A_{10} + A_{14}\lambda^2 + A_{11}\lambda^4) - \{A_{15}\lambda^6 - \lambda^2(A_{16}\lambda^2 + A_{17} - A_{19} - A_{20}\lambda^2 - A_{21}\lambda^4 + A_{22} + A_{23}\lambda + A_{24}\lambda^3)\} - A_{18}]$

Now neglecting λ^2 as compared with unity we may drop k_1 terms entirely and we get,

$$q_3 = \frac{A_7(1-A_{12})}{2A_{11}A_1A_7 + A_{27} - A_{21}} \lambda \dots\dots\dots(13)$$

If we can choose λ arbitrarily, we may choose it as small as we like and thus make q_3 approach zero. This shows that there is no finite buckling load for the infinite shell unless we prevent the buckling mode with $m = 1$.

In this mode the axis of the tube is deformed to a steep helical curve, with the circular cross-sections remain circular and normal to the deformed axis. Since every such cross-section rotates about one of its diameters, this mode may be excluded by preventing such a rotation of the terminal cross-section of a long cylinder.

Particular Case:

Putting $D_v = K_v = 0$ in the equations (2), (6), (9) reduces to the corresponding equations for anisotropic gridwork cylindrical shells which are found identical with the previous result.

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